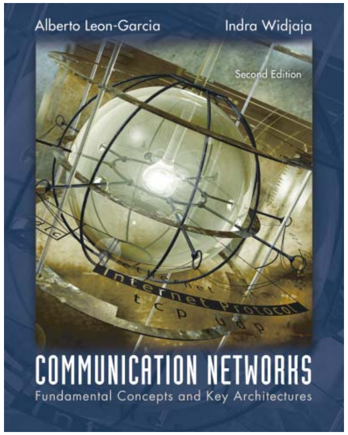
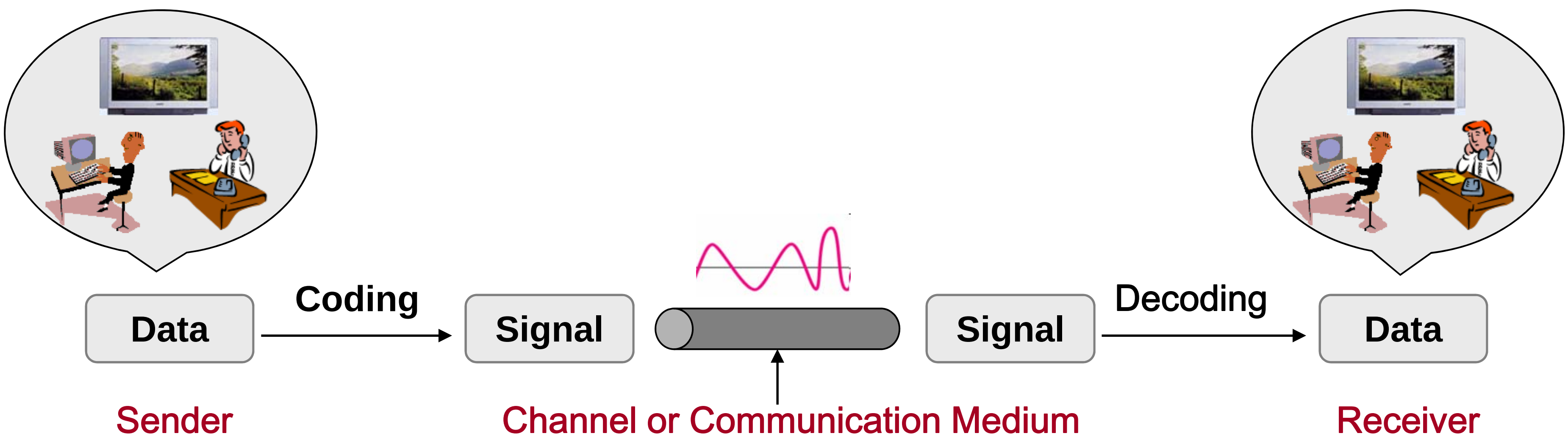
Chapter 3
Digital Transmission Fundamentals



Analog vs. Digital
Digital Representation of Analog Signals
Why Digital Communications?

Data vs. Signal

- **Data:** piece of information formatted in human/machine readable form: voice, music, image, file
- **Signal:** electric or electromagnetic (EM) representation of data; transmission media work by conducting energy along a physical path; thus, to be transmitted, data must be turned into energy in the form of EM signals
- **Transmission :** communication of data through propagation and processing of signals

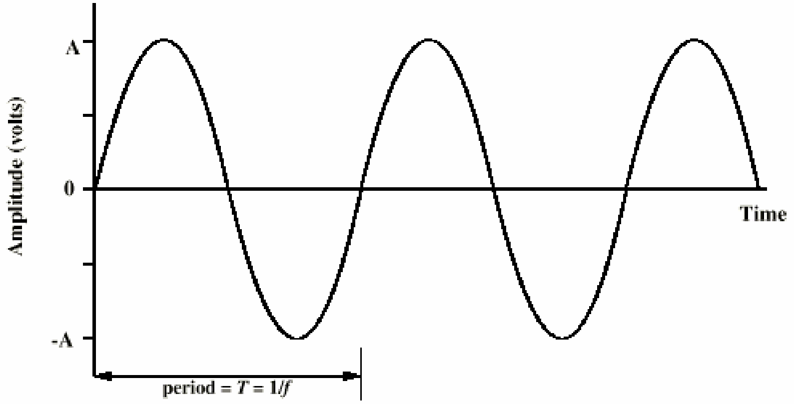


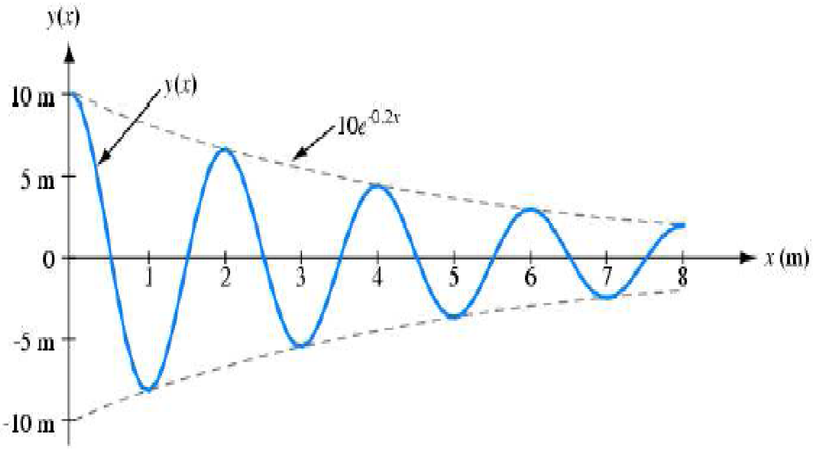
Signal Representation

Signal Representation:

typically in 2D space, function of time, space or frequency

- when the horizontal axis is time, graph displays the value of a signal at one particular point in space as a function of time
- when the horizontal axis is space, graph displays the value of a signal at one particular point in time as a function of space





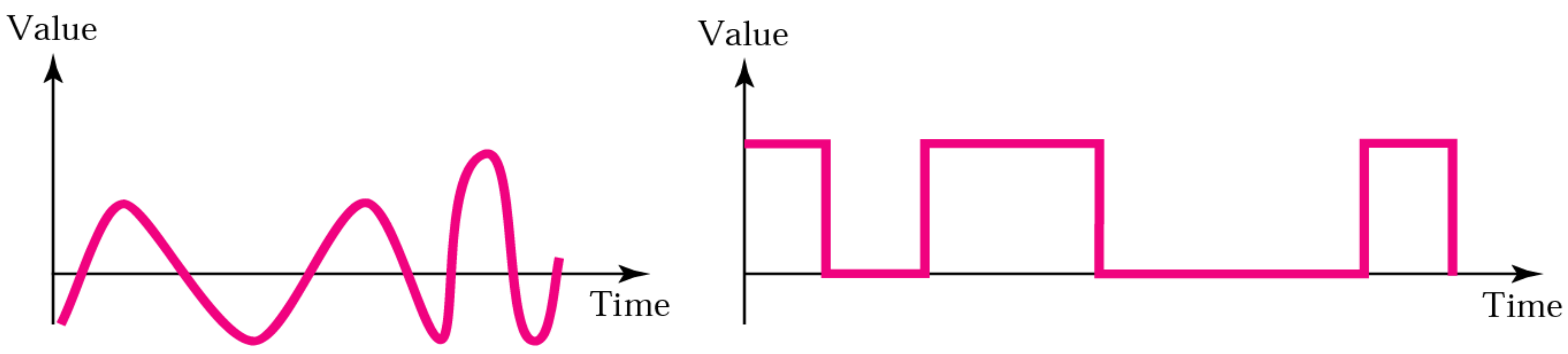
Analog vs. Digital

Analog data: representation variable takes on continuous values in some interval, e.g. voice, temperature, etc.

