- Example. d_1-d_2 = 1.5m, f=100 MHz

Power vs. Amplitude

- Amplitude refers to the field strength of an E or M field
- Power is proportional to the square of the field strength
- So power decaying proportional to r^2 (path loss exponent) means field strength decays proportional to r
- Example. Received amplitude as given last class (Eq. 3) has power

(Eq. 1)

Plane Earth Path Loss

- As a practical example of two-ray multipath fading, consider a signal propagating over a flat plane

(Fig. 1)

- h_T is tx height, h_R is rx height, d is the distance from tx to rx, R is the reflection coefficient (i.e. coefficient multiplied by reflection)
- received signal is

(Eq. 2)

- If d is sufficiently large, R = -1 (from physics)
- Trig identity: sin a – sin b = 2 cos((a+b)/2) sin((a-b)/2)

(Eq. 3)

- $d_2 - d_1$ is … using the approximation $(1+u)^{1/2} \approx 1 + u/2$

(Fig. 2)

(Eq. 4)

- furthermore $\sin u \approx u$ for small u

(Eq. 5)

- so the path loss exponent for plane earth propagation is 4.
- Limitations: Earth needs to be very flat and a very good conductor in order for this to hold (e.g., calm ocean)
- On dry ground, path loss exponent is close to 2 as long as line-of-sight path exists from tx to rx

Rayleigh fading

- What about lots and lots of sinusoids?

- Remember that sin(a+b) = sin a cos b + cos a sin b, so

(Eq. 6)

- The A cos θ and A sin θ terms are random (because path length is random, and phase is random)
- Mean zero, variance small (but the same for both)
- So in the end we get

(Eq. 7)

- Thanks to the central limit theorem, sums of large numbers of random variables approach the Gaussian distribution, so U and V are Gaussian random variables
- Another trig identity: U cos a + V sin a = $(U^2+V^2)^{1/2}\sin(a+b)$, where

(Eq. 8)

- The signal strength is then $(U^2+V^2)^{1/2}$, which has the Rayleigh distribution, which has CDF

(Eq. 9)

- The extra phase doesn't matter – can be tracked
- r^2 is the average power of the sinusoid (given)
- tells us the probability that the signal strength will be less than a given value in an environment with lots of paths

- Example: Average power is 1 W. What is the probability that the signal strength falls below 1?

Flat vs. frequency-selective fading

- How different is fading from frequency to frequency?
- Recall Eq. 3

(Eq. 10)

- if d_1-d_2 is large, a small change in frequency will have a huge effect (frequency-selective fading)
- on the other hand if d_1-d_2 is small, a small change in frequency will have a negligible effect (flat fading)

Delay Spread and Coherence Bandwidth