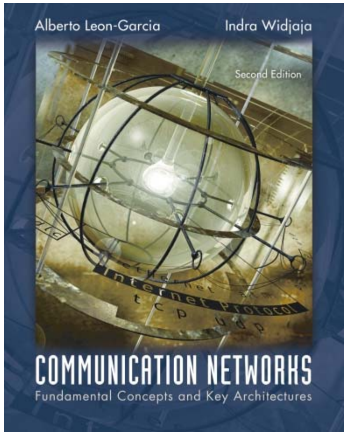
Chapter 3
Digital Transmission Fundamentals

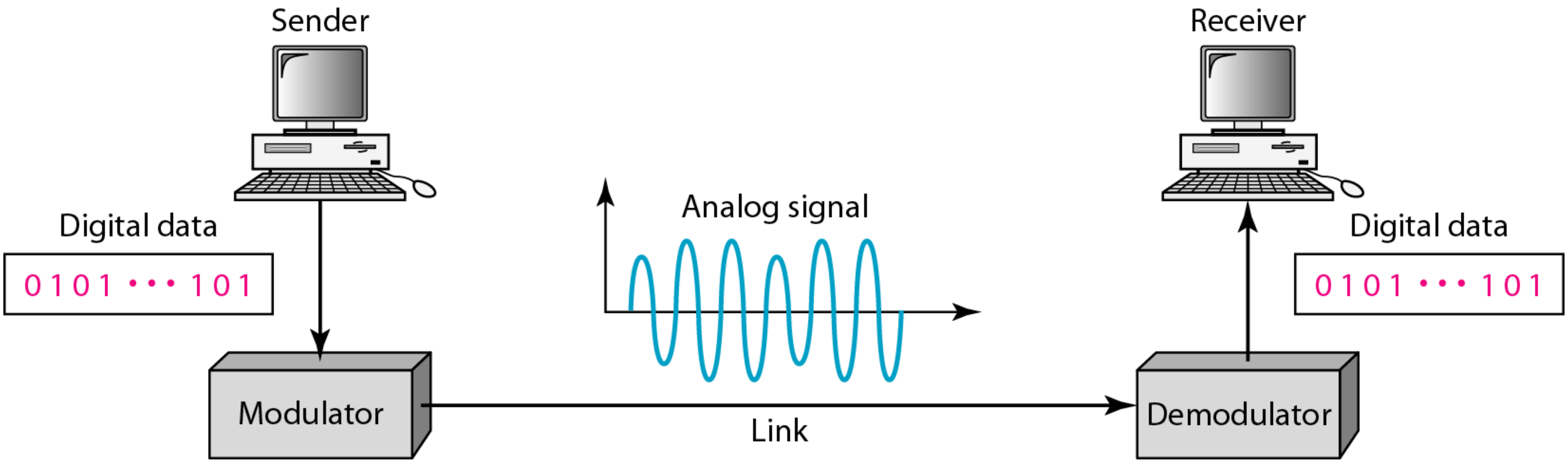


Modems and
Digital Modulation

CSE 3213, Winter 2010

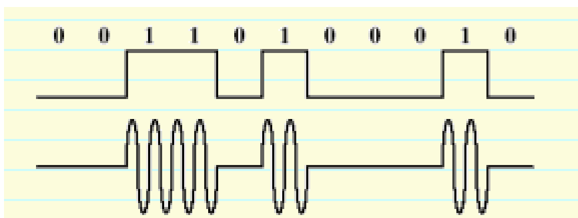
Instructor: Foroohar Foroozan

Modulation of Digital Data

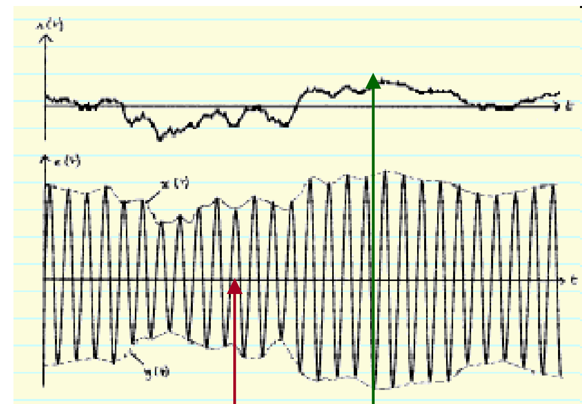


Modulation of Digital Data

Modulation – process of converting digital data or a low-pass analog signal to band-pass (higher-frequency) analog signal



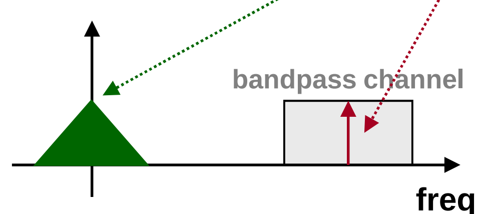
Digital-to-analog modulation.