Digital data : representation variable takes on discrete (a finite & countable number of) values in a given interval, e.g. text, digitized images, etc.

Analog signal: continuous in time and can assume an infinite No. of values in a given range (continuous in time and value)

Discrete (digital) signal: signal that is continuous in time and can assume only a limited number of values (maintains a constant level and then changes to another constant level)

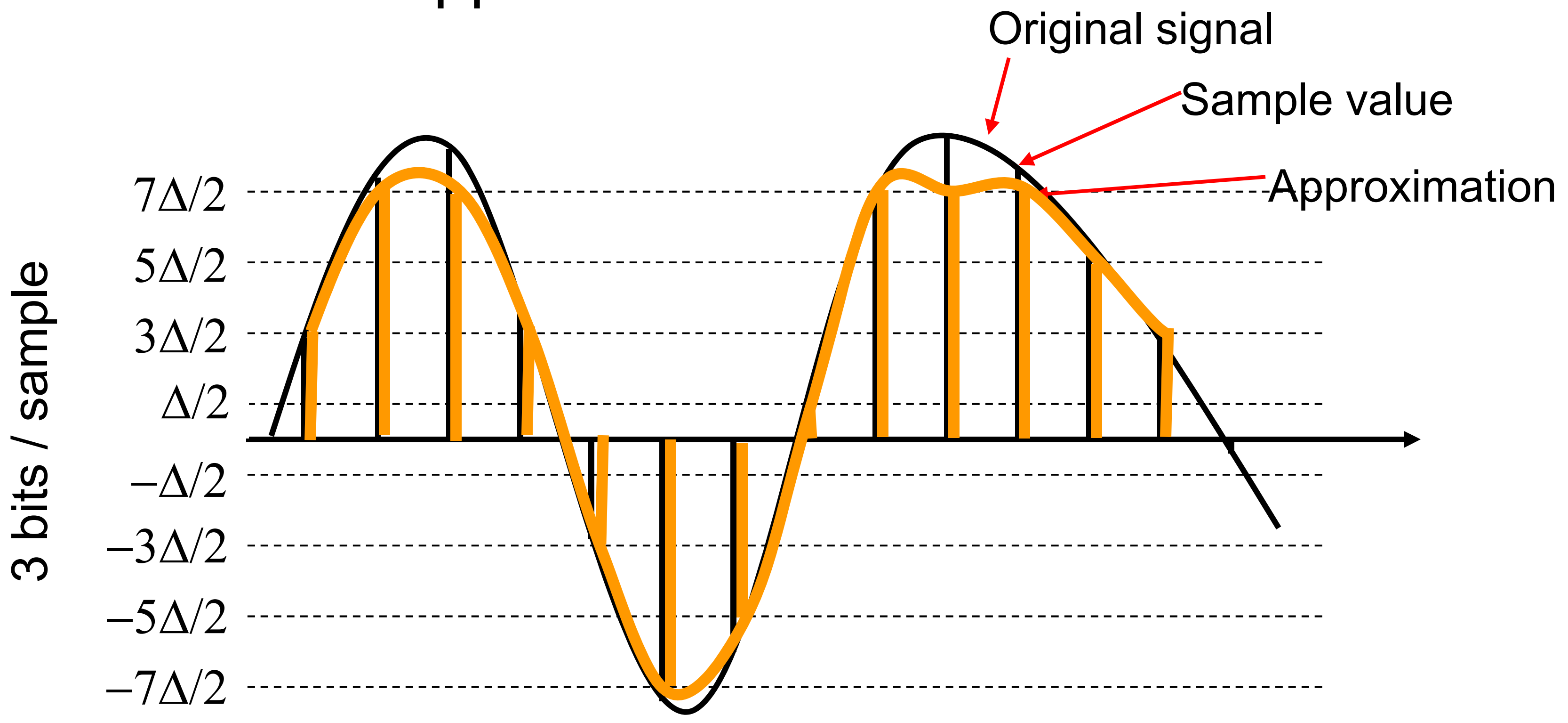


a. Analog signal

b. Digital signal

Digitization of Analog Signal

- Sample analog signal in time and amplitude
- Find closest approximation



R_s = Bit rate = # bits/sample x # samples/second

Example: Voice and Audio

Telephone voice

- W_s = 4 kHz $\rightarrow$ 8000 samples/sec
- 8 bits/sample
- R_s=8 x 8000 = 64 kbps

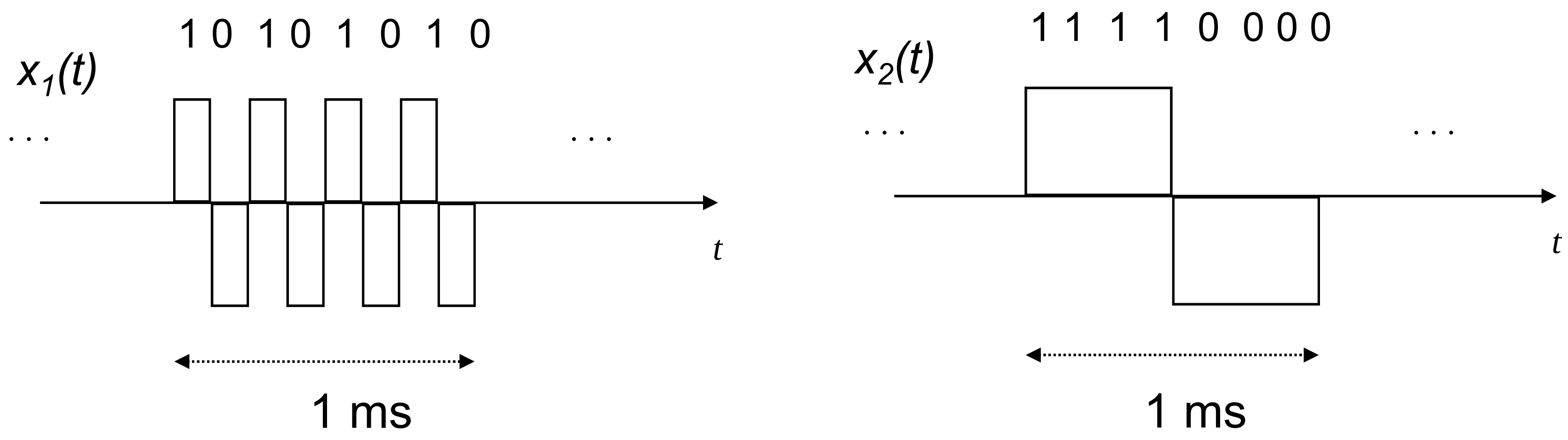
- Cellular phones use more powerful compression algorithms: 8-12 kbps

CD Audio

- W_s = 22 kHertz $\rightarrow$ 44000 samples/sec
- 16 bits/sample
- R_s=16 x 44000= 704 kbps per audio channel
- MP3 uses more powerful compression algorithms: 50 kbps per audio channel

Sampling Rate and Bandwidth

- A signal that varies faster needs to be sampled more frequently
- *Bandwidth* measures how fast a signal varies



- What is the bandwidth of a signal?
- How is bandwidth related to sampling rate?

