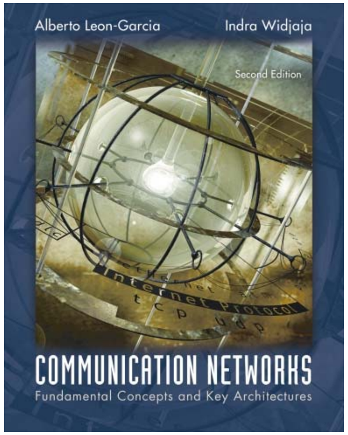
Chapter 3
Digital Transmission Fundamentals



Characterization of Communication Channels
Fundamental Limits in Digital Transmission

CSE 3213, Winter 2010
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Chapter 3
Digital Transmission Fundamentals

Characterization of Communication Channels

(Review)

Communications Channels

- A *physical medium* is an inherent part of a communications system
  - Copper wires, radio medium, or optical fiber
- Communications system includes electronic or optical devices that are part of the path followed by a signal
  - Equalizers, amplifiers, signal conditioners
- By *communication channel* we refer to the combined end-to-end physical medium and attached devices
- Sometimes we use the term *filter* to refer to a channel especially in the context of a specific mathematical model for the channel

Communications Channels

Signal Bandwidth

- In order to transfer data faster, a signal has to vary more quickly.

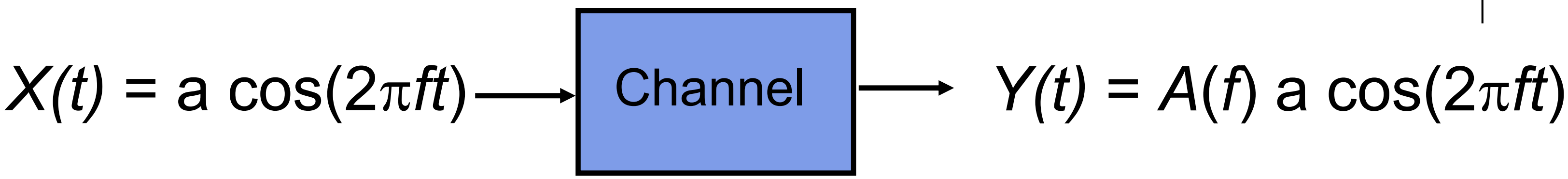
Channel Bandwidth

- A channel or medium has an inherent limit on how fast the signals it passes can vary
- *Limits how tightly input pulses can be packed*

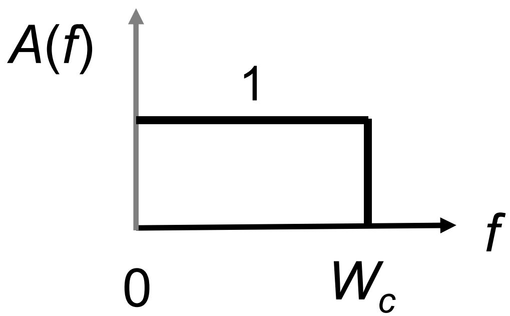
Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals
- *Limits accuracy of measurements on received signal*

Bandwidth of a Channel



- If input is sinusoid of frequency f, then
  - output is a sinusoid of same frequency f
  - Output is attenuated by an amount A(f) that depends on f
  - A(f)≈1, then input signal passes readily
  - A(f)≈0, then input signal is blocked
- Bandwidth W_c is range of frequencies passed by channel



Ideal low-pass channel

How good is a channel?

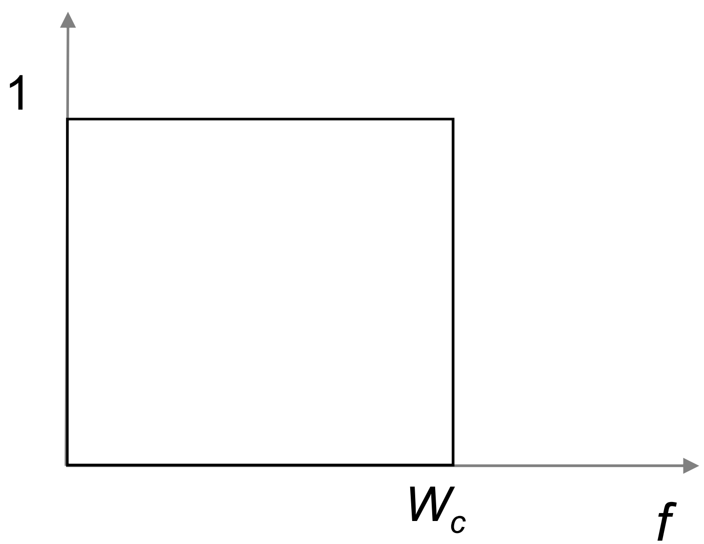
- Performance: What is the maximum reliable transmission speed?
  - Speed: Bit rate, *R* bps
  - Reliability: Bit error rate, BER=10^{-k}
  - Focus of this section
- Cost: What is the cost of alternatives at a given level of performance?
  - Wired vs. wireless?
  - Electronic vs. optical?
  - Standard A vs. standard B?

