Signals

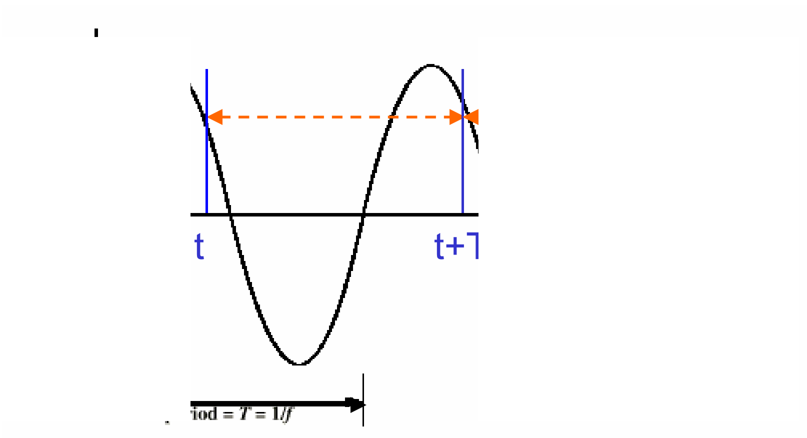
Periodic vs. Aperiodic Signals

- **periodic signal – completes a pattern within some measurable time frame, called a period (T), and then repeats that pattern over subsequent identical periods**

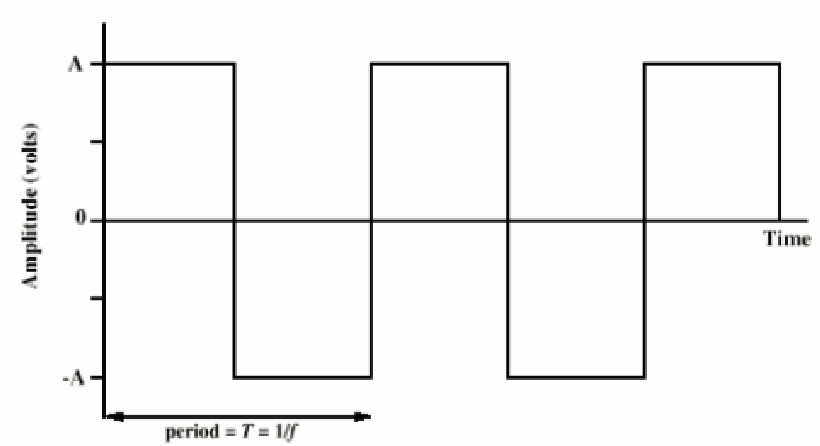
$$\exists T \in R \quad s.t. \quad s(t+T) = s(t), \ \forall t \in \langle -\infty, +\infty \rangle$$

  - **T is the smallest value that satisfies the equation**
  - **T is (typically) expressed in seconds**

- **aperiodic signal – changes without exhibiting a pattern that repeats over time**



periodic analog signal



periodic digital signal

Analog Signals

Classification of Analog Signals

(1) **Simple Analog Signal** – cannot be decomposed into simpler signals

- sinewave – most fundamental form of periodic analog signal – mathematically described with 3 parameters

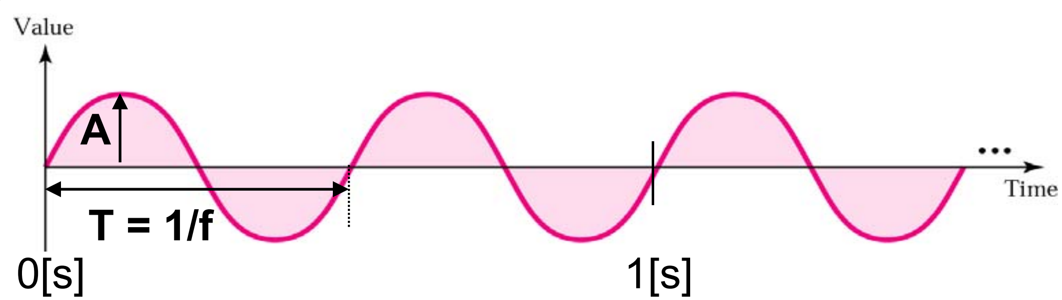
$$s(t) = A \cdot \sin(2\pi f t + \varphi)$$