Periodic Signals

- A periodic signal with period T can be represented as sum of sinusoids using Fourier Series:

$$x(t) = a_0 + a_1\cos(2\pi f_0 t + \phi_1) \quad + \quad a_2\cos(2\pi 2 f_0 t + \phi_2) \quad + \ldots$$
$$+ \, a_k\cos(2\pi k f_0 t + \phi_k) + \ldots$$

"DC"
long-term
average

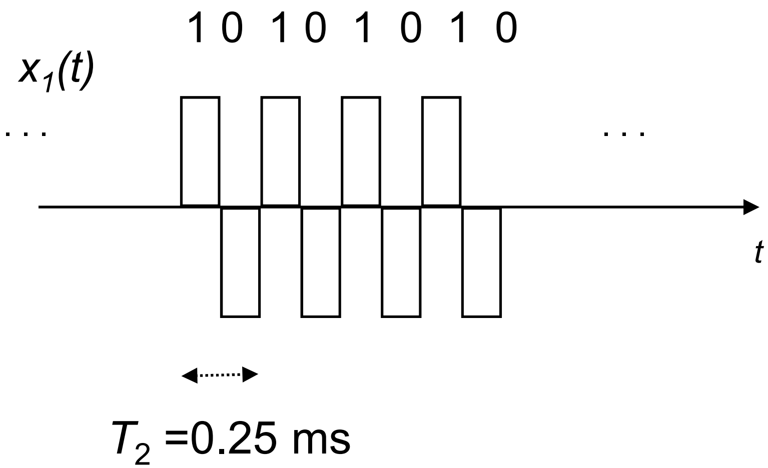
fundamental
frequency $f_0 = 1/T$
first harmonic

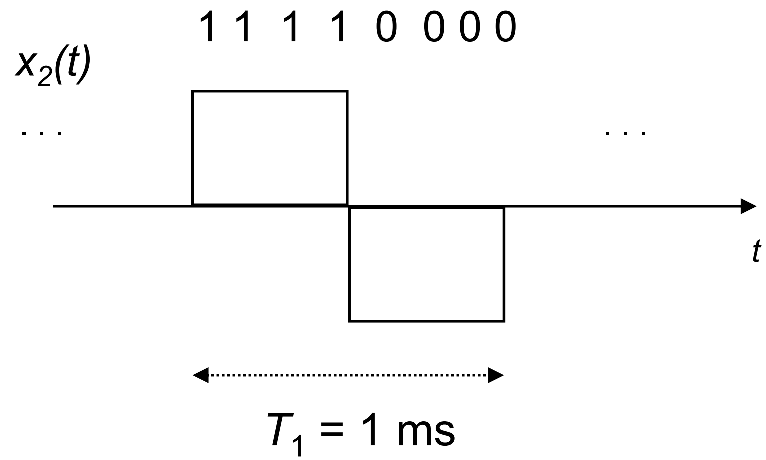
kth harmonic

•$|a_k|$ determines amount of power in kth harmonic

•Amplitude specturm $|a_0|$, $|a_1|$, $|a_2|$, …

Example Fourier Series





$$x_1(t) = 0 + \frac{4}{\pi}\cos(2\pi 4000t) + \frac{4}{3\pi}\cos(2\pi 3(4000)t) + \frac{4}{5\pi}\cos(2\pi 5(4000)t) + \ldots$$

$$x_2(t) = 0 + \frac{4}{\pi}\cos(2\pi 1000t) + \frac{4}{3\pi}\cos(2\pi 3(1000)t) + \frac{4}{5\pi}\cos(2\pi 5(1000)t) + \ldots$$

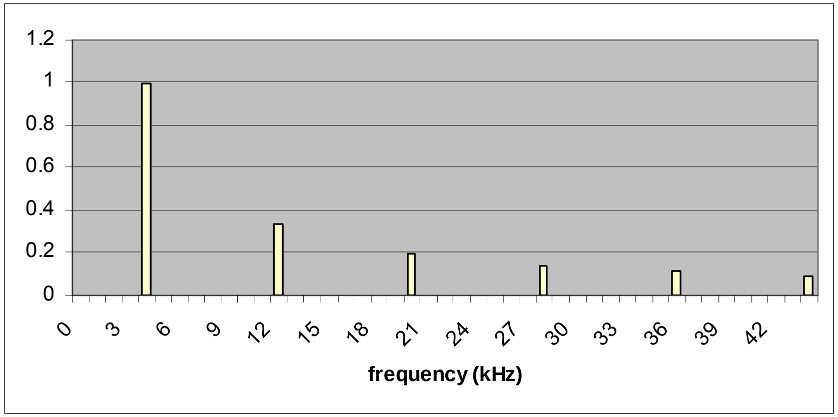
Only odd harmonics have power

http://www.nst.ing.tu-bs.de/schaukasten/fourier/en_idx.html

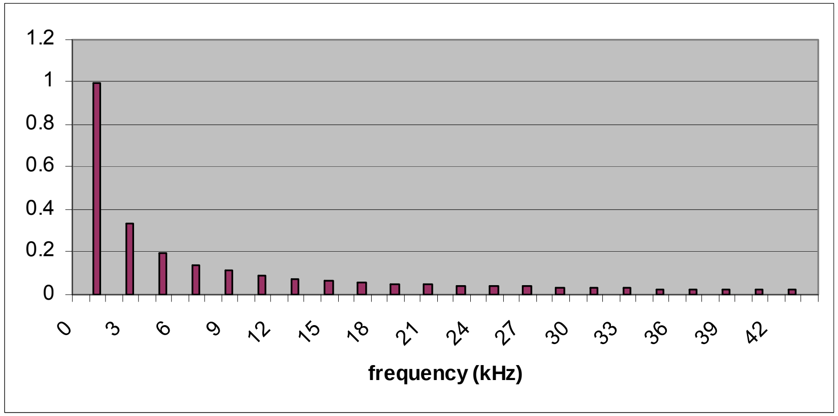
Spectra & Bandwidth

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- *$x_1(t)$* varies faster in time & has more high frequency content than *$x_2(t)$*
- Bandwidth W_s is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power

