Chapter 3
Digital Transmission Fundamentals

Characterization of Communication Channels
Fundamental Limits in Digital Transmission

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Characterization of Communication Channels (Review)
Communications Channels

- A *physical medium* is an inherent part of a communications system
  - Copper wires, radio medium, or optical fiber
- Communications system includes electronic or optical devices that are part of the path followed by a signal
  - Equalizers, amplifiers, signal conditioners
- By *communication channel* we refer to the combined end-to-end physical medium and attached devices
- Sometimes we use the term *filter* to refer to a channel especially in the context of a specific mathematical model for the channel
Communications Channels

**Signal Bandwidth**
- In order to transfer data faster, a signal has to vary more quickly.

**Channel Bandwidth**
- A channel or medium has an inherent limit on how fast the signals it passes can vary
  - *Limits how tightly input pulses can be packed*

**Transmission Impairments**
- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals
  - *Limits accuracy of measurements on received signal*
Bandwidth of a Channel

If input is sinusoid of frequency $f$, then
- output is a sinusoid of same frequency $f$
- Output is attenuated by an amount $A(f)$ that depends on $f$
  - $A(f) \approx 1$, then input signal passes readily
  - $A(f) \approx 0$, then input signal is blocked

Bandwidth $W_c$ is range of frequencies passed by channel
How good is a channel?

- **Performance:** What is the maximum reliable transmission speed?
  - Speed: Bit rate, $R$ bps
  - Reliability: Bit error rate, $BER=10^{-k}$
  - Focus of this section

- **Cost:** What is the cost of alternatives at a given level of performance?
  - Wired vs. wireless?
  - Electronic vs. optical?
  - Standard A vs. standard B?
Ideal Low-Pass Filter

- Ideal filter: all sinusoids with frequency $f<W_c$ are passed without attenuation and delayed by $\tau$ seconds; sinusoids at other frequencies are blocked.

$$y(t)=A_{in}\cos \left(2\pi ft - 2\pi f\tau\right)=A_{in}\cos \left(2\pi f(t - \tau)\right) = x(t-\tau)$$

**Amplitude Response**

- $\text{Amplitude Response}$
  - $y(t)$ is constant for $f < W_c$.
  - $y(t)$ decreases as $f$ approaches $W_c$.

**Phase Response**

- $\phi(f) = -2\pi ft$
  - $\phi(f)$ decreases linearly with $f$.
  - $\phi(f) = 0$ at $f = 0$.
  - $\phi(f) = -\frac{1}{2\pi}$ at $f = \frac{1}{2\pi}$.
Example: Low-Pass Filter

- Simplest non-ideal circuit that provides low-pass filtering
  - Inputs at different frequencies are attenuated by different amounts
  - Inputs at different frequencies are delayed by different amounts

Amplitude Response

\[
A(f) = \frac{1}{(1+4\pi^2f^2)^{1/2}}
\]

Phase Response

\[
\phi(f) = \tan^{-1} 2\pi f
\]
Example: Bandpass Channel

- Some channels pass signals within a band that excludes low frequencies
  - Telephone modems, radio systems, …
Channel Distortion

\[ x(t) = \sum a_k \cos (2\pi f_k t + \theta_k) \]

- Let \( x(t) \) corresponds to a digital signal bearing data information
- How well does \( y(t) \) follow \( x(t) \)?

\[ y(t) = \sum A(f_k) a_k \cos (2\pi f_k t + \theta_k + \Phi(f_k)) \]

- Channel has two effects:
  - If amplitude response is not flat, then different frequency components of \( x(t) \) will be transferred by different amounts
  - If phase response is not flat, then different frequency components of \( x(t) \) will be delayed by different amounts
- In either case, the shape of \( x(t) \) is altered
Example: Amplitude Distortion

- Let \( x(t) \) input to ideal lowpass filter that has zero delay and \( W_c = 1.5 \) kHz, 2.5 kHz, or 4.5 kHz

\[
x(t) = -0.5 + \frac{4}{\pi} \sin\left(\frac{\pi}{4}\right)\cos(2\pi 1000t) + \frac{4}{\pi} \sin\left(\frac{2\pi}{4}\right)\cos(2\pi 2000t) + \frac{4}{\pi} \sin\left(\frac{3\pi}{4}\right)\cos(2\pi 3000t) + \ldots
\]

- \( W_c = 1.5 \) kHz passes only the first two terms
- \( W_c = 2.5 \) kHz passes the first three terms
- \( W_c = 4.5 \) kHz passes the first five terms
Amplitude Distortion

- (a) 1 Harmonic
- (b) 2 Harmonics
- (c) 4 Harmonics

As the channel bandwidth increases, the output of the channel resembles the input more closely.
Digital Binary Signal

Bit rate = 1 bit / T seconds

For a given communications medium:
- How do we increase transmission speed?
- How do we achieve reliable communications?
- Are there limits to speed and reliability?
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Fundamental Limits in Digital Transmission
Data Rate Limits in Digital Transmission

Max Data Rate [bps] Over a Channel?