(1.1) **peak amplitude** (A) – absolute value of signal's highest intensity – unit: volts [V]

(1.2) **frequency** (f) – number of periods in one second – unit: hertz [Hz] = [1/s] – period and frequency are inverses of each other!

(1.3) **phase** (φ) – absolute position of the waveform relative to an arbitrary origin – unit: degrees [º] or radians [rad]

The origin is usually taken as the last previous passage through zero from the negative to the positive direction.

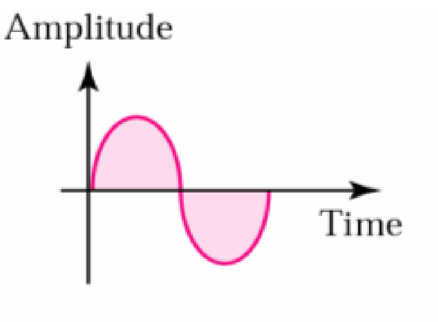


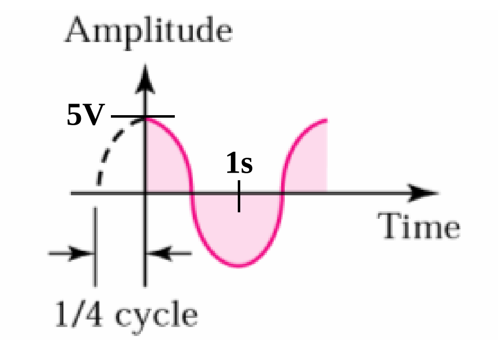
(2) **Composite Analog Signal** – composed of multiple sinewaves

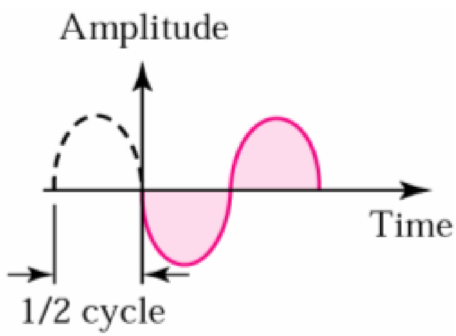
Simple Analog Signals

Phase in Simple Analog Signals – **measured in degrees or radians**

- **360º = 2π rad**
- **1º = 2π/360 rad**
- **1 rad = (360/2π)º** = 57.29578°
- **phase shift of 360º = shift of 1 complete period**
- **phase shift of 180º = shift of 1/2 period**
- **phase shift of 90º = shift of 1/4 period**







φ = 0º or 360º

Analog Signals

Example [period and frequency]

Unit	Equivalent	Unit	Equivalent
seconds (s)	1 s	hertz (Hz)	1 Hz
milliseconds (ms)	10^{-3} s		
microseconds (μs)	10^{-6} s		
nanoseconds (ns)	10^{-9} s		
picoseconds (ps)	10^{-12} s		

units of period and respective frequency

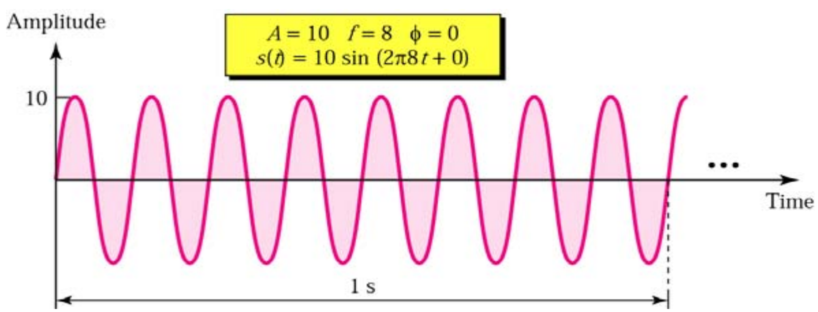
(a) Express a period of 100 ms in <u>micro</u>seconds.