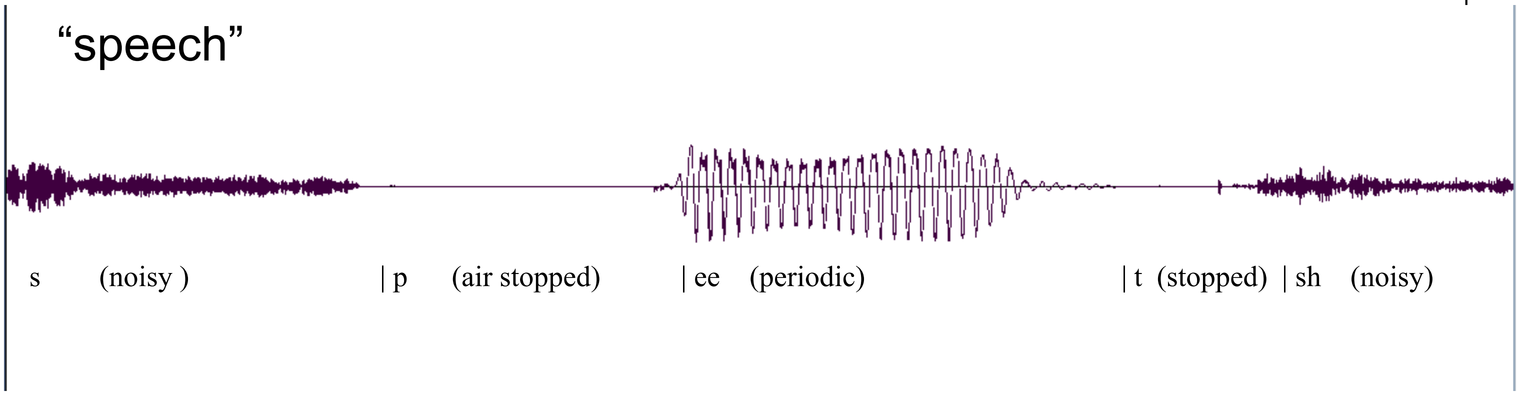
Spectrum of *$x_1(t)$*



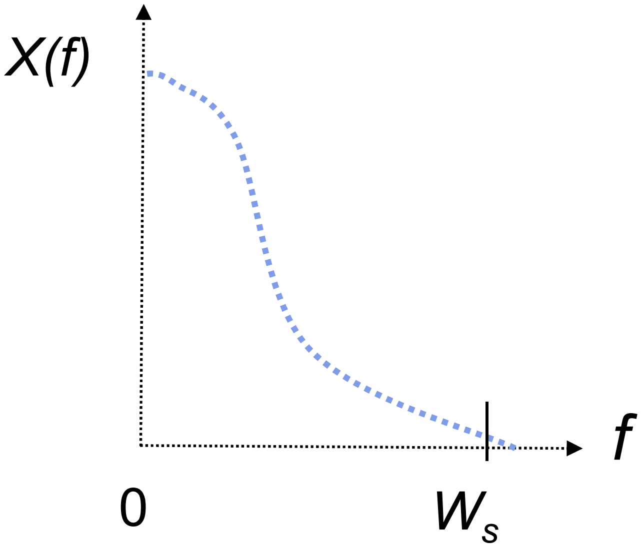
Spectrum of *$x_2(t)$*



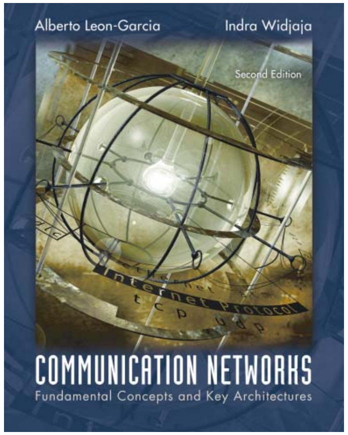
Bandwidth of General Signals



- Not all signals are periodic
  - E.g. voice signals varies according to sound
  - Vowels are periodic, “s” is noiselike
- Spectrum of long-term signal
  - Averages over many sounds, many speakers
  - Involves Fourier transform
- Telephone speech: 4 kHz
- CD Audio: 22 kHz

