

CSE 2021

Computer Organization

Hugh Chesser, CSEB
1012U



SPIM Simulator



1. SPIM is a software simulator for running MIPS program
2. SPIM's name is just MIPS spelled backwards
3. There are different versions for different machines:
 - Unix: spim, xspim
 - PC: PCspim (download instructions available on course homepage)
4. SPIM provides additional features not available in MIPS like system calls
 - Systems calls are operating-system like calls for inputting variables, displaying results, etc.
 - Format for system calls is:

```
place value of input argument in $a0
place value of system-call-code in $v0
syscall
```

System Calls



Example # 1: Print a string

```
.data
    str:    .asciiz "the answer is"
.text
    addi $v0,$zero,4
    la $a0,str    # pseudoinstruction
    syscall
```

Example # 2: Input an integer

```
addi $v0,$zero,5
syscall
```

Example # 3: Print an integer

```
addi $v0,$zero,1
addi $a0,$s0,$zero
syscall
```

Example # 4: Read String

```
addi $v0,$zero,8
la $a0, Buff    # $a0=address of Buff
addi $a1,$zero,60 # $a1=max. len.
syscall
```

Service	System Call Code (\$v0)	Arguments	Result
print_int	1	\$a0 = int	
print_float	2	\$f12 = float	
print_double	3	\$f12 = double	
print_string	4	\$a0 = string address	
read_int	5		int (in \$v0)
read_float	6		float (in \$f0)
read_double	7		double (in \$f0)
read_string	8	\$a0 = buffer \$a1 = length	
sbrk	9	\$a0 = amount	address (in \$v0)
exit	10		terminate prog

Putting it all together (2)



Activity: Write a MIPS program which does the following:

1. Accepts an integer N using the following prompt
Please input a value for $N =$
2. Computes the sum of integers from 1 to N , i.e., $(1 + 2 + \dots + N)$ if $N > 0$
3. Displays the result (X) as
The sum of the integers from 1 to N is X
4. Waits for the next number N .
5. If $N \leq 0$, the program exits with the following farewell
Chao - Have a good day

Run the program in the spim simulator to verify the results



Agenda for Today

1. Addition, Subtraction
2. Overflow
3. Multiplication
4. Division
5. Floating Point: IEEE 754 single and double precision formats

Patterson: Sections 3.1 – 3.5

Addition and Subtraction



In MIPS, addition and subtraction for signed numbers use 2's complement arithmetic

Example 1: Add 10_{ten} and 15_{ten}

Example 2: Subtract 15_{ten} from 10_{ten}

bit 1	bit 2	Prev. Carry	Sum	Next Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Table 1: Truth Table for addition

Addition and Subtraction



In MIPS, addition and subtraction for signed numbers use 2's complement arithmetic

Example 1: Add 10_{ten} and 15_{ten}

Step 1: Represent the operands in 2's complement

$$10_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1010_{\text{two}}$$

$$15_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1111_{\text{two}}$$

Step 2: Perform bit by bit addition using table 1.

$$\begin{array}{r} 10_{\text{ten}} + 15_{\text{ten}} \\ \hline = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 1001_{\text{two}} \\ = 25_{\text{ten}} \end{array}$$

Example 2: Subtract 15_{ten} from 10_{ten}

The problem is reduced to $(10_{\text{ten}} + (-15_{\text{ten}}))$

$$10_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1010_{\text{two}}$$

$$-15_{\text{ten}} = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 0001_{\text{two}}$$

$$\begin{array}{r} 10_{\text{ten}} - 15_{\text{ten}} \\ = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1011_{\text{two}} \\ = -5_{\text{ten}} \end{array}$$

bit 1	bit 2	Prev. Carry	Sum	Next Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Table 1: Truth Table for addition



Overflow (1)

Recall that:

$$\begin{aligned} \text{Smallest signed integer:} & \quad 1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000_{\text{two}} \\ & \quad = -(2^{31})_{\text{ten}} = -2,147,483,648_{\text{ten}} \end{aligned}$$

$$\begin{aligned} \text{Largest signed integer:} & \quad 0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{\text{two}} \\ & \quad = (2^{31} - 1)_{\text{ten}} = 2,147,483,647_{\text{ten}} \end{aligned}$$

What happens if the result of an operation is more than the largest signed integer or less than the smallest signed integer?

Example: Add $2,147,483,640_{\text{ten}}$ and 28_{ten}

$$\begin{array}{r} 28_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 1100_{\text{two}} \\ 2,147,483,640_{\text{ten}} = 0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1000_{\text{two}} \\ \hline 28_{\text{ten}} + 2,147,483,640_{\text{ten}} = 1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 0100_{\text{two}} \\ = -2,147,483,628_{\text{ten}} \end{array}$$

Overflow caused the value to be perceived as a negative integer

Overflow (2)



When can overflow occur?

Operation	Operand A	Operand B	Result indicating overflow
$A + B$	$A \geq 0$	$B \geq 0$	< 0
$A + B$	$A < 0$	$B < 0$	≥ 0
$A - B$	$A \geq 0$	$B < 0$	< 0
$A - B$	$A < 0$	$B \geq 0$	≥ 0



Integer Multiplication, Division

Both operations really imply a series of additions and subtractions

Example: Multiply 10_{ten} and 3_{ten} :

$$10_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1010_{\text{two}}$$

$$3_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0011_{\text{two}}$$

$$10_{\text{ten}} * 1_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1010_{\text{two}}$$

$$10_{\text{ten}} * 2_{\text{ten}} = \underline{0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 0100}_{\text{two}}$$

$$= 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 1110_{\text{two}} = 0x1e = 30_{\text{ten}}$$