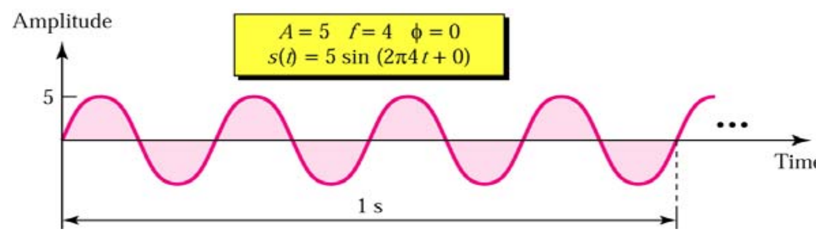
(b) Express the corresponding frequency in <u>kilo</u>hertz.

Simple Analog Signals

Frequency in Simple Analog Signals – rate of signal change with respect to time

- change in a short span of time ⇒ high frequency
- change over a long span of time ⇒ low frequency
- signal does not change at all ⇒ zero frequency
  (signal never completes a cycle T= ∞ ⇒ f=0) – DC signal
- signal changes instantaneously ⇒ ∞ frequency
  (signal completes a cycle in T=0 ⇒ f=∞)





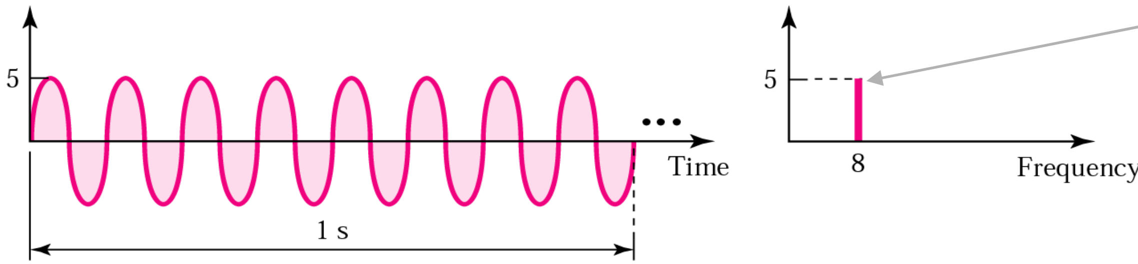
Time Domain Plot – specifies signal amplitude at each instant of time

- does NOT express explicitly signal's phase and frequency

Frequency Domain Plot – specifies peak amplitude with respect to frequency

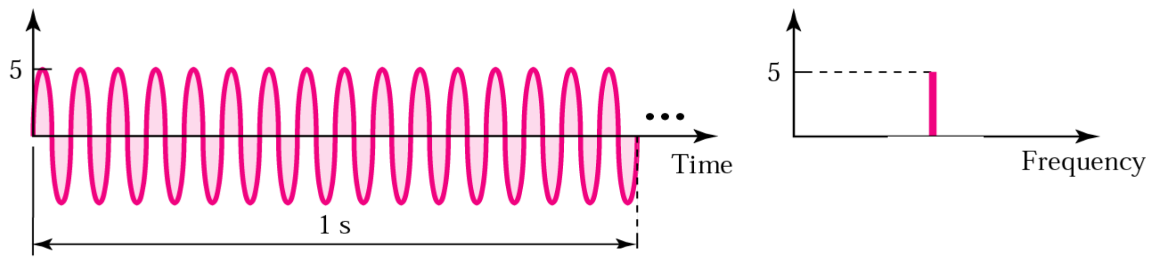
- phase CANNOT be shown in the frequency domain