- depends on three factors:
  - bandwidth available
  - # of levels in digital signal
  - quality of channel – level of noise

Nyquist Theorem

- defines theoretical max bit rate in noiseless channel [1924]
  - even perfect (noiseless) channels have limited capacity

Shannon Theorem

- Nyquist Theorem extended - defines theoretical max bit rate in noisy channel [1949]
  - if random noise is present, situation deteriorates rapidly!
Intersymbol Interference — the inevitable filtering effect of any practical channel will cause spreading of individual data symbols that pass through the channel

- this spreading causes part of symbol energy to overlap with neighbouring symbols causing intersymbol interference (ISI)

- ISI can significantly degrade the ability of the data detector to differentiate a current symbol from the diffused energy of the adjacent symbols

As the channel bandwidth $B$ increases, the width of the impulse response decreases

$⇒$ pulses can be input in the system more closely spaced, i.e. at a higher rate.
**Impulse Response** — response of a low-pass channel (of bandwidth $B$) to a narrow pulse $h(t)$, aka Nyquist pulse:

$$s(t) = \frac{\sin(2\pi B t)}{2\pi B t}$$

- zeros: where $\sin(2\pi B t) = 0 \Rightarrow t = \frac{1}{2B}$

What is the minimum pulse/bit duration time to avoid significant ISI?!
Example [system response to binary input 110]

Assume: channel bandwidth = max analog frequency passed = B [Hz].

New pulse is sent every $T_s$ sec $\Rightarrow$ data rate $= 1/T_s$ [bps] $= 2B$ [bps]

The combined signal has the correct values at $t = 0, 1, 2$.

$$r_{\text{max}} = \frac{1 \text{ pulse}}{T_s \text{ second}} = 2W = 2B \left[ \frac{\text{pulses}}{\text{second}} \right]$$

Maximum **signaling rate** that is achievable through an ideal low-pass channel.
Nyquist Law — max rate at which digital data can be transmitted over a communication channel of bandwidth $B$ [Hz] is

$$C_{\text{noiseless}} = 2 \cdot B \cdot \log_2 M \text{ [bps]}$$

- $M$ – number of discrete levels in digital signal
- $M \uparrow \Rightarrow C \uparrow$, however this places increased burden on receiver
  - instead of distinguishing one of two possible signals, now it must distinguish between $M$ possible signals
  - especially complex in the presence of noise

if spacing between levels becomes too small, noise signal can cause receiver to make wrong decision
Example [ multilevel digital transmission ]

2-level encoding: \( C=2B \) [bps]  
one pulse – one bit

4-level encoding: \( C=2 \times 2=4B \) [bps]  
100110100011010010  
\( \Rightarrow \)  
one pulse – two bits

8-level encoding: \( C=2 \times 3=6B \) [bps]  
100110100011010010  
\( \Rightarrow \)  
one pulse – three bits
**Data Rate Limits: Shannon Law**

**Shannon Law** — maximum transmission rate over a channel with bandwidth $B$, with Gaussian distributed noise, and with signal-to-noise ratio $\text{SNR}=S/N$, is

$$C_{\text{noisy}} = B \cdot \log_2 (1 + \text{SNR}) \text{ [bps]}$$

- **theoretical limit** — there are numerous impairments in every real channel besides those taken into account in Shannon's Law (e.g. attenuation, delay distortion, or impulse noise)

- **no indication of levels** — no matter how many levels we use, we cannot achieve a data rate higher than the capacity of the channel

- **in practice** we need to use both methods (Nyquist & Shannon) to find what data rate and signal levels are appropriate for each particular channel:

The Shannon capacity gives us the upper limit!  
The Nyquist formula tells us how many levels we need!
Data Rate Limits

Example  [ data rate over telephone line ]

What is the theoretical highest bit rate of a regular telephone line?
A telephone line normally has a bandwidth of 3000 Hz (300 Hz to 3300 Hz). The signal-to-noise ratio is usually 35 dB (3162) on up-link channel (user-to-network).

Solution:

We can calculate the theoretical highest bit rate of a regular telephone line as

\[ C = B \log_2 (1 + \text{SNR}) = \]
\[ = 3000 \log_2 (1 + 3162) = \]
\[ = 3000 \log_2 (3163) \]

\[ C = 3000 \times 11.62 = 34,860 \text{ bps} \]
Example  [data rate / number of levels]

We have a channel with a 1 MHz bandwidth. The SNR for this channel is 63; what is the appropriate bit rate and number of signal level?

**Solution:**

First use Shannon formula to find the upper limit on the channel’s data-rate

\[ C = B \log_2 (1 + SNR) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 (64) = 6 \text{ Mbps} \]

Although the Shannon formula gives us 6 Mbps, this is the upper limit. For better performance choose something lower, e.g. 4 Mbps.

Then use the Nyquist formula to find the number of signal levels.

\[ C = 2 \cdot B \cdot \log_2 M \text{ [bps]} \]

4 Mbps = 2 × 1 MHz × log₂ L  \[\Rightarrow\] L = 4