Ideal Low-Pass Filter

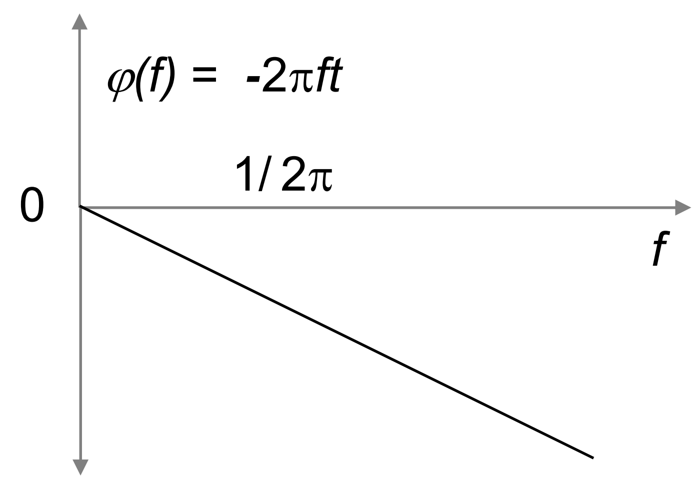
- Ideal filter: all sinusoids with frequency $f<W_c$ are passed without attenuation and delayed by τ seconds; sinusoids at other frequencies are blocked

$$y(t)=A_{in}\cos(2\pi ft - 2\pi f\tau) = A_{in}\cos(2\pi f(t-\tau)) = x(t-\tau)$$