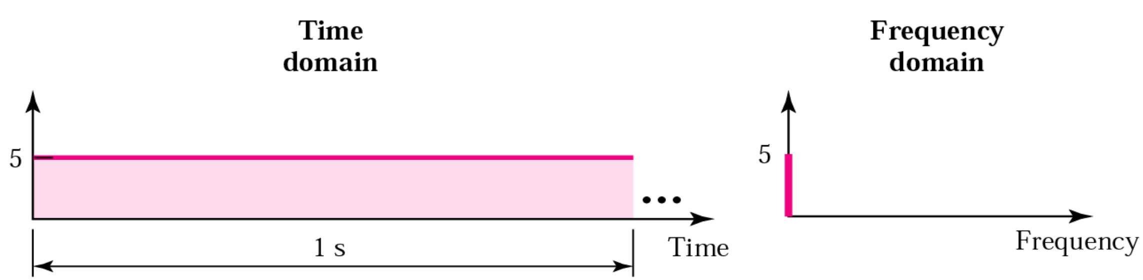
Simple Analog Signals



b. A signal with frequency 8

One 'spike' in frequency domain shows two characteristics of the signal:
spike position = signal frequency,
spike height = peak amplitude.





Analog signals are best represented in the frequency domain.

Composite Analog Signals

Fourier Analysis – **any composite signal can be represented as a combination of simple sine waves with different frequencies, phases and amplitudes**

$$s(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) + \ldots$$

- **periodic composite signal (period=T, frequency = f_0=1/T) can be represented as a sum of simple sines/cosines known as *Fourier series*:**

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t) \right]$$

With the aid of good table of integrals, it is easy to determine the frequency-domain nature of many signals. →

$$A_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) dt, \quad n = 0,1,2, \ldots$$

$$B_n = \frac{2}{T} \int_0^T s(t) \sin(2\pi n f_0 t) dt, \quad n = 1,2,3, \ldots$$

- **f_0 is referred to as ‘fundamental frequency’**
- **integer multiples of f_0 are referred to as ‘harmonics’**

Composite Analog Signals (cont.)

Angular Frequency – **aka radian frequency – number of 2π revolutions during a single period of a given signal**

$$\omega = \frac{2\pi}{T} = 2\pi \cdot T$$

- **simple multiple of ordinary frequency**

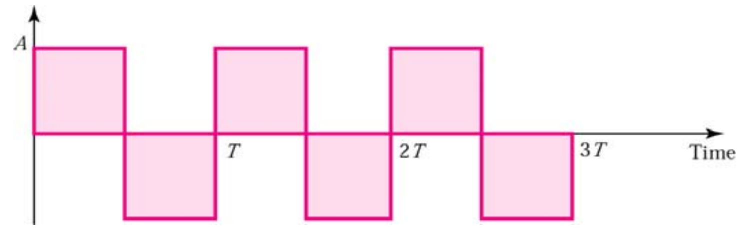
$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)\right]$$

$$A_n = \frac{2}{T}\int_0^T s(t)\cos(n\omega_0 t)dt, \qquad n = 0,1,2,...$$

$$B_n = \frac{2}{T}\int_0^T s(t)\sin(n\omega_0 t)dt, \qquad n = 1,2,...$$

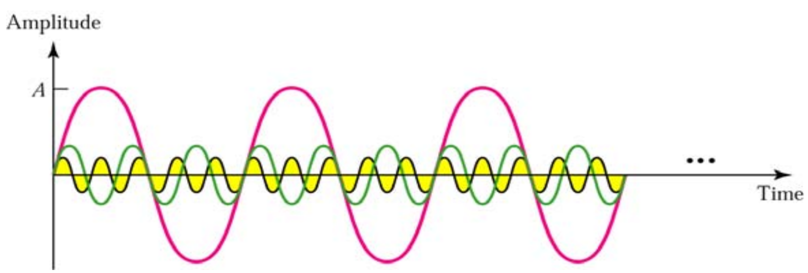
Composite Analog Signals (cont.)