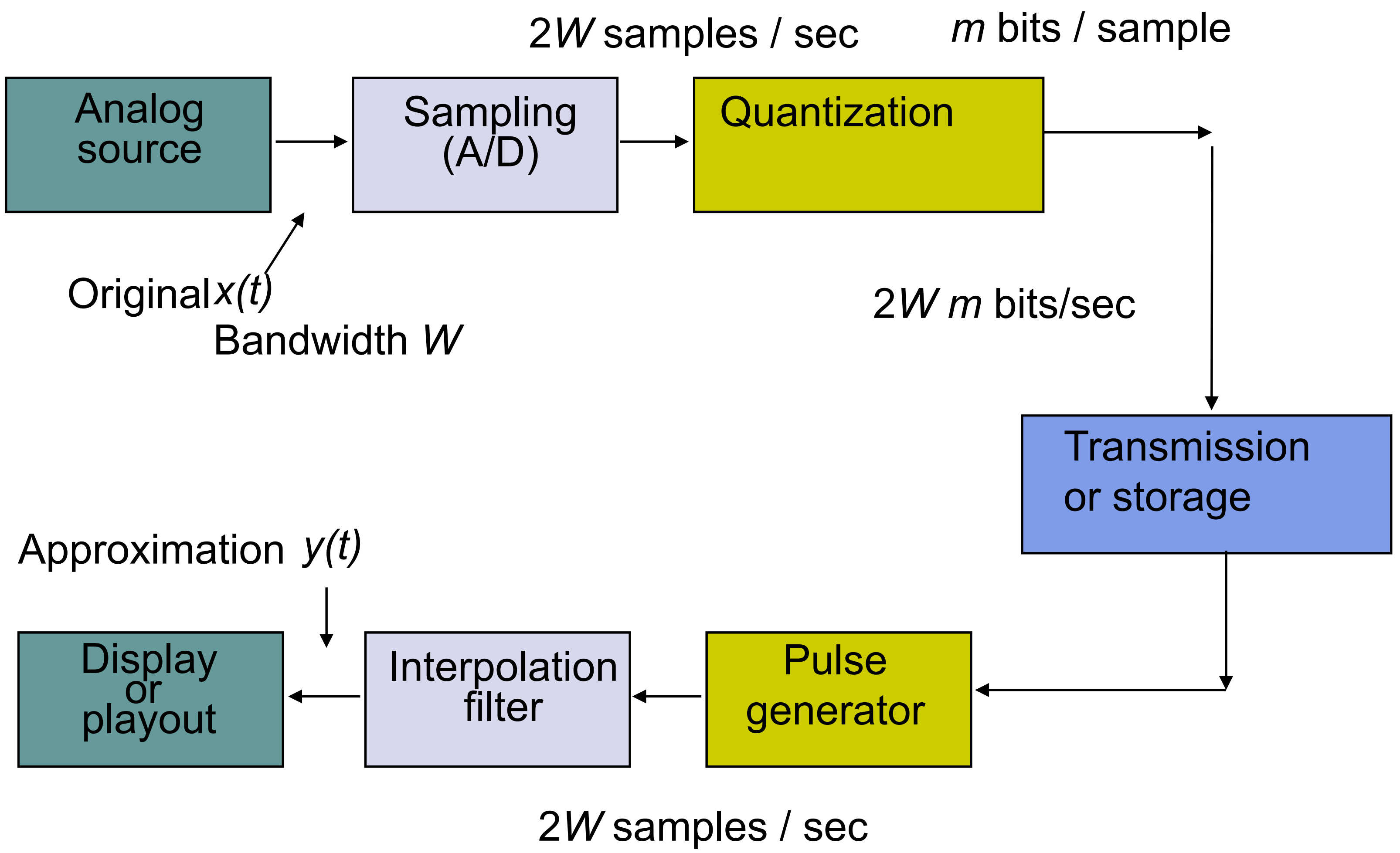


Chapter 3
Communication Networks and Services



Digital Representation of Analog Signals

Digital Transmission of Analog Information

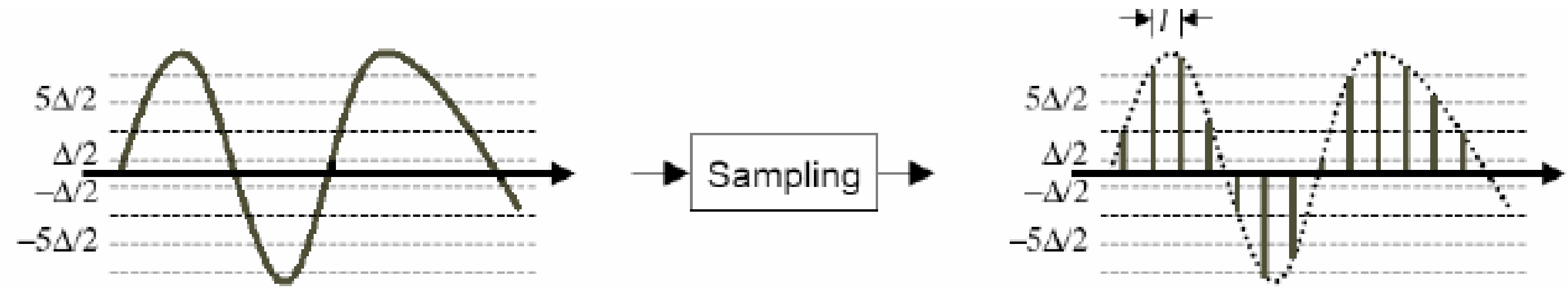


Digital Transmission of Analog Signals (Cont.)

Digitization Procedure consists of two steps:

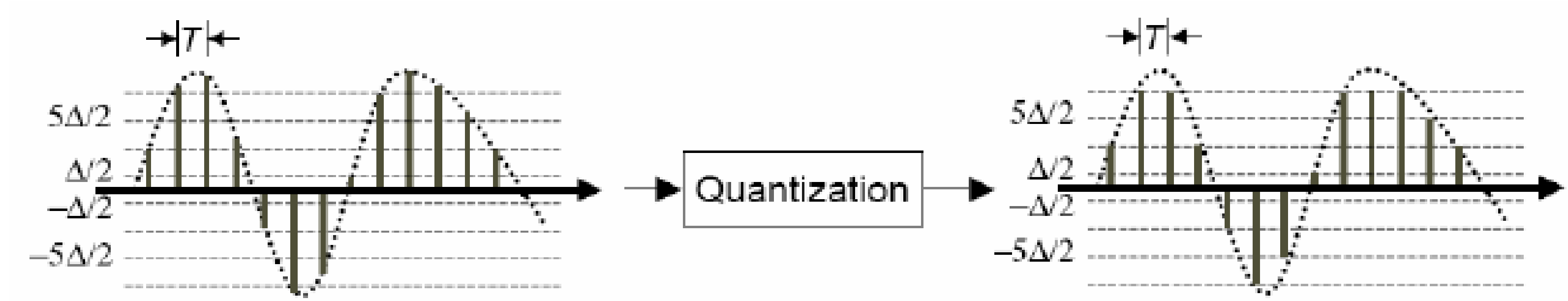
(1) sampling – obtain signal values at equal intervals (T)

(2) quantization – approximate samples to certain values



Analog signal (continuous-time, continuous-amplitude)

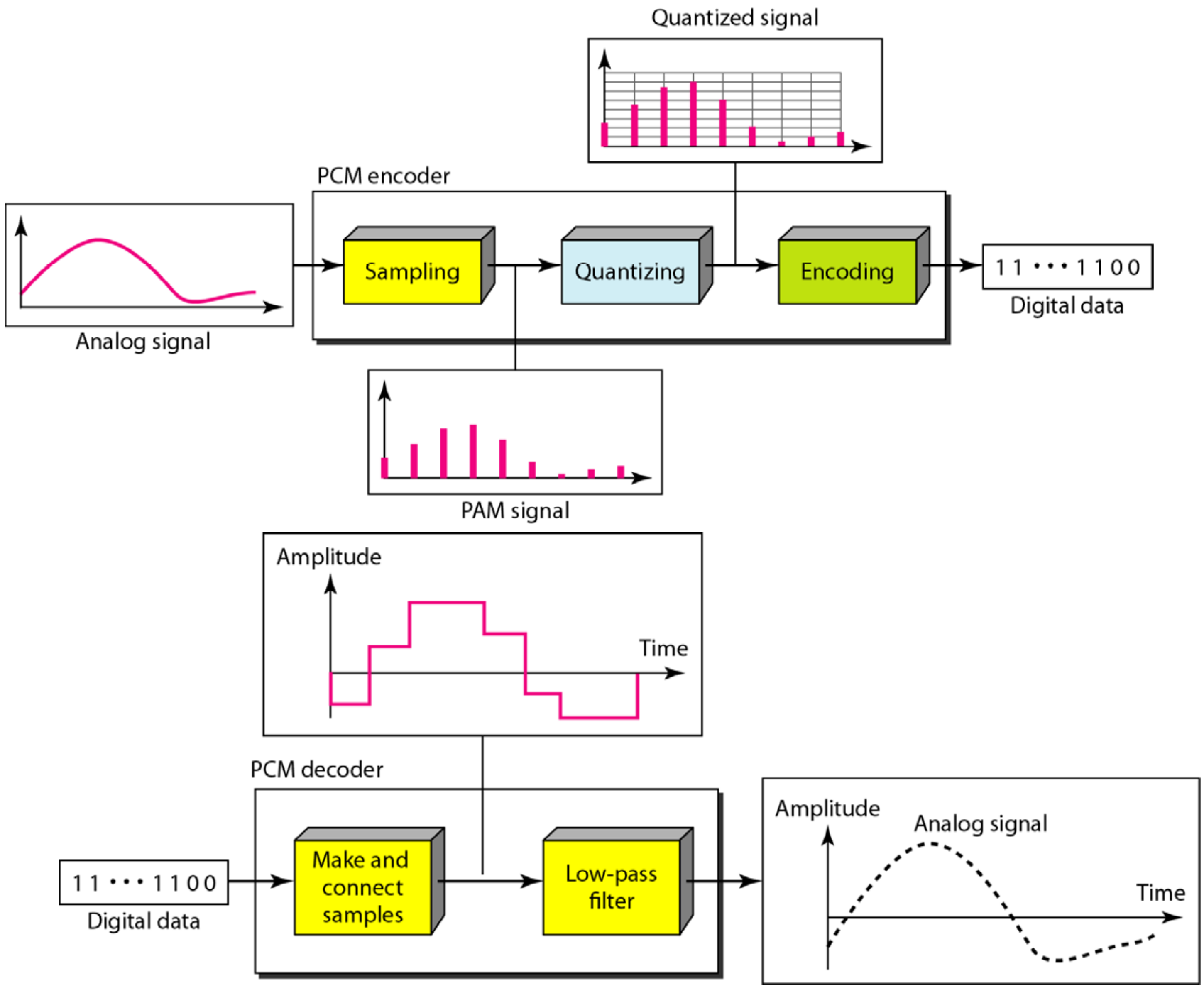
Discrete time signal (discrete -time, continuous-amplitude)