Amplitude Response

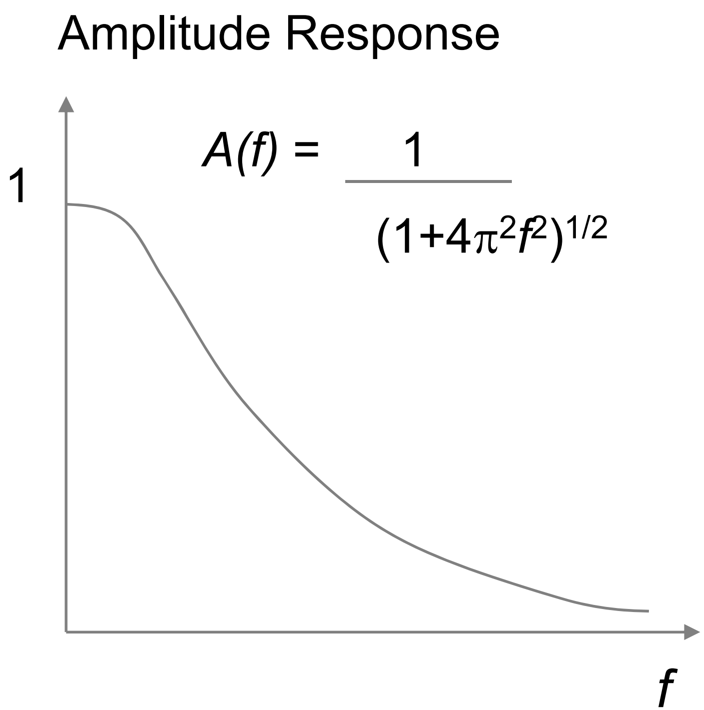


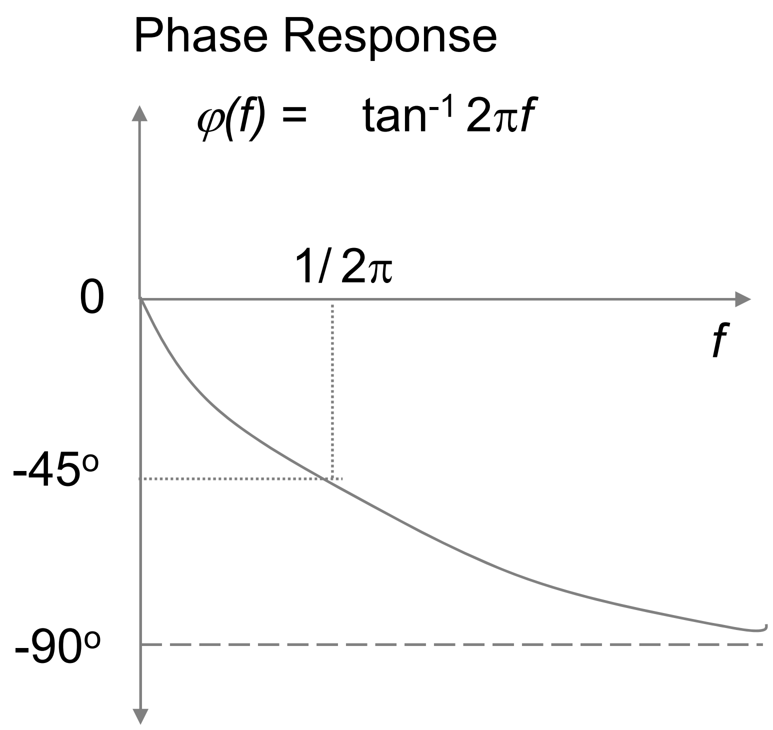
Phase Response



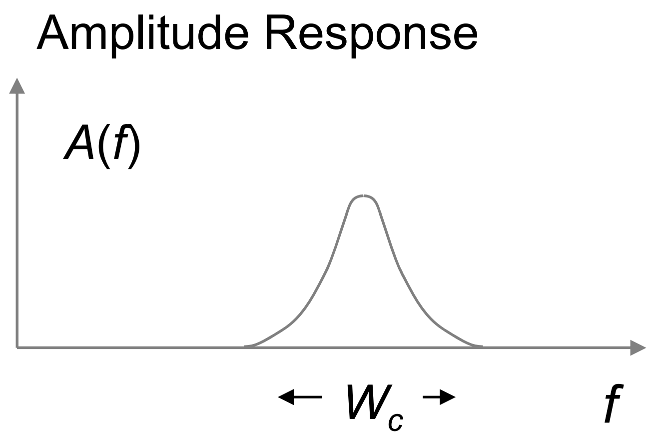
Example: Low-Pass Filter

- Simplest non-ideal circuit that provides low-pass filtering
  - Inputs at different frequencies are attenuated by different amounts
  - Inputs at different frequencies are delayed by different amounts





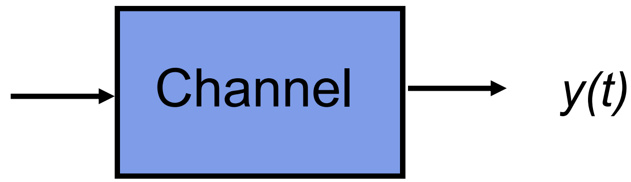
Example: Bandpass Channel



- Some channels pass signals within a band that excludes low frequencies
  - Telephone modems, radio systems, …

Channel Distortion

$$x(t) = \sum a_k \cos(2\pi f_k t + \theta_k)$$

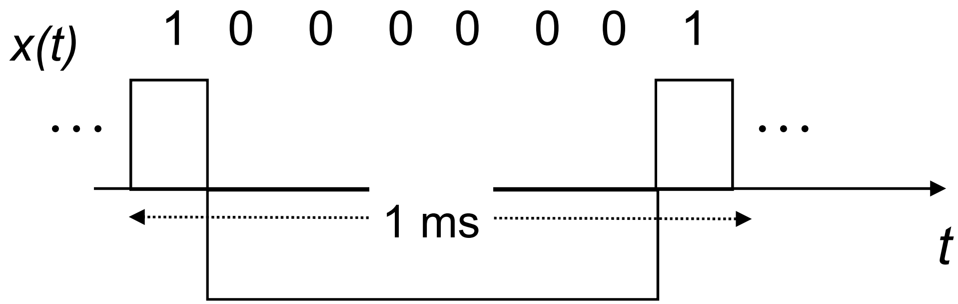


- Let *x(t)* corresponds to a digital signal bearing data information
- How well does *y(t)* follow *x(t)*?

$$y(t) = \sum A(f_k)\, a_k \cos(2\pi f_k t + \theta_k + \Phi(f_k))$$

- Channel has two effects:
  - If amplitude response is not flat, then different frequency components of *x(t)* will be transferred by different amounts
  - If phase response is not flat, then different frequency components of *x(t)* will be delayed by different amounts
- In either case, the shape of *x(t)* is altered