Example: Divide 28_{ten} by 9_{ten} :

$$28_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001\ 1100_{\text{two}}$$

$$9_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1001_{\text{two}}$$

$$\begin{array}{r}
 11 \\
 1001 \overline{) 11100} \\
 \underline{1001} \\
 1010 \\
 \underline{1001} \\
 1
 \end{array}$$



Floating Point: Single Precision

1. In MIPS, decimal numbers are represented with the [IEEE 754 binary representation](#) that uses the **normalized** standard scientific binary notation defined as

$$(-1)^S \times (1 + \text{fraction})_{\text{two}} \times 2^{\text{exponent} - \text{bias}}$$

2. A number in [normalized scientific notation](#) has a mantissa that has no leading 0's and must be of the form $(1 + \text{fraction})$. For example, the binary representations 2.0×2^{-5} , 0.5×2^{-3} , 4.0×2^{-6} , and 1.0×2^{-4} are all equivalent but only 1.0×2^{-4} is the normalized scientific binary notation.
3. MIPS allows for two floating point representations: Single precision and double precision.
4. [Single precision](#) has a bias of 127 while double precision has a bias of 1023.
5. In single precision, the floating point representation is 32 bit long and has the following form

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
S	exponent								fraction																						
	(8 bits)								(23 bits)																						

where S represents the sign bit, which is 1 for negative numbers and 0 for positive numbers.

Activity 2:

Represent -0.75_{ten} , 1.4_{ten} in single precision of IEEE 754 binary representation.



Floating Point: Double Precision

1. In **double precision**, the value of bias in

$$(-1)^S \times (1 + \text{fraction})_{\text{two}} \times 2^{\text{exponent} - \text{bias}}$$

is 1023.

2. In single precision, the floating point representation is 64 bit long and has the following form

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
S		exponent											fraction																		
		(11 bits)											(Total of 52 bits)																		

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
fraction (continued)																															

Activity 3:

Represent -0.75_{ten} in double precision of IEEE 754 binary representation.

Activity 4:

Show that the largest magnitude that can be represented using single precision is $\pm 6.8_{\text{ten}} \times 10^{38}$, while the smallest fraction that can be represented is $\pm 5.9_{\text{ten}} \times 10^{-39}$.

Floating Point Registers



Name	Example	Comments
32 floating point registers each is 32 bits long	<code>\$f0, \$f1, \$f2, \$f3, \$f4, ..., \$f31</code>	MIPS floating point registers are used in pairs for double precision numbers
Memory w/ 2^{30} words	Memory[0], Memory[4], ... Memory[4294967292]	Memory is accessed one floating point (single or double precision) at a time

The following is the established register usage convention for the floating point registers:

- `$f0, $f1, $f2, $f3:` Function-returned values
- `$f4, $f5, ..., $f11:` Temporary values
- `$f12, $f13, $f14, $f15:` Arguments passed into a function
- `$f16, $f17, $f18, $f19:` More Temporary values
- `$f20, $f21, ..., $f31:` Saved values

Floating Point Instructions



Category	Instruction	Example	Meaning	Comments
Arithmetic	FP add single	<code>add.s \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 + \$f6$	Single Prec.
	FP subtract single	<code>sub.s \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 - \$f6$	Single Prec.
	FP multiply single	<code>mul.s \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 \times \$f6$	Single Prec.
	FP divide single	<code>div.s \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 / \$f6$	Single Prec.
	FP add double	<code>add.d \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 + \$f6$	Double Prec.
	FP subtract double	<code>sub.d \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 - \$f6$	Double Prec.
	FP multiply double	<code>mul.d \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 \times \$f6$	Double Prec.
	FP divide double	<code>div.d \$f2,\$f4,\$f6</code>	$\$f2 \leftarrow \$f4 / \$f6$	Double Prec.
Data Transfer	load word copr.1	<code>lwcl \$f2,100(\$s2)</code>	$\$f2 \leftarrow \text{Mem}[\$s2+100]$	Single Prec.
	store word copr.1	<code>swcl \$f2,100(\$s2)</code>	$\text{Mem}[\$s2+100] \leftarrow \$f2$	Single Prec.
Conditional branch	FP compare single (eq, ne, lt, le, gt, ge)	<code>c.lt.s \$f2,\$f4</code>	if($\$f2 < \$f4$) cond = 1, else cond = 0	Single Prec.
	FP compare double (eq, ne, lt, le, gt, ge)	<code>c.lt.d \$f2,\$f4</code>	if($\$f2 < \$f4$) cond = 1, else cond = 0	Double Prec.
	Branch on FP true	<code>bc1t 25</code>	if cond==1 go to PC+100+4	Single/ Double Prec.
	Branch on FP false	<code>bc1f 25</code>	if cond==0 go to PC+100+4	Single/ Double Prec.

Example



```
# calculate area of a circle
```

```
    .data
Ans:    .asciiz    "The area of the circle is: "
Ans_add:.word     Ans                # Pointer to String (Ans)
Pi:     .double   3.1415926535897924
Rad:    .double   12.345678901234567
Rad_add:.word     Rad                # Pointer to float (Rad)
    .text
main:  lw $a0, Ans_add($0)           # load address of Ans into $a0
      addi $v0, $0, 4                # Sys Call 4 (Print String)
      syscall

#-----
      la $s0, Pi                     # load float (Assembler Instruction)
      ldc1 $f2, 0($s0)              # load address of Pi into $s0
      # $f2 = Pi

#-----
      lw $s0, Rad_add($0)           # load float (MIPS Instruction)
      ldc1 $f4, 0($s0)              # load address of Rad into $s0
      # $f4 = Rad
      mul.d $f12, $f4, $f4
      mul.d $f12, $f12, $f2
      addi $v0, $0, 3                # Sys Call 3 (Print Double)
      syscall
exit:  jr $ra
```