Discrete time signal (discrete -time, continuous-amplitude)

Digital signal (discrete -time, discrete -amplitude)

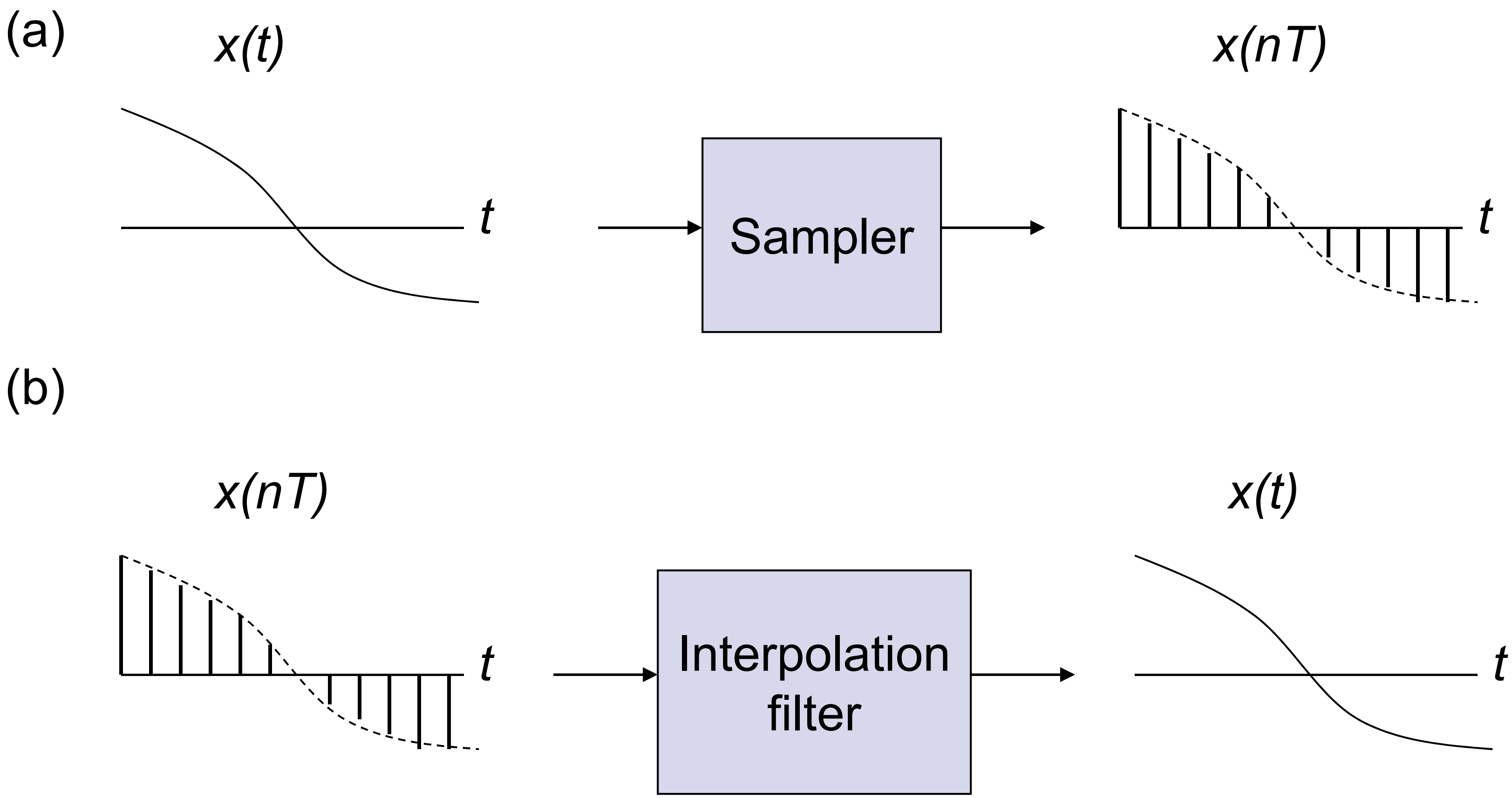
Pulse Code Modulation (PCM)



Sampling Theorem

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

Nyquist: Perfect reconstruction if sampling rate $1/T > 2W_s$



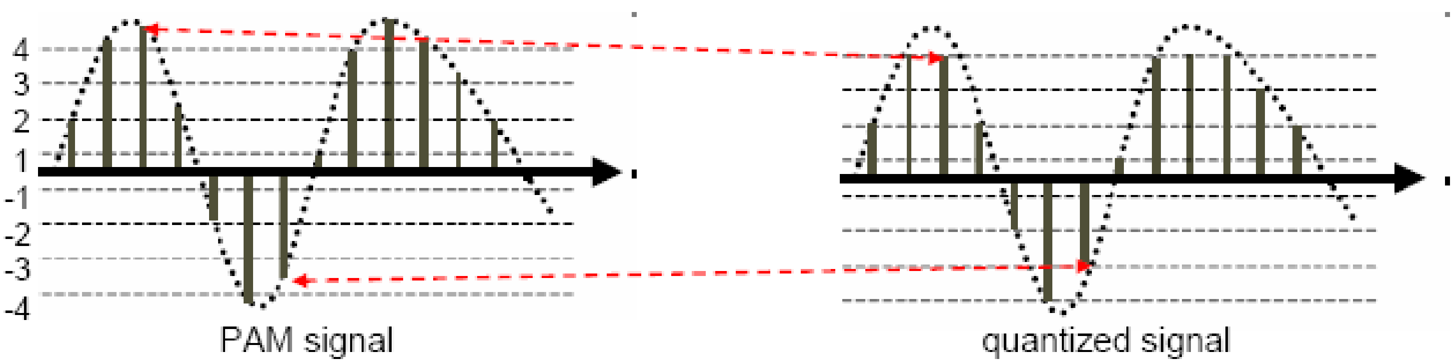
Quantization

❑ PAM signal samples have amplitudes of '∞ precision" –direct encoding of such amplitudes would require ∞ number of bits (digital pulses) per sample

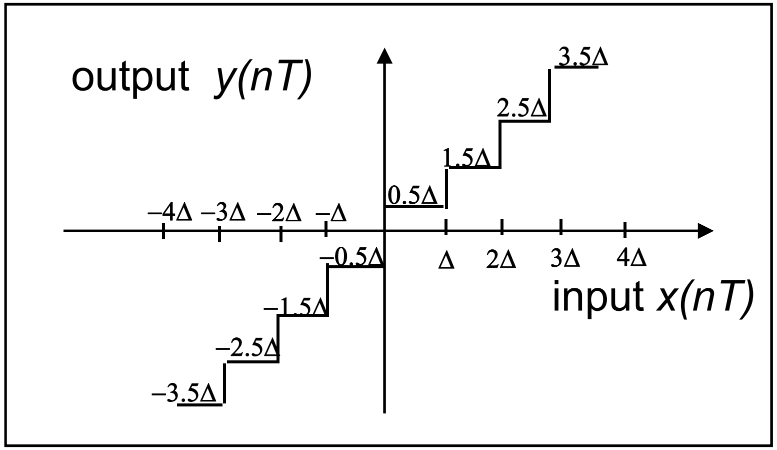
❑ to convert PAM signal to digital signal (that is practical for transmission), each sample has to be 'rounded up' to the nearest of M possible quantization levels

