Chapter 3
Digital Transmission Fundamentals

Analog vs. Digital
Digital Representation of Analog Signals
Why Digital Communications?
Data vs. Signal

- **Data**: piece of information formatted in human/machine readable form: voice, music, image, file

- **Signal**: electric or electromagnetic (EM) representation of data; transmission media work by conducting energy along a physical path; thus, to be transmitted, data must be turned into energy in the form of EM signals

- **Transmission**: communication of data through propagation and processing of signals
Signal Representation:

- Typically in 2D space, function of time, space or frequency.
  - When the horizontal axis is time, the graph displays the value of a signal at one particular point in space as a function of time.
  - When the horizontal axis is space, the graph displays the value of a signal at one particular point in time as a function of space.
Analog vs. Digital

**Analog data**: representation variable takes on continuous values in some interval, e.g. *voice*, *temperature*, etc.

**Digital data**: representation variable takes on discrete (a finite & countable number of) values in a given interval, e.g. *text*, *digitized images*, etc.

**Analog signal**: *continuous in time* and can assume an infinite No. of values in a given range (continuous in time and value)

**Discrete (digital) signal**: signal that is *continuous in time* and can assume only a limited number of values (maintains a constant level and then changes to another constant level)
Digitization of Analog Signal

- Sample analog signal in time and amplitude
- Find closest approximation

$$R_s = \text{Bit rate} = \# \text{ bits/sample} \times \# \text{ samples/second}$$
Example: Voice and Audio

**Telephone voice**
- $W_s = 4 \text{ kHz} \rightarrow 8000$ samples/sec
- 8 bits/sample
- $R_s = 8 \times 8000 = 64 \text{ kbps}$

- Cellular phones use more powerful compression algorithms: 8-12 kbps

**CD Audio**
- $W_s = 22 \text{ kHertz} \rightarrow 44000$ samples/sec
- 16 bits/sample
- $R_s = 16 \times 44000 = 704 \text{ kbps per audio channel}$

- MP3 uses more powerful compression algorithms: 50 kbps per audio channel
Sampling Rate and Bandwidth

- A signal that varies faster needs to be sampled more frequently.
- *Bandwidth* measures how fast a signal varies.

What is the bandwidth of a signal?
How is bandwidth related to sampling rate?
Periodic Signals

- A periodic signal with period $T$ can be represented as sum of sinusoids using Fourier Series:

$$
x(t) = a_0 + a_1 \cos(2\pi f_0 t + \phi_1) + a_2 \cos(2\pi 2f_0 t + \phi_2) + \ldots + a_k \cos(2\pi kf_0 t + \phi_k) + \ldots
$$

- $|a_k|$ determines amount of power in $k$th harmonic
- Amplitude spectrum $|a_0|, |a_1|, |a_2|, \ldots$
Example Fourier Series

\[ x_1(t) = 0 + \frac{4}{\pi} \cos(2\pi 4000t) \]
\[ + \frac{4}{3\pi} \cos(2\pi 3(4000)t) \]
\[ + \frac{4}{5\pi} \cos(2\pi 5(4000)t) + \ldots \]

\[ x_2(t) = 0 + \frac{4}{\pi} \cos(2\pi 1000t) \]
\[ + \frac{4}{3\pi} \cos(2\pi 3(1000)t) \]
\[ + \frac{4}{5\pi} \cos(2\pi 5(1000)t) + \ldots \]

Only odd harmonics have power

http://www.nst.ing.tu-bs.de/schaukasten/fourier/en_idx.html
Spectra & Bandwidth

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- $x_1(t)$ varies faster in time & has more high frequency content than $x_2(t)$
- Bandwidth $W_s$ is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power
Bandwidth of General Signals

- Not all signals are periodic
  - E.g. voice signals varies according to sound
  - Vowels are periodic, “s” is noiselike
- Spectrum of long-term signal
  - Averages over many sounds, many speakers
  - Involves Fourier transform
- Telephone speech: 4 kHz
- CD Audio: 22 kHz
Chapter 3

Communication Networks and Services

Digital Representation of Analog Signals
Digital Transmission of Analog Information

- Analog source
  - Original \( x(t) \)
  - Bandwidth \( W \)
- Sampling (A/D)
- Quantization
- Transmission or storage
- Pulse generator
- Interpolation filter
- Display or playout

- 2\(W\) samples / sec
- \(m\) bits / sample
- 2\(W\) \(m\) bits/sec
- \(2W\) samples / sec
Digitization Procedure consists of two steps:
(1) sampling – obtain signal values at equal intervals (T)
(2) quantization – approximate samples to certain values