Example: Amplitude Distortion

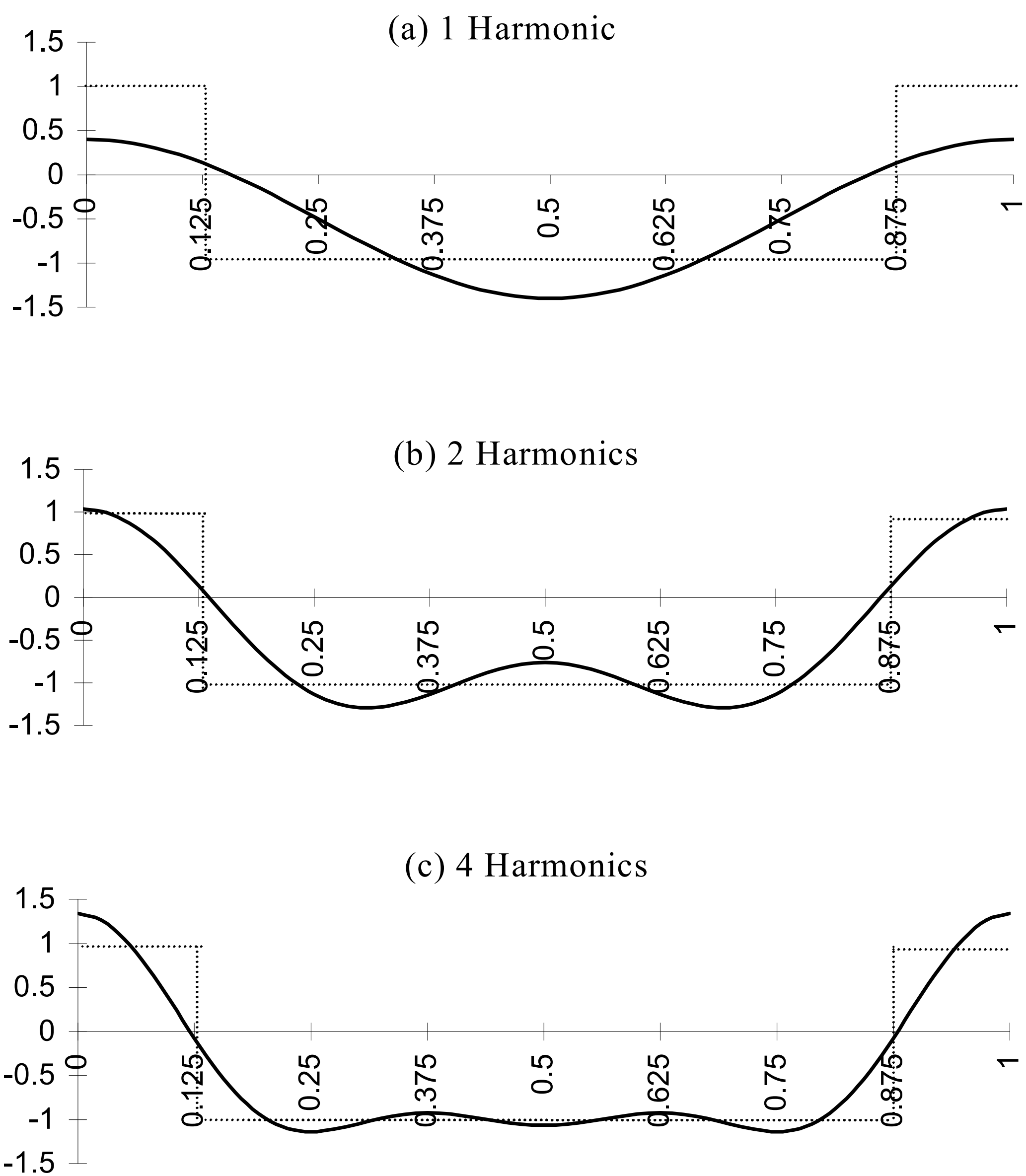


- Let *x(t)* input to ideal lowpass filter that has zero delay and W_c = 1.5 kHz, 2.5 kHz, or 4.5 kHz

$$x(t) = -0.5 + \frac{4}{\pi}\sin\left(\frac{\pi}{4}\right)\cos(2\pi 1000t) + \frac{4}{\pi}\sin\left(\frac{2\pi}{4}\right)\cos(2\pi 2000t) + \frac{4}{\pi}\sin\left(\frac{3\pi}{4}\right)\cos(2\pi 3000t) + \ldots$$

