CSE 4214 :: Problem Set 2

1. Let $h[k] = [1, 2, 3, 2, 1]$ for $k = [1, 2, 3, 4, 5]$, and $h[k] = 0$ elsewhere; also let $g[k] = [1, 1, 1]$ for $k = [1, 2, 3]$ and $g[k] = 0$ elsewhere. Find and sketch the discrete-time convolution $h[k] \star g[k]$.

2. Let x be a Gaussian random variable with mean μ and variance σ^2. For some constant z, express the probability that x is greater than z in terms of the complementary error function (erfc).

3. Let $s[k]$ represent a signal, zero everywhere except from $k = 1$ to n_b inclusive, and let $h[k]$ represent the impulse response of the detection filter. If $h[k] = s[n_b - k]$, show that

$$[s[k] \star h[k]]_{n_b} = \sum_{i=1}^{n_b} s[i]^2.$$

4. Let $x(t)$ be a zero-mean random process with power spectral density

$$S_x(j\omega) = \begin{cases} 1+\omega, & -1 \le \omega < 0, \\ 1-\omega, & 0 \le \omega \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find the variance of the process $x(t)$.

5. Let

$$s_0[k] = \begin{cases} 1, & 1 \le k \le 4, \\ -1, & 5 \le k \le 8, \\ 0 & \text{elsewhere,} \end{cases}$$

   and let $s_1[k] = -s_0[k]$. Also let the decision threshold $z = 0$. Note that $n_b = 8$. If $h[k] = s_0[n_b - k]$, find an expression for the probability of error in terms of erfc.