M quantization levels : $m = \log_2(M)$ bits per level

M↑ ⇒ better precision , more bits per sample
M↓ ⇒ poor precision , fewer bits per sample

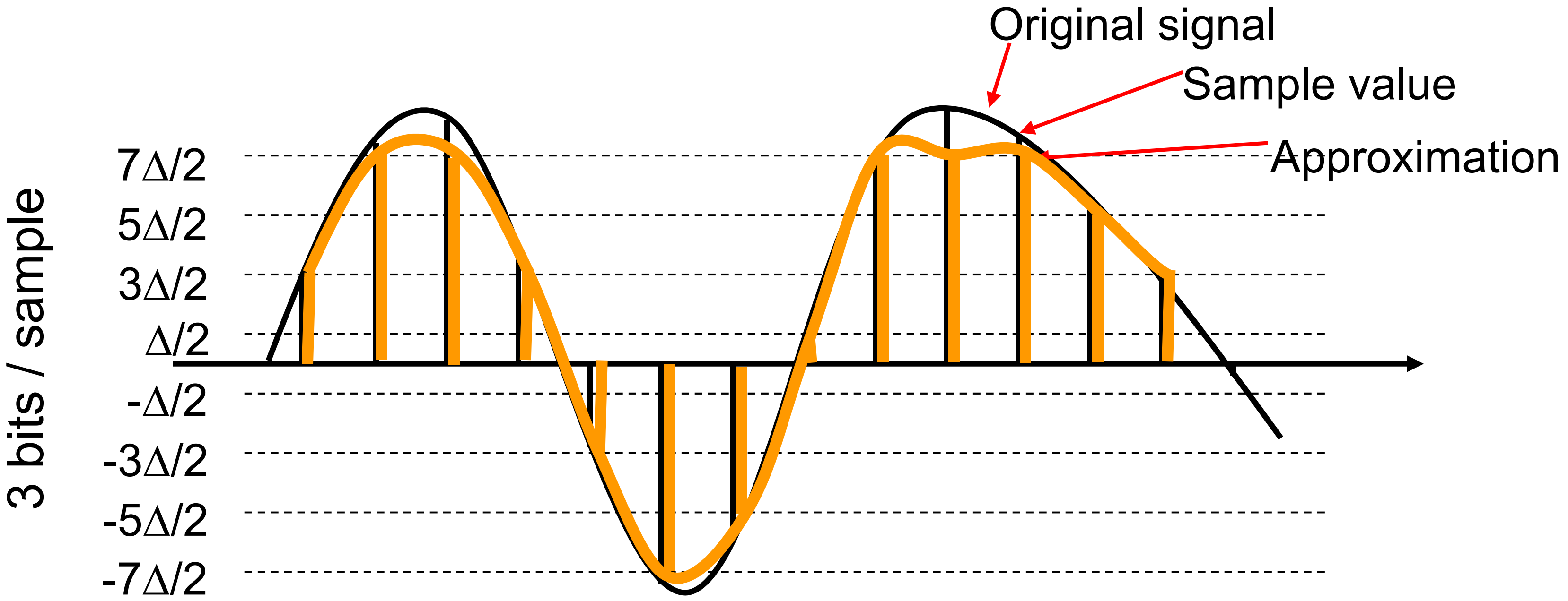


Quantization



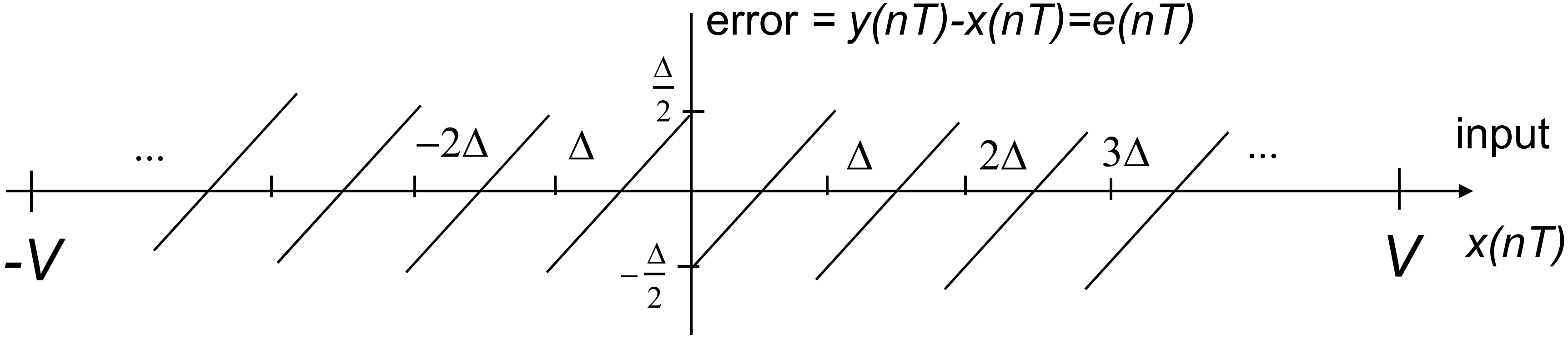
Quantizer maps input into closest of 2^m representation values

Quantization error: "noise" = $x(nT) - y(nT)$



Quantizer Performance

$M = 2^m$ levels, Dynamic range($-V$, V) $\Delta = 2V/M$



If the number of levels M is large, then the error is approximately uniformly distributed between (-Δ/2, Δ2)

Average Noise Power = Mean Square Error:

$$\sigma_e^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \frac{1}{\Delta} dx = \frac{\Delta^2}{12}$$

Quantizer Performance

Figure of Merit:

Signal-to-Noise Ratio = Avg signal power / Avg noise power

Let σ_x^2 be the signal power, then

$$SNR = \frac{\sigma_x^2}{\Delta^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3\left(\frac{\sigma_x}{V}\right)^2 M^2 = 3\left(\frac{\sigma_x}{V}\right)^2 2^{2m}$$

The ratio $V/\sigma_x \approx 4$

The SNR is usually stated in decibels:

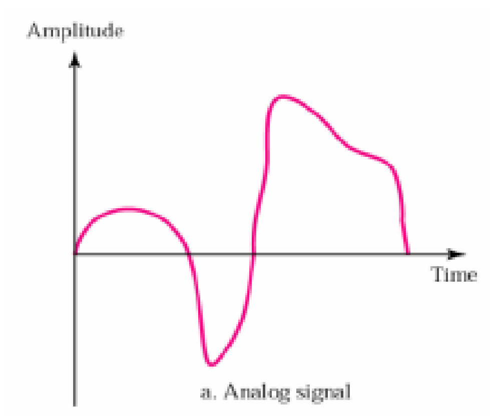
SNR dB = $10 \log_{10} \sigma_x^2/\sigma_e^2 = 6\text{m} + 10 \log_{10} 3\sigma_x^2/V^2$

SNR dB = 6*m* - 7.27 dB for $V/\sigma_x = 4$.