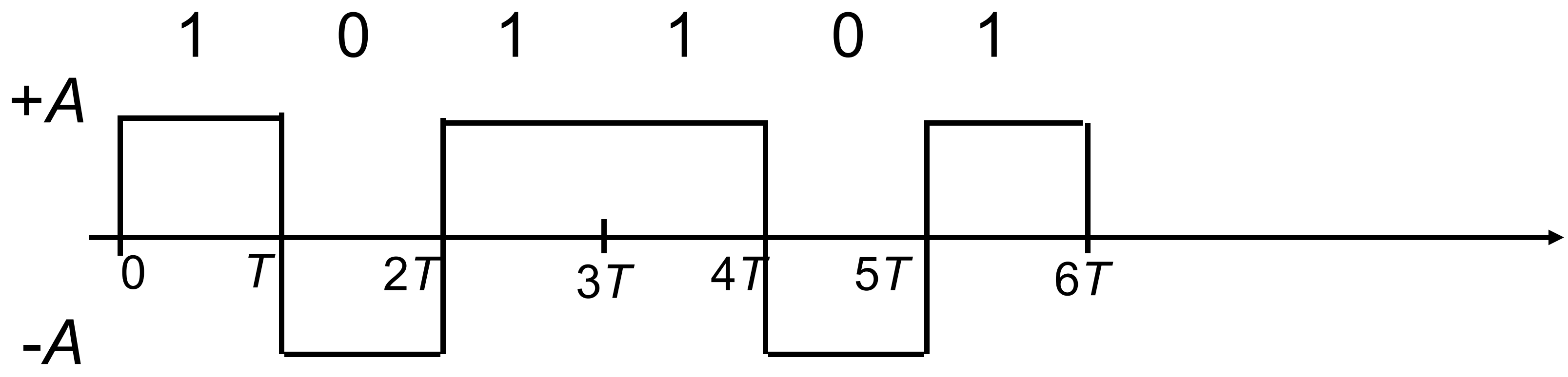
- W_c = 1.5 kHz passes only the first two terms
- W_c = 2.5 kHz passes the first three terms
- W_c = 4.5 kHz passes the first five terms

Amplitude Distortion



- As the channel bandwidth increases, the output of the channel resembles the input more closely

Digital Binary Signal



Bit rate = 1 bit / T seconds

For a given communications medium:

- How do we increase transmission speed?
- How do we achieve reliable communications?
- Are there limits to speed and reliability?

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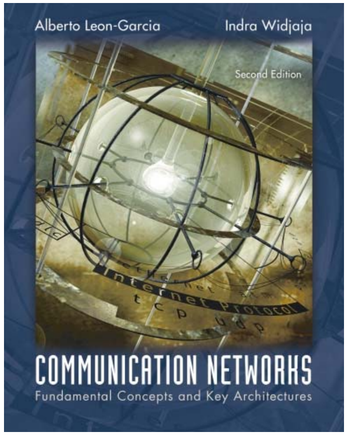


Fundamental Limits in Digital Transmission

Data Rate Limits in Digital Transmission

Max Data Rate [bps] Over a Channel? – depends on three factors:

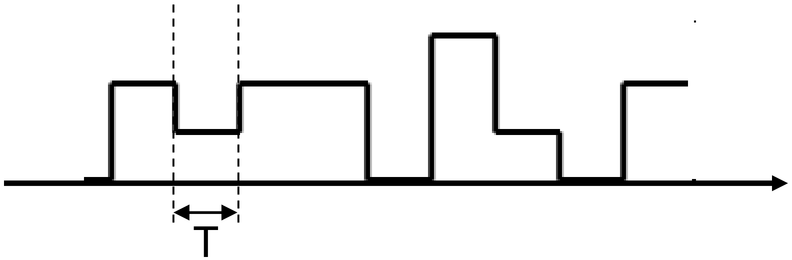
- bandwidth available
- # of levels in digital signal
- quality of channel – level of noise

Nyquist Theorem – defines theoretical max bit rate in noiseless channel [1924]

- even perfect (noiseless) channels have limited capacity

Shannon Theorem – Nyquist Theorem extended - defines theoretical max bit rate in noisy channel [1949]

