CSE 4214 :: Problem Set 5

1. Let
$$\mathbf{s} = \frac{1}{\sqrt{4}}[1, 1, 1, 1]. \tag{1}$$
Consider a 4-ary system using $\mathbf{s}$ as a basis vector, where $\mathbf{s}_i = \alpha_i \mathbf{s}$, and $\alpha_i = +3, +1, -1, -3$ for $i = 0, 1, 2, 3$, respectively. Express the probability of symbol error in terms of erfc.

2. Let
$$\mathbf{s}_x = \frac{1}{\sqrt{4}}[1, 1, 1, 1], \text{and} \tag{2}$$
$$\mathbf{s}_y = \frac{1}{\sqrt{4}}[1, 1, -1, -1]. \tag{3}$$
Show that $\mathbf{s}_x$ and $\mathbf{s}_y$ form an orthonormal basis, and find the energy per bit of a 4-ary system where each coordinate can take values of either $+2$ or -2.

3. Show that
$$\sin(2\pi f_c t) + \cos(2\pi f_c t) = \sqrt{2}\sin(2\pi f_c t + \pi/4), \tag{4}$$
$$\sin(2\pi f_c t) - \cos(2\pi f_c t) = \sqrt{2}\sin(2\pi f_c t + 3\pi/4), \tag{5}$$
$$-\sin(2\pi f_c t) - \cos(2\pi f_c t) = \sqrt{2}\sin(2\pi f_c t + 5\pi/4), \text{ and} \tag{6}$$
$$-\sin(2\pi f_c t) + \cos(2\pi f_c t) = \sqrt{2}\sin(2\pi f_c t + 7\pi/4). \tag{7}$$
Using these results, explain why transmitting a bit simultaneously with sin and cos is called *quadrature phase shift keying*.

4. Consider the feedback shift register in the diagram below. Determine whether it produces an M-sequence. If so, give the M-sequence; if not, explain why not.