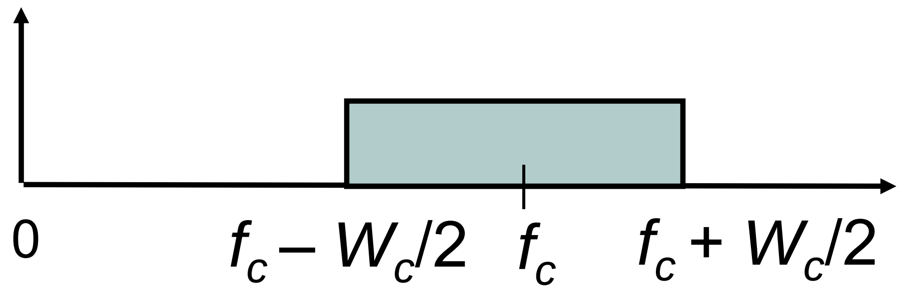
Analog-to-analog modulation.

– aka carrier frequency - modulated signal - high frequency signal that acts as a basis for the information signal

- information signal is called modulating signal



Bandpass Channels

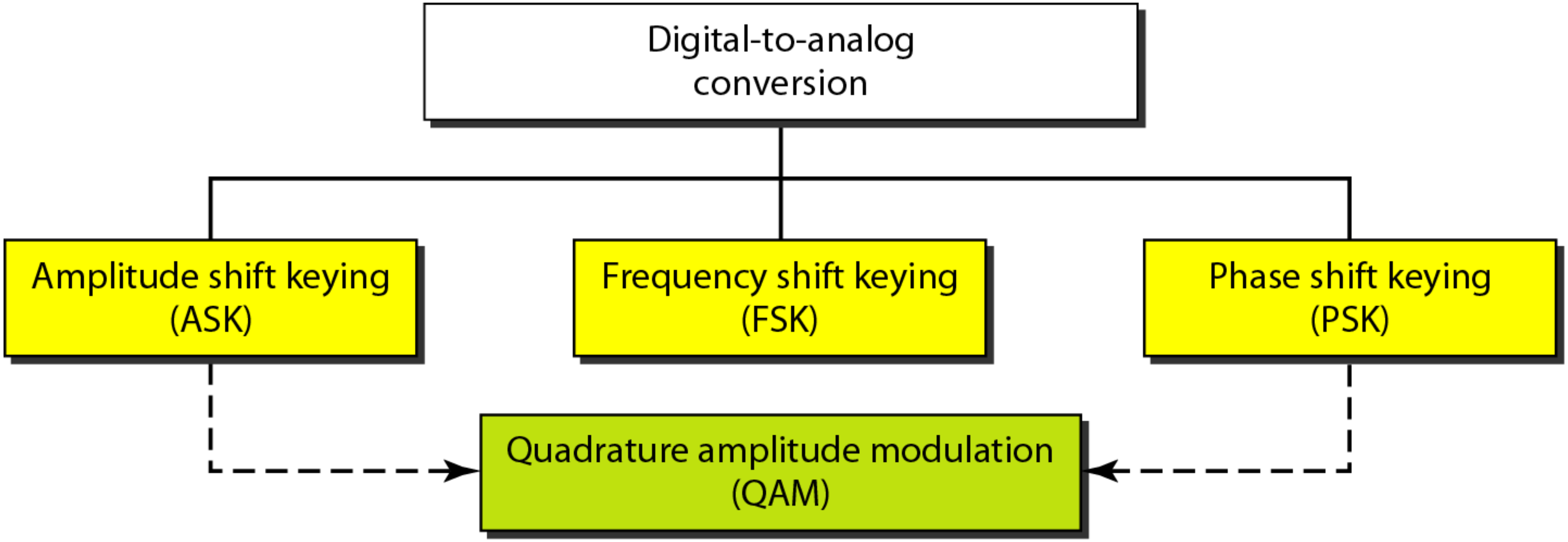


- Bandpass channels pass a range of frequencies around some center frequency f_c
  - Radio channels, telephone & DSL modems
- Digital modulators embed information into waveform with frequencies passed by bandpass channel
- Sinusoid of frequency f_c is centered in middle of bandpass channel
- Modulators embed information into a sinusoid