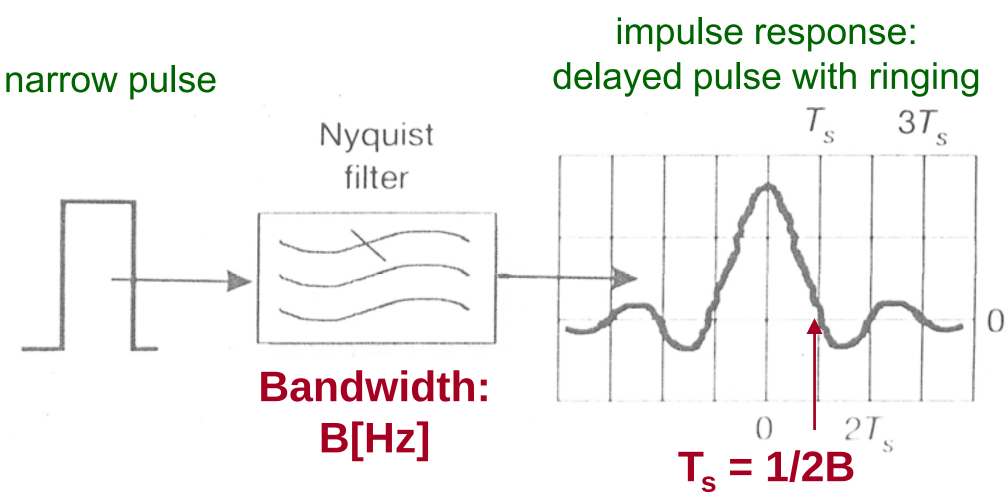
- if random noise is present, situation deteriorates rapidly!

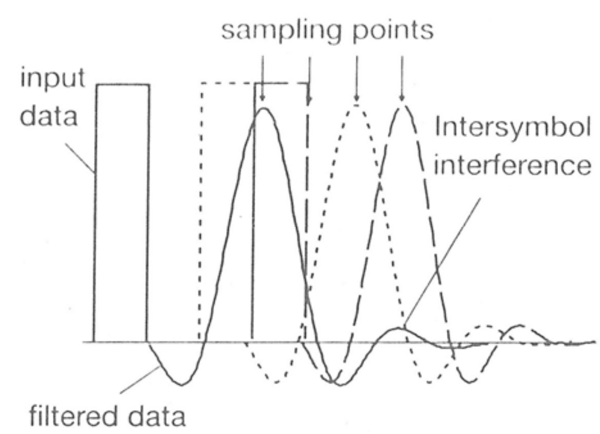


Data Rate Limits: Nyquist Theorem

Intersymbol Interference – the inevitable filtering effect of any practical channel will cause spreading of individual data symbols that pass through the channel

- this spreading causes part of symbol energy to overlap with neighbouring symbols causing intersymbol interference (ISI)
- ISI can significantly degrade the ability of the data detector to differentiate a current symbol from the diffused energy of the adjacent symbols





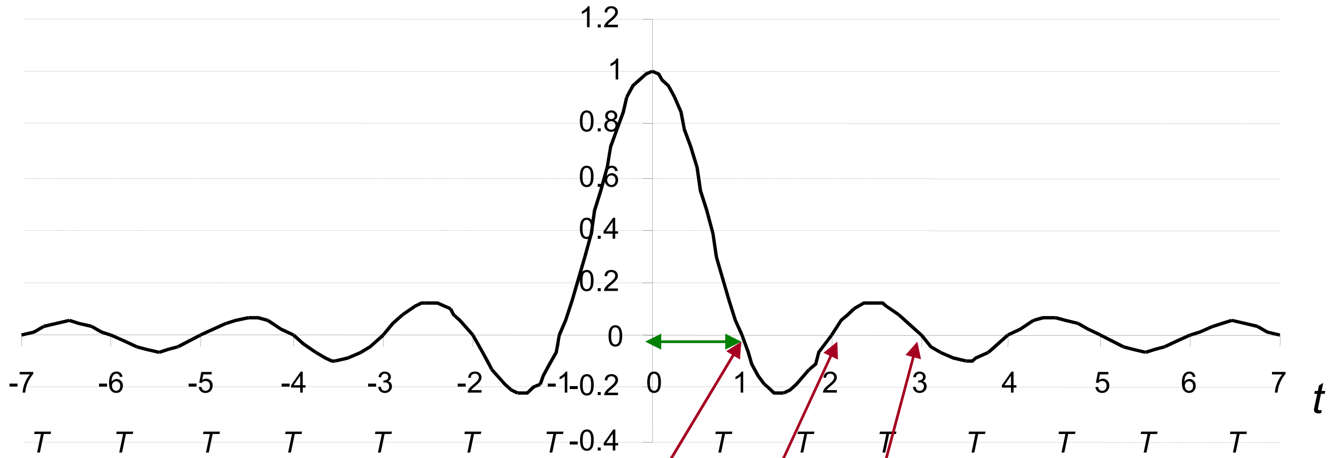
As the channel bandwidth B increases, the width of the impulse response decreases
⇒ pulses can be input in the system more closely spaced, i.e. at a higher rate.