Example [periodic square wave]

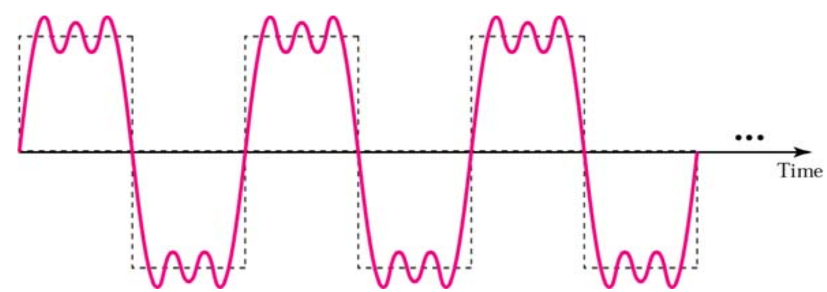


No DC component!!!

$$s(t)=\frac{4A}{\pi}\sin(2\pi ft)+\frac{4A}{3\pi}\sin(2\pi(3f)t)+\frac{4A}{5\pi}\sin(2\pi(5f)t)+\ldots$$



three harmonics



adding three harmonics

With three harmonics we get an approximation of a square wave.
To get the actual square, all harmonics up to ∞ should be added.

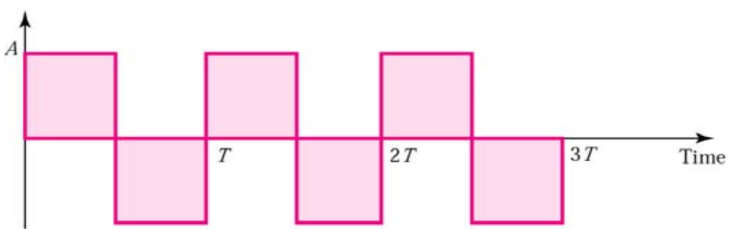
http://www.nst.ing.tu-bs.de/schaukasten/fourier/en_idx.html
http://www.phy.ntnu.edu.tw/java/sound/sound.html

Composite Analog Signals (cont.)

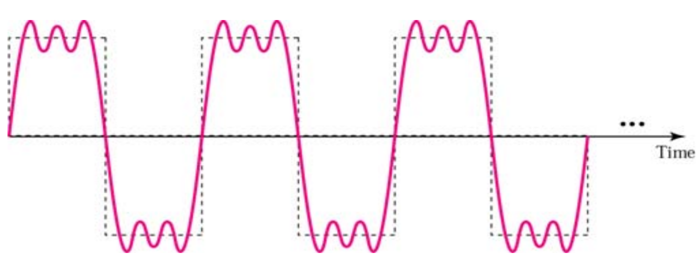
Frequency Spectrum of Analog Signal – range (set) of frequencies that signal contains

Absolute Bandwidth of Analog Signal – width of signal spectrum: $B = f_{highest} - f_{lowest}$

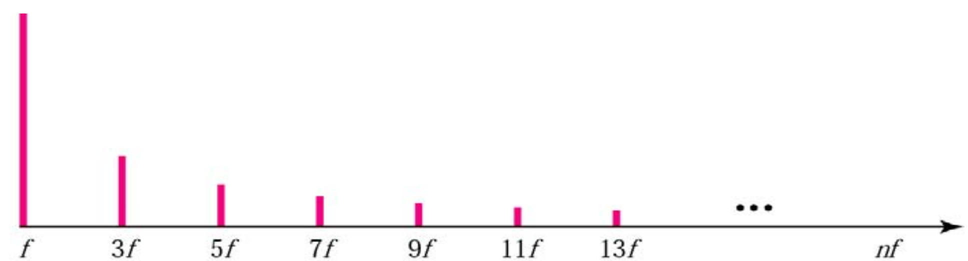
Effective Bandwidth of Analog Signal – range of frequencies where signal contains most of its power/energy