Digital To Analog Modulation

– process of changing one of the characteristics of an analog signal (typically a sinewave) based on the information in a digital signal

- sinewave is defined by three characteristics (amplitude, frequency, and phase) ⇒ digital data (binary 0 and 1) can be represented by varying any of the three
- application: transmission of digital data over telephone wire (modem)

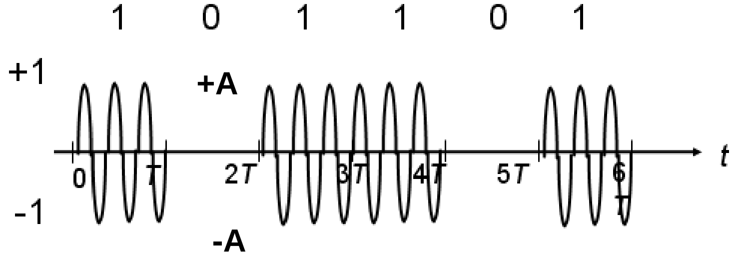


Amplitude Modulation

ASK – strength of the carrier signal is varied to represent binary 1 or 0

- both frequency and phase remain constant while the amplitude changes
- commonly, one of the amplitudes is zero

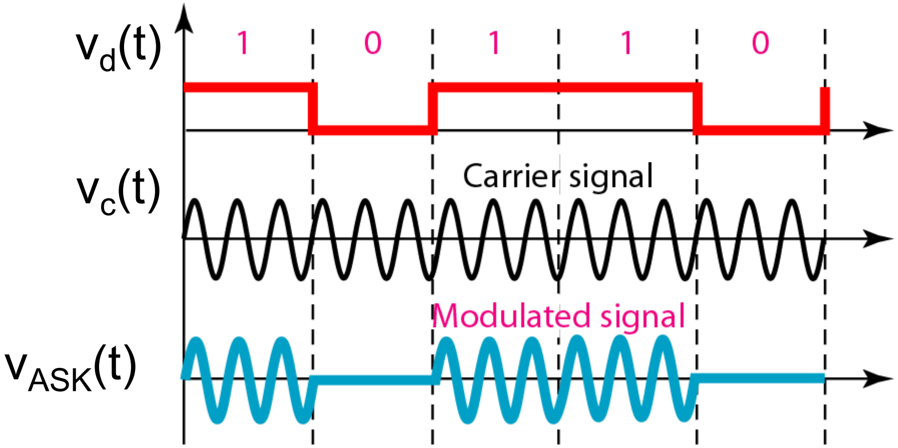
$$s(t)=\begin{cases}A_0\cos(2\pi f_c t), & \text{binary } 0\\ A_1\cos(2\pi f_c t), & \text{binary } 1\end{cases} = \begin{cases}0, & \text{binary } 0\\ A\cos(2\pi f_c t), & \text{binary } 1\end{cases}$$

