1. Consider an analog repeater system in which the signal has power $\sigma_x^2$ and each stage adds noise with power $\sigma_n^2$. For simplicity assume that each repeater recovers the original signal without distortion but that the noise accumulates. Find the SNR after n repeater links. Write the expression in decibels: \[ \text{SNR dB} = 10 \log_{10} \text{SNR} \]

\[ \text{SNR dB} = 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} \]

\[ = 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} - 10 \log_{10} n \]

2. Suppose a baseband transmission system is constrained to a maximum signal level of $\pm 1$ volt and that the additive noise that appears in the receiver is uniformly distributed between $[-1/16, 1/16]$. How many levels of pulses can this transmission system use before the noise starts introducing errors?

\[ \text{Solution:} \]
If two adjacent signal levels are separated by more than $2/16$ then the noise cannot translate one adjacent signal into the next. The maximum range that the signal can span is $+1 - (-1) = 2$, so the maximum number of levels is $2/(1/8) = 16$.

3. Assume a pulse code modulation (PCM) scheme that uses 3 bits to differentiate between 8 different levels of a PAM (i.e. analog) signal.

The following bit string, generated with the given PCM, has been received at time $t=1$:

000001010011100100011010

\[ \text{Sketch the analog signal that is represented by the string.} \]
4. A link is to be operated at a bandwidth efficiency of $B=9$, i.e. at a rate of 9 bps for each Hz of bandwidth. Obtain the minimum SNR required at the receiver to allow, in theory, error-free transmission with this bandwidth efficiency. Express your answer in dB’s.

Shannon formula for the ultimate capacity $C$ of a communication link with $W$ Hz bandwidth is $C = W \log_2(1+S/N)$ bps. Accordingly,

$$B = \frac{C}{W} = \log_2(1+S/N) = 9$$

$$2^9 = 1 + \frac{S}{N} \Rightarrow$$

$$S/N \approx 511 \Rightarrow$$

$$S/N = 10 \log 511 \ [\text{dB}] \approx 27 \ [\text{dB}]$$
5. What is the maximum reliable bit rate possible over a telephone channel with the following parameters?

a. $W = 2.4$ kHz $\text{SNR} = 40$ dB

\[ C = 2400 \log_2 (1 + 10000) = 31890 \text{ bps}. \]

b. $W = 3.0$ kHz $\text{SNR} = 20$ dB

\[ C = 3000 \log_2 (1 + 100) = 19974 \text{ bps}. \]

c. $W = 3.0$ kHz $\text{SNR} = 40$ dB

\[ C = 3000 \log_2 (1 + 10000) = 39863 \text{ bps}. \]

6. Suppose we wish to transmit at a rate of 64 kbps over a 3 kHz telephone channel. What is the minimum SNR required to accomplish this?

**Solution:**
We know that $R = 64$ kbps and $W = 3$ kHz. What we need to find is $\text{SNR}_{\text{min}}$. The channel capacity is:
\[ C = W \log_2 (1 + \text{SNR}), \quad C \geq C_{\text{min}} = 64 \text{ kbps} \]
\[ C_{\text{min}} = W \log_2 (1 + \text{SNR}_{\text{min}}) \Rightarrow \log_2 (1 + \text{SNR}_{\text{min}}) = \frac{64}{3} \Rightarrow 1 + \text{SNR}_{\text{min}} = 2^{64/3} \]
\[ \Rightarrow \text{SNR}_{\text{min}} = 2.64 \times 10^6 \]

in dB: $\text{SNR}_{\text{min}} = 10 \log_{10} (2.64 \times 10^6) = 64.2$ dB :: a very clean channel

7. Suppose that a low-pass communications system has a 1 MHz bandwidth. What bit rate is attainable using 8-level pulses? What is the Shannon capacity of this channel if the SNR is 20 dB? 40 dB?

**Solution:**
Nyquist pulses can be sent over this system at a rate of 2 million pulses per second. Eight-level signaling carries 3 bits per pulse, so the bit rate is 6 Mbps.
The Shannon capacities are:
\[ C = 1000000 \log_2 (1 + 100) = 6.6 \text{ Mbps}. \]
\[ C = 1000000 \log_2 (1 + 10000) = 13.3 \text{ Mbps}. \]