Data Rate Limits: Nyquist Theorem

Impulse Response – response of a low-pass channel (of bandwidth B) to a narrow pulse h(t), aka Nyquist pulse:

$$s(t) = \frac{\sin(2\pi Bt)}{2\pi Bt}$$

- zeros: where $\sin(2\pi Bt)=0 \Rightarrow t = \frac{1}{2B}$

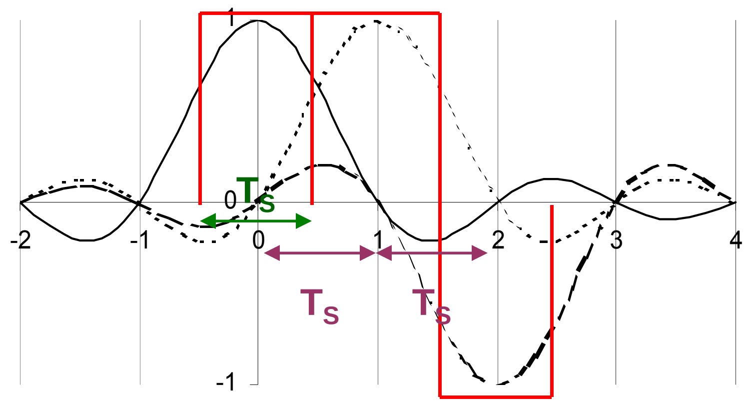


$T_S = \frac{1}{2B}$ $\frac{2}{2B}$ $\frac{3}{2B}$

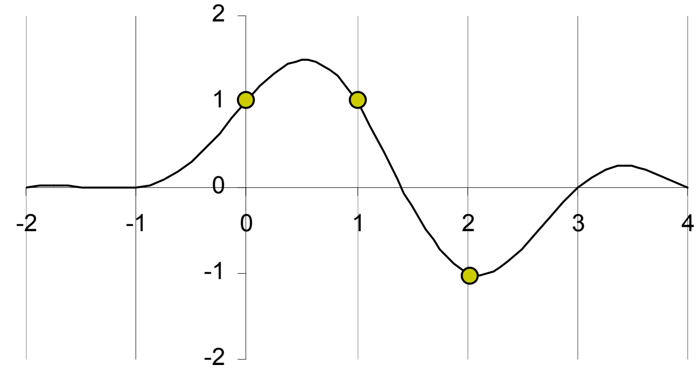
What is the minimum pulse/bit duration time to avoid significant ISI?!

Data Rate Limits: Nyquist Theorem

Example [system response to binary input 110]



three separate pulses



combined signal

Assume: channel bandwidth = max analog frequency passed = B [Hz].

New pulse is sent every T_S sec $\Rightarrow$ data rate = $1/T_S$ [bps] = 2B [bps]

The combined signal has the correct values at t = 0, 1, 2.

$$r_{max} = \frac{1}{T_S}\frac{\text{pulse}}{\text{second}} = 2W = 2B\left[\frac{\text{pulses}}{\text{second}}\right]$$

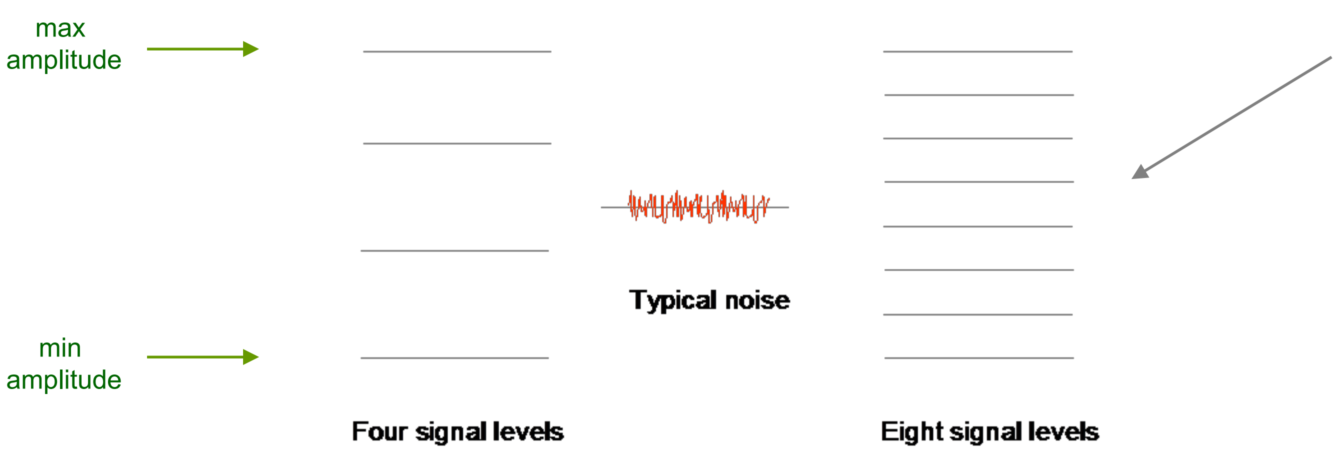
Maximum <u>signaling rate</u> that is achievable through an ideal low-pass channel.