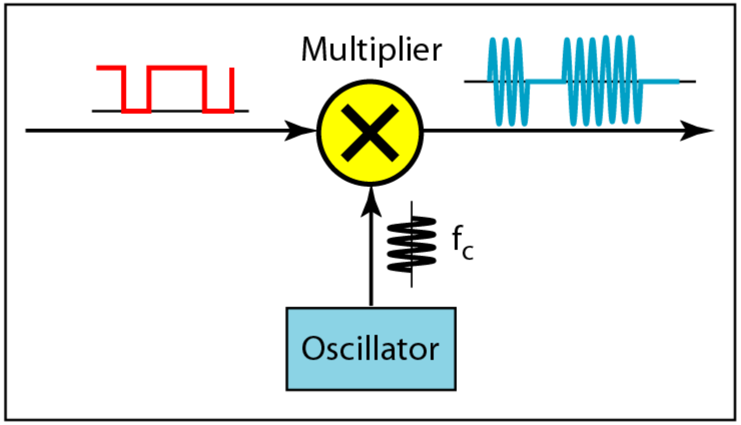


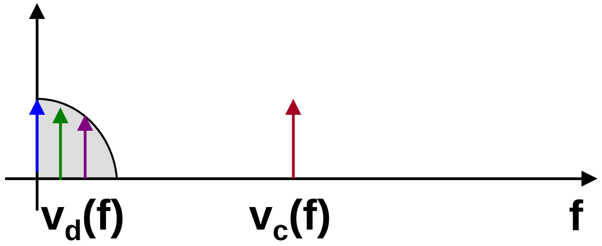
- demodulation: only the presence or absence of a sinusoid in a given time interval needs to be determined
- advantage: simplicity
- disadvantage: ASK is very susceptible to noise interference – noise usually (only) affects the amplitude, therefore ASK is the modulation technique most affected by noise
- application: ASK is used to transmit digital data over optical fiber

Amplitude Modulation (Cont.)

Example [ASK]







How does the frequency spectrum of $v_{ASK}(t)$ look like!?

Amplitude Modulation (Cont.)

ASK-Modulated Signal: Frequency Spectrum

$$\cos A \cdot \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

Carrier signal: $v_c(t) = \cos(2\pi f_c t) = \cos(\omega_c t)$ **, where $2\pi f_c = \omega_c$**

Digital signal: **(unipolar!!!)**

$$v_d(t) = A \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right]$$

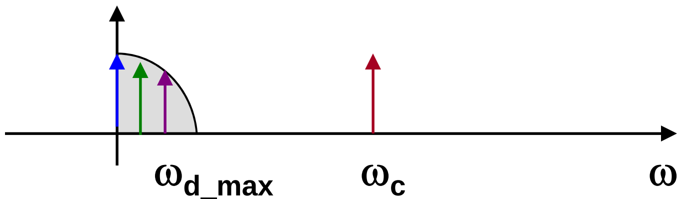
Modulated signal: $v_{ASK}(t) = v_c(t) \cdot v_d(t) =$

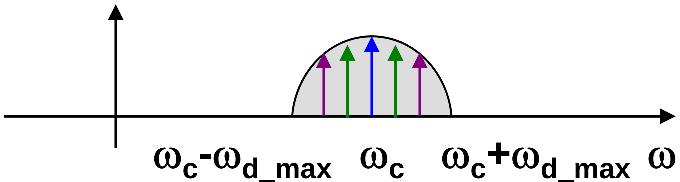
$$= \cos\omega_c t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] =$$

$$= \frac{1}{2}\cos\omega_c t + \frac{2}{\pi}\cos\omega_c t \cdot \cos\omega_0 t - \frac{2}{3\pi}\cos\omega_c t \cdot \cos 3\omega_0 t + \ldots =$$

$$= \frac{1}{2}\cos\omega_c t + \frac{1}{\pi}\left[\cos(\omega_c - \omega_0)t + \cos(\omega_c + \omega_0)t\right] -$$

$$- \frac{1}{3\pi}\left[\cos(\omega_c - 3\omega_0)t + \cos(\omega_c + 3\omega_0)t\right] + \ldots$$



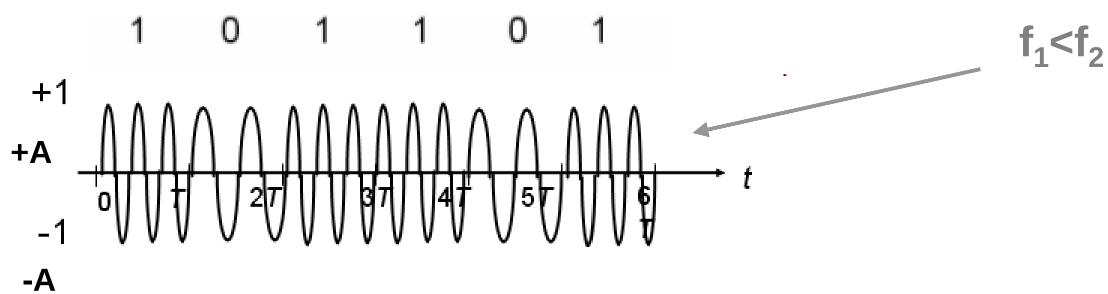


Frequency Modulation

FSK – frequency of the carrier signal is varied to represent binary 1 or 0

- both peak amplitude and phase remain constant during each bit interval

$$s(t) = \begin{cases} A\cos(2\pi f_1 t), & \text{binary } 0 \\ A\cos(2\pi f_2 t), & \text{binary } 1 \end{cases}$$