Analog signal (continuous-time, continuous-amplitude)

Discrete time signal (discrete -time, continuous-amplitude)

Discrete time signal (discrete -time, continuous-amplitude)

Digital signal (discrete -time, discrete -amplitude)
Pulse Code Modulation (PCM)

1. Analog signal
2. Sampling
3. Quantizing
4. Encoding
5. Quantized signal
6. Digital data
7. PAM signal
8. PCM decoder
9. Make and connect samples
10. Low-pass filter
11. Digital data
12. Analog signal
13. Time
14. Amplitude
Sampling Theorem

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

Nyquist: Perfect reconstruction if sampling rate $1/T > 2W_s$

(a) $x(t)$

(b) $x(nT)$
Quantization

- PAM signal samples have amplitudes of ‘∞ precision’ – direct encoding of such amplitudes would require ∞ number of bits (digital pulses) per sample.

- To convert PAM signal to digital signal (that is practical for transmission), each sample has to be ‘rounded up’ to the nearest of M possible quantization levels.

\[ m = \log_2(M) \text{ bits per level} \]

\[ M \uparrow \Rightarrow \text{better precision, more bits per sample} \]
\[ M \downarrow \Rightarrow \text{poor precision, fewer bits per sample} \]
Quantization

Quantizer maps input into closest of $2^m$ representation values

Quantization error: “noise” = $x(nT) - y(nT)$

Original signal
Sample value
Approximation

3 bits / sample

Quantization
Quantizer Performance

\( M = 2^m \text{ levels}, \quad \text{Dynamic range} \ ( -V, V ) \Delta = 2V/M \)

If the number of levels \( M \) is large, then the error is approximately uniformly distributed between \((-\Delta/2, \Delta2)\)

Average Noise Power = Mean Square Error:

\[
\sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} x^2 \frac{1}{\Delta} \, dx = \frac{\Delta^2}{12}
\]
Quantizer Performance

Figure of Merit:
Signal-to-Noise Ratio = Avg signal power / Avg noise power
Let $\sigma_x^2$ be the signal power, then

$$SNR = \frac{\sigma_x^2}{\Delta^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3 \left( \frac{\sigma_x}{V} \right)^2 M^2 = 3 \left( \frac{\sigma_x}{V} \right)^2 2^{2m}$$

The ratio $V/\sigma_x \approx 4$

The SNR is usually stated in decibels:
SNR dB = $10 \log_{10} \frac{\sigma_x^2}{\sigma_e^2} = 6m + 10 \log_{10} \frac{3\sigma_x^2}{V^2}$

SNR dB = 6$m$ - 7.27 dB for $V/\sigma_x = 4$. 
Quantization (Cont.)

Example [ Quantization of PAM Signal ]
Assume an analog signal, as shown below, has to be quantized using at most 8-bits per sample. How many different quantization levels are allowed / should be used?
Example [ voice signal in telephone system ]
Natural human voice occupies the range of 80 – 4000 [Hz]. Human ear can tolerate SNR of 40 [dB]. Assume we want to transmit human voice in digitized form. What bit rate [bps] should be supported by the channel to enable such transmission?

(1) Sampling rate?!
Based on Nyquist Sampling Theorem:
max frequency = 4 [kHz] ⇒ sampling rate = 2*4 [kHz] = 8000 [samples/sec]

(2) # of bits per sample?!
Based on SNR formula:
40 [dB] = 6*m - 7.76 ⇒ # bits per sample = 8 ⇒ # of levels = $2^8 = 256$

data rate = # samples per second * # bits per sample = 64 kbps
Digital Signals

- sequence of voltage pulses (DC levels) – each pulse represents a signal element

- binary data are transmitted using only 2 types of signal elements (1 = positive voltage, 0 = negative voltage)

- key digital-signals terms:
  - bit interval – time required to send one single bit – unit: [sec]
  - bit rate – number of bit intervals per second – unit: [bps]

Most digital signals are aperiodic, so it is not appropriate / correct to talk about their period.
Digital Signals (Cont.)

Digital Signal as a Composite Analog Signal

- digital signal, with all its sudden changes, is actually a composite signal having an infinite number of frequencies

- a digital signal is a composite signal with an infinite bandwidth

- if a medium has a wide bandwidth, a digital signal can be sent through it

- some frequencies will be weakened or blocked; still, enough frequencies will be passed to preserve a decent signal shape

- what is the minimum required bandwidth \( B \) [Hz] of a band-limited medium if we want to send \( n \) [bps]?

**FIGURE 4.6** Frequency Components of a Square Wave \( (T = 1/f_s) \).