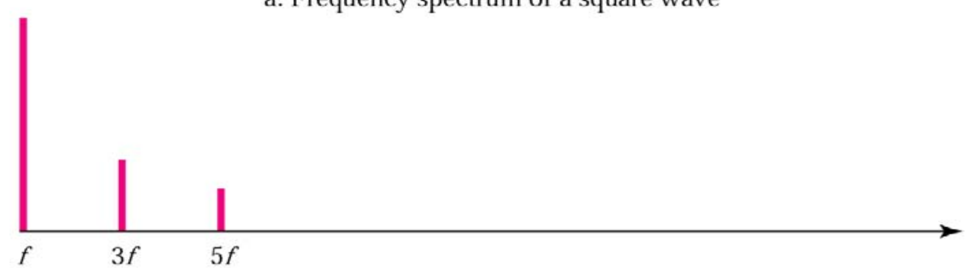
square wave



3-harmonic representation



a. Frequency spectrum of a square wave



b. Frequency spectrum of an approximation with only three harmonics

Composite Analog Signals (cont.)

Example **[frequency spectrum and bandwidth of analog signal]**

A periodic signal is composed of five sinewaves with frequencies of 100, 300, 500, 700 and 900 Hz.

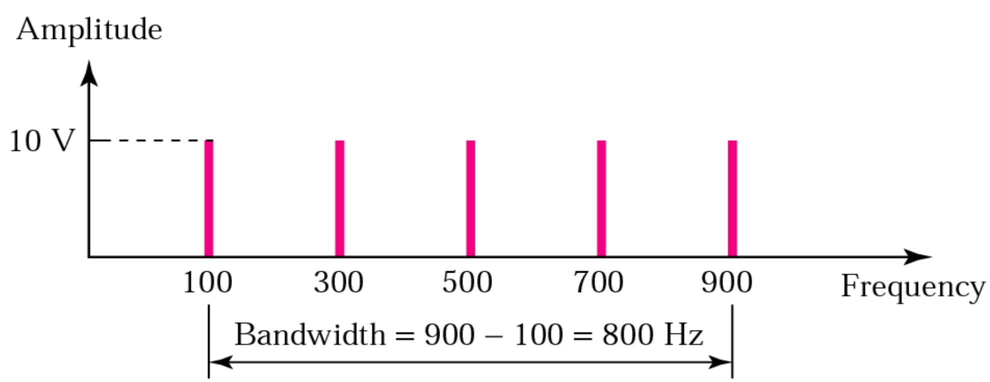
What is the bandwidth of this signal?

Draw the frequency spectrum, assuming all components have a max amplitude of 10V.

Solution:

$B = f_{highest} - f_{lowest} = 900 - 100 = 800$ Hz

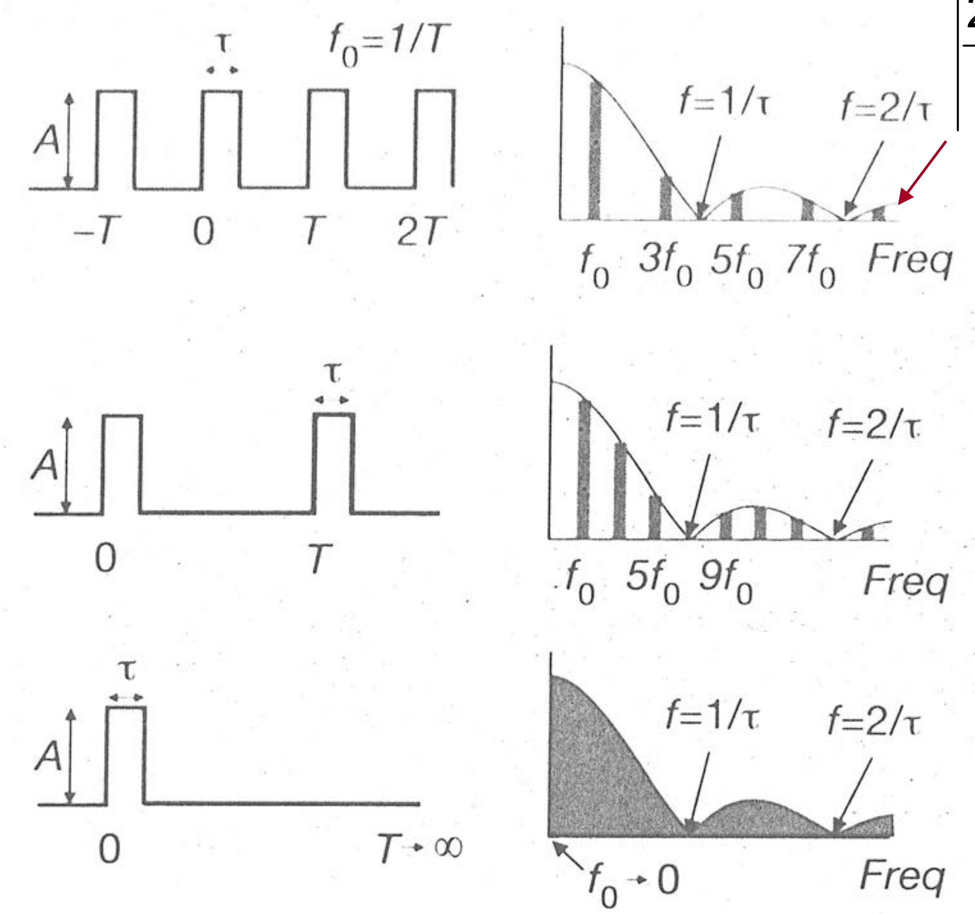
The spectrum has only five spikes, at 100, 300, 500, 700, and 900.



Composite Analog Signals (cont.)

Example **[frequency spectrum of a data pulse]**

$$\left|\frac{2A\tau}{T}\cdot\frac{\sin\left(\pi n \tau/T\right)}{\pi n \tau/T}\right| \approx \left|\frac{\sin(\pi\cdot f\cdot\tau)}{\pi\cdot f\cdot\tau}\right| \text{ or } \left|\frac{\sin(x)}{x}\right|$$



envelope of frequency spectrum

periodic signal ⇒ discrete frequency spectrum

aperiodic signal ⇒ continuous frequency spectrum

What happens if $\tau \to 0$???

Exercise

1. **Before data can be transmitted, they must be transformed to _______________.**
   (a) periodic signals
   (b) electromagnetic signals
   (c) aperiodic signals
   (d) low-frequency sinewaves

2. **In a frequency-domain plot, the vertical axis measures the _______________.**
   (a) peak amplitude
   (b) frequency
   (c) phase
   (d) slope

3. **In a time-domain plot, the vertical axis measures the _______________.**
   (a) peak amplitude
   (b) amplitude
   (c) frequency
   (d) time

4. **If the bandwidth of a signal is 5 KHz and the lowest frequency is 52 KHz, what is the highest frequency _______________.**
   (a) 5 KHz
   (b) 10 KHz
   (c) 47 KHz
   (d) 57 KHz

Exercise

5. **If one of the components of a signal has a frequency of zero, the average amplitude of the signal ________________.**
   (a) is greater than zero
   (b) is less than zero
   (c) is zero
   (d) (a) or (b)

6. **Give two sinewaves A and B, if the frequency of A is twice that of B, then the period of B is _________________ that of A.**
   (a) one-half
   (b) twice
   (c) the same as
   (d) indeterminate from

7. **A device is sending out data at the rate of 1000 bps.**
   (a) How long does it take to send out 10 bits?
   (b) How long does it take to send out a single character (8 bits)?
   (c) How long does it take to send a file of 100,000 characters?