Data Rate Limits: Nyquist Theorem

Nyquist Law – **max rate at which digital data can be transmitted over a communication channel of bandwidth B [Hz] is**

$$C_{noiseless} = 2 \cdot B \cdot \log_2 M \ \text{[bps]}$$

- **M – number of discrete levels in digital signal**
- **$M \uparrow \Rightarrow C \uparrow$, however this places increased burden on receiver**
  - **instead of distinguishing one of two possible signals, now it must distinguish between M possible signals**
    - **especially complex in the presence of noise**



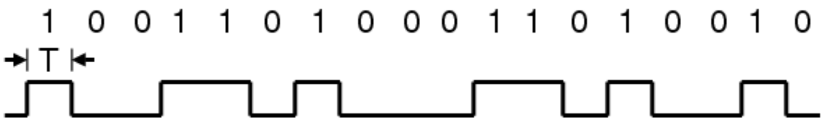
if spacing between levels becomes too small, noise signal can cause receiver to make wrong decision

Data Rate Limits: Nyquist Theorem

Example [multilevel digital transmission]

2-level encoding: C=2B [bps]

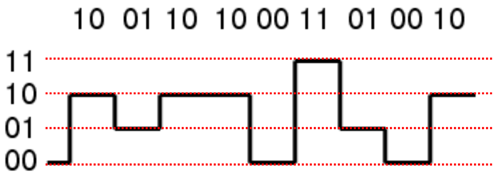
one pulse – one bit



4-level encoding: C=2*2=4B [bps]

one pulse – two bits

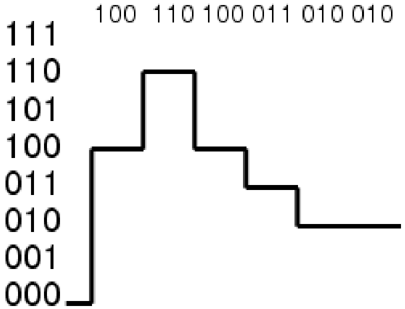
100110100011010010 ⇒



8-level encoding: C=2*3=6B [bps]

one pulse – three bits

100110100011010010 ⇒



Data Rate Limits: Shannon Law

Shannon Law – maximum transmission rate over a channel with bandwidth B, with Gaussian distributed noise, and with signal-to-noise ratio SNR=S/N, is

$$C_{noisy} = B \cdot \log_2(1+SNR) \;\; [bps]$$

- theoretical limit – there are numerous impairments in every real channel besides those taken into account in Shannon's Law (e.g. attenuation, delay distortion, or impulse noise)
- no indication of levels – no matter how many levels we use, we cannot achieve a data rate higher than the capacity of the channel
- in practice we need to use both methods (Nyquist & Shannon) to find what data rate and signal levels are appropriate for each particular channel:

The Shannon capacity gives us the upper limit!

The Nyquist formula tells us how many levels we need!

Data Rate Limits

Example [data rate over telephone line]

What is the theoretical highest bit rate of a regular telephone line?
A telephone line normally has a bandwidth of 3000 Hz (300 Hz to 3300 Hz). The signal-to-noise ratio is usually 35 dB (3162) on up-link channel (user-to-network).

Solution:

We can calculate the theoretical highest bit rate of a regular telephone line as

$$C = B \log_2 (1 + SNR) =$$
$$= 3000 \log_2 (1 + 3162) =$$
$$= 3000 \log_2 (3163)$$

$$C = 3000 \times 11.62 = 34{,}860 \text{ bps}$$

Data Rate Limits

Example [data rate / number of levels]

We have a channel with a 1 MHz bandwidth. The SNR for this channel is 63; what is the appropriate bit rate and number of signal level?

Solution:

First use Shannon formula to find the upper limit on the channel's data-rate

$$\mathbf{C = B \log_2 (1 + SNR) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 (64) = 6\ Mbps}$$

Although the Shannon formula gives us 6 Mbps, this is the upper limit. For better performance choose something lower, e.g. 4 Mbps.

Then use the Nyquist formula to find the number of signal levels.

$$C = 2 \cdot B \cdot \log_2 M \ \ [bps]$$

$$\mathbf{4\ Mbps = 2 \times 1\ MHz \times \log_2 \mathit{L} \ \Rightarrow \ \mathit{L} = 4}$$