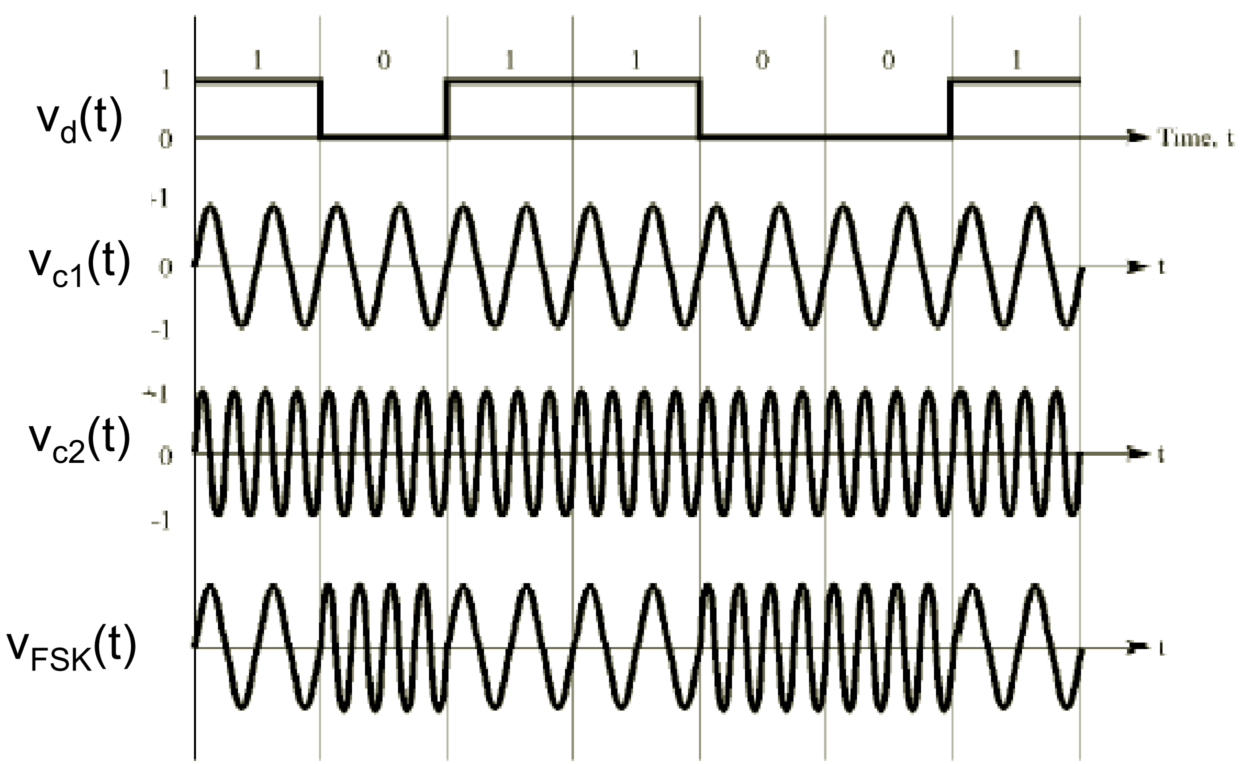
- demodulation: demodulator must be able to determine which of two possible frequencies is present at a given time

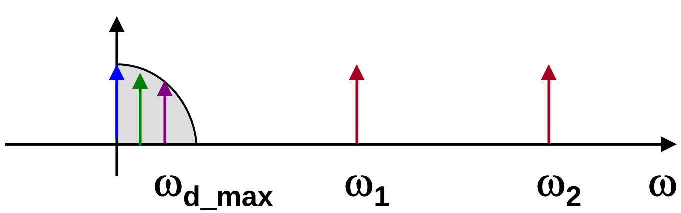
- advantage: FSK is less susceptible to errors than ASK – receiver is looking for specific frequency changes over a number of intervals, so voltage (noise) spikes can be ignored

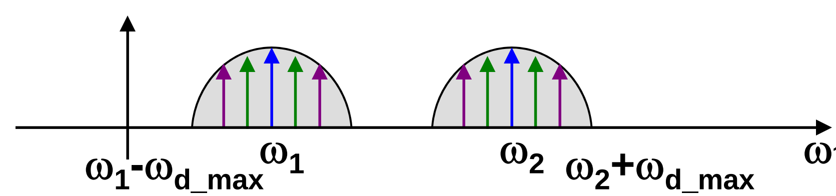
- disadvantage: FSK spectrum is 2 x ASK spectrum
- application: over voice lines, in high-frequency radio transmission, etc.