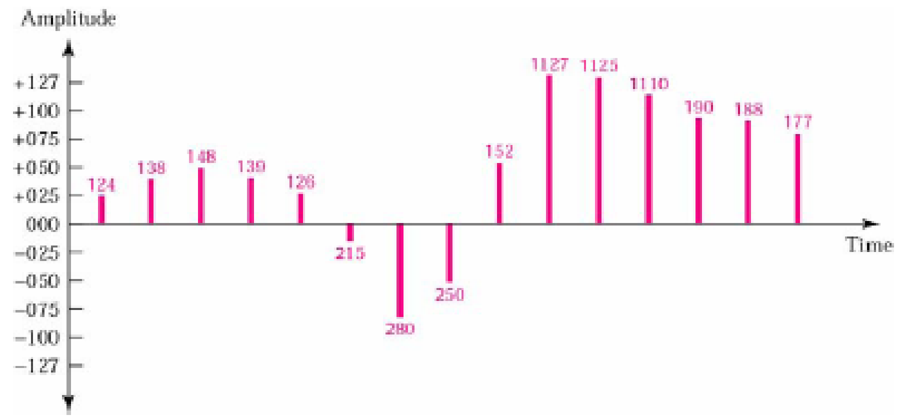
Quantization (Cont.)

Example [Quantization of PAM Signal]

Assume an analog signal, as shown below, has to be quantized using at most 8-bits per sample. How many different quantization levels are allowed / should be used?



a. Analog signal



+024	00011000	−015	10001111	+125	01111101
+038	00100110	−080	11010000	+110	01101110
+048	00110000	−050	10110010	+090	01011010
+039	00100111	+052	00110110	+088	01011000
+026	00011010	+127	01111111	+077	01001101

Sign bit
+ is 0 − is 1

Quantization (Cont.)

Example [voice signal in telephone system]

Natural human voice occupies the range of 80 – 4000 [Hz]. Human ear can tolerate SNR of 40 [dB]. Assume we want to transmit human voice in digitized form.
What bit rate [bps] should be supported by the channel to enable such transmission?

(1) Sampling rate?!

Based on Nyquist Sampling Theorem:

max frequency = 4 [kHz] $\Rightarrow$ sampling rate = 2*4 [kHz] = 8000 [samples/sec]

(2) # of bits per sample?!

Based on SNR formula:

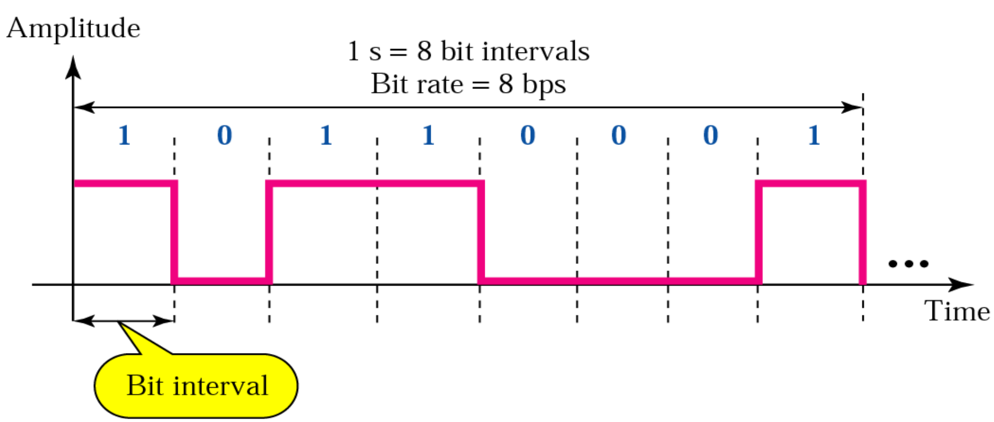
40 [dB] = 6*m - 7.76 $\Rightarrow$ # bits per sample = 8 $\Rightarrow$ # of levels = 2^8 = 256

data rate = # samples per second * # bits per sample = 64 kbps

Digital Signals

– sequence of voltage pulses (DC levels) – each pulse represents a *signal element*

- binary data are transmitted using only 2 types of signal elements (1 = positive voltage, 0 = negative voltage)
- key digital-signals terms:
  - bit interval – time required to send one single bit – unit: [sec]
  - bit rate – number of bit intervals per second – unit: [bps]



Most digital signals are aperiodic,
so it is not appropriate / correct to talk about their period.

Digital Signals (Cont.)

Digital Signal as a Composite Analog Signal

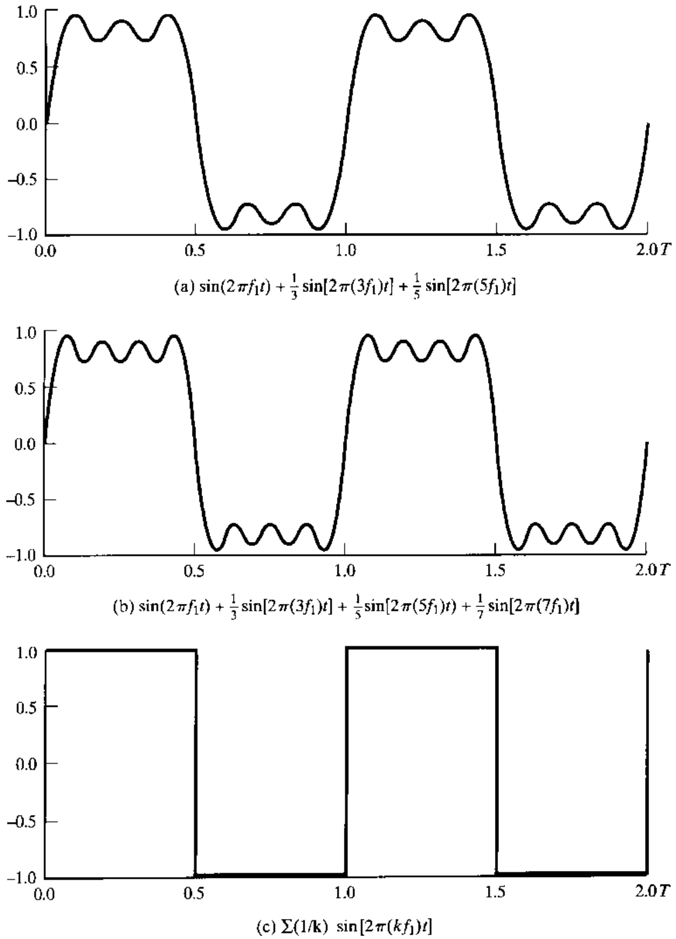


FIGURE 4.6 Frequency Components of a Square Wave ($T = 1/f_1$).

– digital signal, with all its sudden changes, is actually a composite signal having an infinite number of frequencies

- a digital signal is a composite signal with an infinite bandwidth
- if a medium has a wide bandwidth, a digital signal can be sent through it
- some frequencies will be weakened or blocked; still, enough frequencies will be passed to preserve a decent signal shape
- what is the minimum required bandwidth B [Hz] of a band-limited medium if we want to send n [bps]?