Winter 2010 CSE3213 Communication Networks
Assignment # 2
Instructor: Foroohar Foroozan

Review chapter 3 (Sections 3.1- 3.5) Garcia before attempting the assignment.

1. Consider an analog repeater system in which the signal has power σ_x^2 and each stage adds noise with power σ_n^2. For simplicity assume that each repeater recovers the original signal without distortion but that the noise accumulates. Find the SNR after n repeater links. Write the expression in decibels: SNR dB = $10 \log_{10}$SNR.

Solution:

After n stages, the signal power is σ_x^2 and the noise power is $n\sigma_n^2$, so the SNR is:

SNR dB = $10 \log_{10}\sigma_x^2/n\sigma_n^2 = 10 \log_{10}\sigma_x^2/\sigma_n^2 + 10 \log_{10}1/n = 10 \log_{10}\sigma_x^2/\sigma_n^2 - 10 \log_{10}n$

2. Suppose a baseband transmission system is constrained to a maximum signal level of ± 1 volt and that the additive noise that appears in the receiver is uniformly distributed between [–1/16, 1/16]. How many levels of pulses can this transmission system use before the noise starts introducing errors?

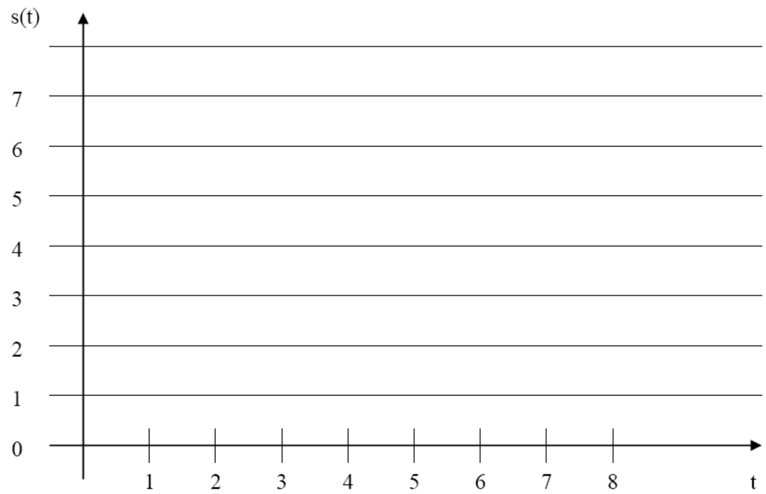
Solution:

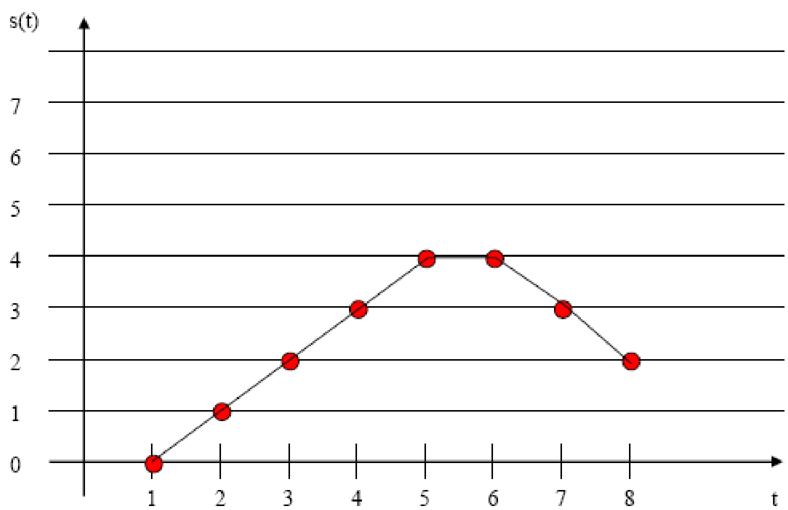
If two adjacent signal levels are separated by more than 2/16 then the noise cannot translate one adjacent signal into the next. The maximum range that the signal can span is +1 – (–1) = 2, so the maximum number of levels is 2/(1/8) = 16.

3. Assume a pulse code modulation (PCM) scheme that uses 3 bits to differentiate between 8 different levels of a PAM (i.e. analog) signal.
The following bit string, generated with the given PCM, has been received at time t=1:

000001010011100100011010

Sketch the analog signal that is represented by the string.





4. A link is to be operated at a bandwidth efficiency of B=9, i.e. at a rate of 9 bps for each Hz of bandwidth. Obtain the minimum SNR required at the receiver to allow, in theory, error-free transmission with this bandwidth efficiency. Express your answer in dB's.

Shannon formula for the ultimate capacity C of a communication link with W Hz bandwidth is $C=W\log_2(1+S/N)$ bps. Accordingly,

$B = C/W = \log_2(1+S/N) = 9 \Rightarrow$

$2^9 = 1 + S/N \Rightarrow$

$S/N \approx 511 \Rightarrow$

$S/N = 10 \log 511$ [dB] ≈ 27 [dB]

5. What is the maximum reliable bit rate possible over a telephone channel with the following parameters?

a. W = 2.4 kHz SNR = 40 dB

$C = 2400 \log_2 (1 + 10000) = 31890$ bps.

b. W = 3.0 kHz SNR = 20 dB

$C = 3000 \log_2 (1 + 100) = 19974$ bps.

c. W = 3.0 kHz SNR = 40 dB

$C = 3000 \log_2 (1 + 10000) = 39863$ bps.

6. Suppose we wish to transmit at a rate of 64 kbps over a 3 kHz telephone channel. What is the minimum SNR required to accomplish this?

Solution:

We know that R = 64 kbps and W = 3 kHz. What we need to find is SNR_{min}. The channel capacity is:

$C = W \log_2 (1 + SNR)$, $C \geq C_{min} = 64$ kbps

$C_{min} = W \log_2 (1 + SNR_{min}) \Rightarrow \log_2 (1 + SNR_{min}) = 64/3 \Rightarrow 1 + SNR_{min} = 2^{64/3}$

$\Rightarrow SNR_{min} = 2.64 \times 10^6$

in dB: $SNR_{min} = 10 \log_{10} (2.64 \times 10^6) = 64.2$ dB ∴ a very clean channel

7. Suppose that a low-pass communications system has a 1 MHz bandwidth. What bit rate is attainable using 8-level pulses? What is the Shannon capacity of this channel if the SNR is 20 dB? 40 dB?

Solution:

Nyquist pulses can be sent over this system at a rate of 2 million pulses per second. Eight-level signaling carries 3 bits per pulse, so the bit rate is 6 Mbps.

The Shannon capacities are:

$C = 1000000 \log_2 (1 + 100) = 6.6$ Mbps.

$C = 1000000 \log_2 (1 + 10000) = 13.3$ Mbps.