Frequency Modulation (Cont.)

Example [FSK]







Frequency Modulation (Cont.)

FSK-Modulated Signal: Frequency Spectrum

Digital signal: $v_d(t)$ **- modulated with ω_1, and**

$v_d'(t) = 1 - v_d(t)$ **- modulated with ω_2**

Modulated signal:

$$v_{FSK}(t) = \cos\omega_1 t \cdot v_d(t) + \cos\omega_2 t \cdot (1 - v_d(t)) =$$

$$= \cos\omega_1 t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] +$$

$$+ \cos\omega_2 t \cdot \left[\frac{1}{2} - \frac{2}{\pi}\cos\omega_0 t + \frac{2}{3\pi}\cos 3\omega_0 t - \frac{2}{5\pi}\cos 5\omega_0 t - \ldots\right] =$$

$$= \ldots$$

$$= \frac{1}{2}\cos\omega_1 t + \frac{1}{\pi}\left[\cos(\omega_1 - \omega_0)t + \cos(\omega_1 + \omega_0)t\right] -$$

$$- \frac{1}{3\pi}\left[\cos(\omega_1 - 3\omega_0)t + \cos(\omega_1 + 3\omega_0)t\right] + \ldots +$$

$$\frac{1}{2}\cos\omega_2 t - \frac{1}{\pi}\left[\cos(\omega_2 - \omega_0)t + \cos(\omega_2 + \omega_0)t\right] -$$

$$+ \frac{1}{3\pi}\left[\cos(\omega_2 - 3\omega_0)t + \cos(\omega_2 + 3\omega_0)t\right] + \ldots +$$

Phase Modulation

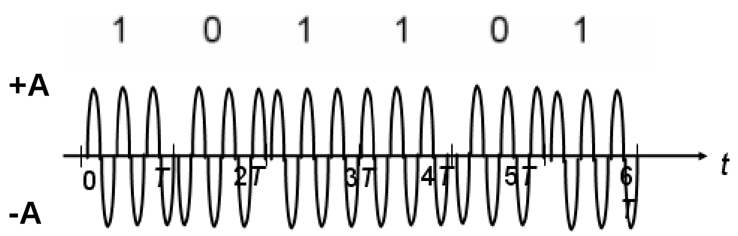
PSK – phase of the carrier signal is varied to represent binary 1 or 0

- peak amplitude and frequency remain constant during each bit interval
- example: binary 1 is represented with a phase of 0º, while binary 0 is represented with a phase of 180º=πrad ⇒ PSK is equivalent to multiplying the carrier signal by +1 when the information is 1, and by -1 when the information is 0

2-PSK, or Binary PSK, since only 2 different phases are used.

$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary 1} \\ A\cos(2\pi f_c t + \pi), & \text{binary 0} \end{cases}$$

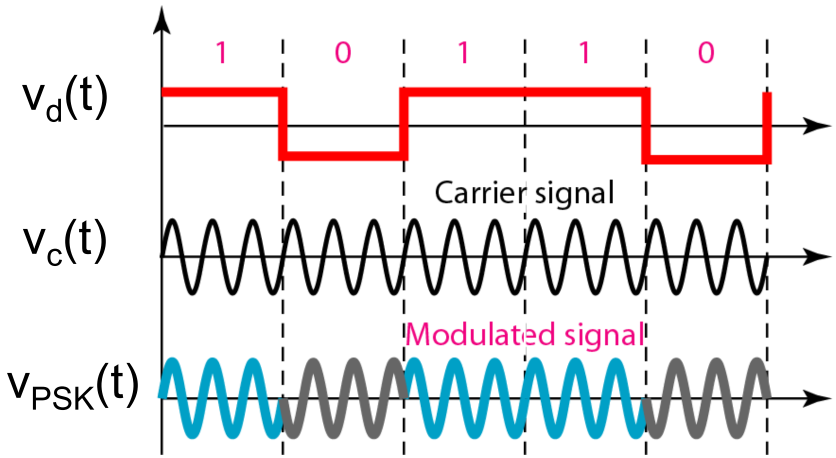
$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary 1} \\ -A\cos(2\pi f_c t), & \text{binary 0} \end{cases}$$

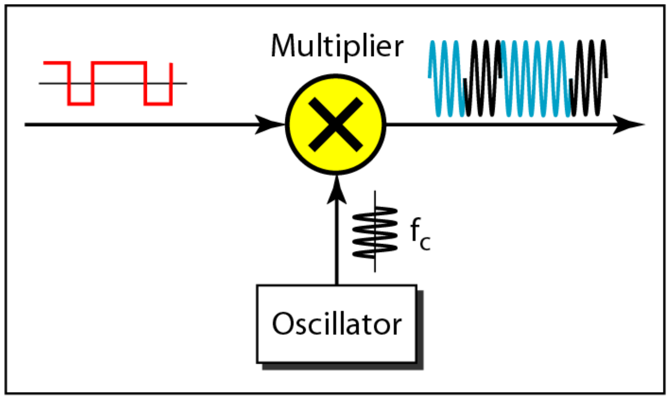


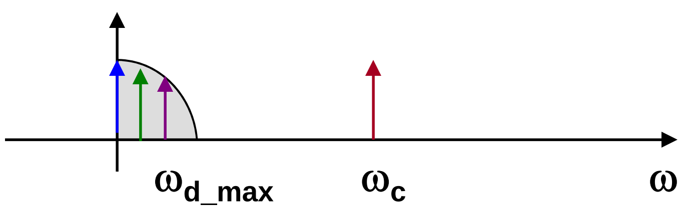
- demodulation: demodulator must be able to determine the phase of received sinusoid with respect to some reference phase
- advantage:
  - PSK is less susceptible to errors than ASK, while it requires/occupies the same bandwidth as ASK
  - more efficient use of bandwidth (higher data-rate) are possible, compared to FSK !!!
- disadvantage: more complex signal detection / recovery process, than in ASK and FSK

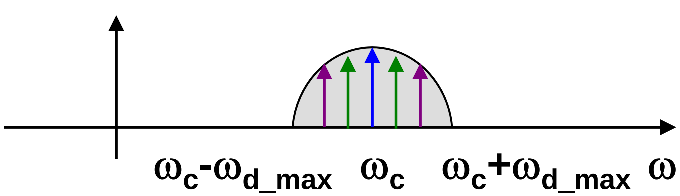
Phase Modulation (Cont.)

Example **[PSK]**









Phase Modulation (Cont.)

PSK Detection / Recovery

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

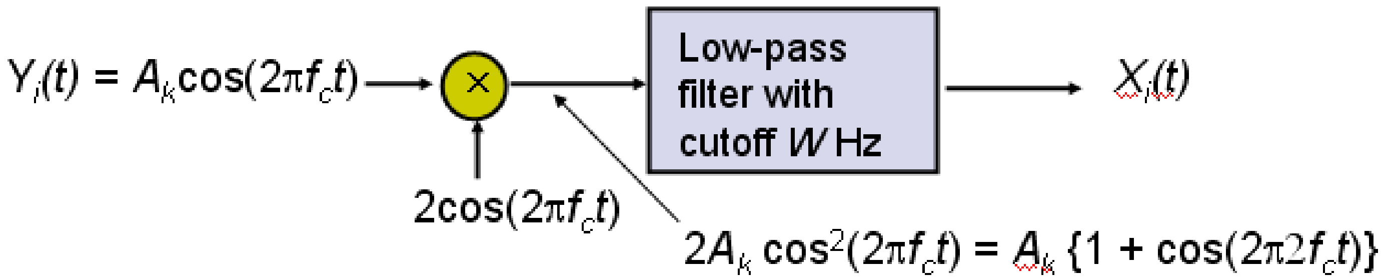
– multiply the received / modulated signal $\pm A\cos(2\pi f_c t)$ by $2*\cos(2\pi f_c t)$

- resulting signal

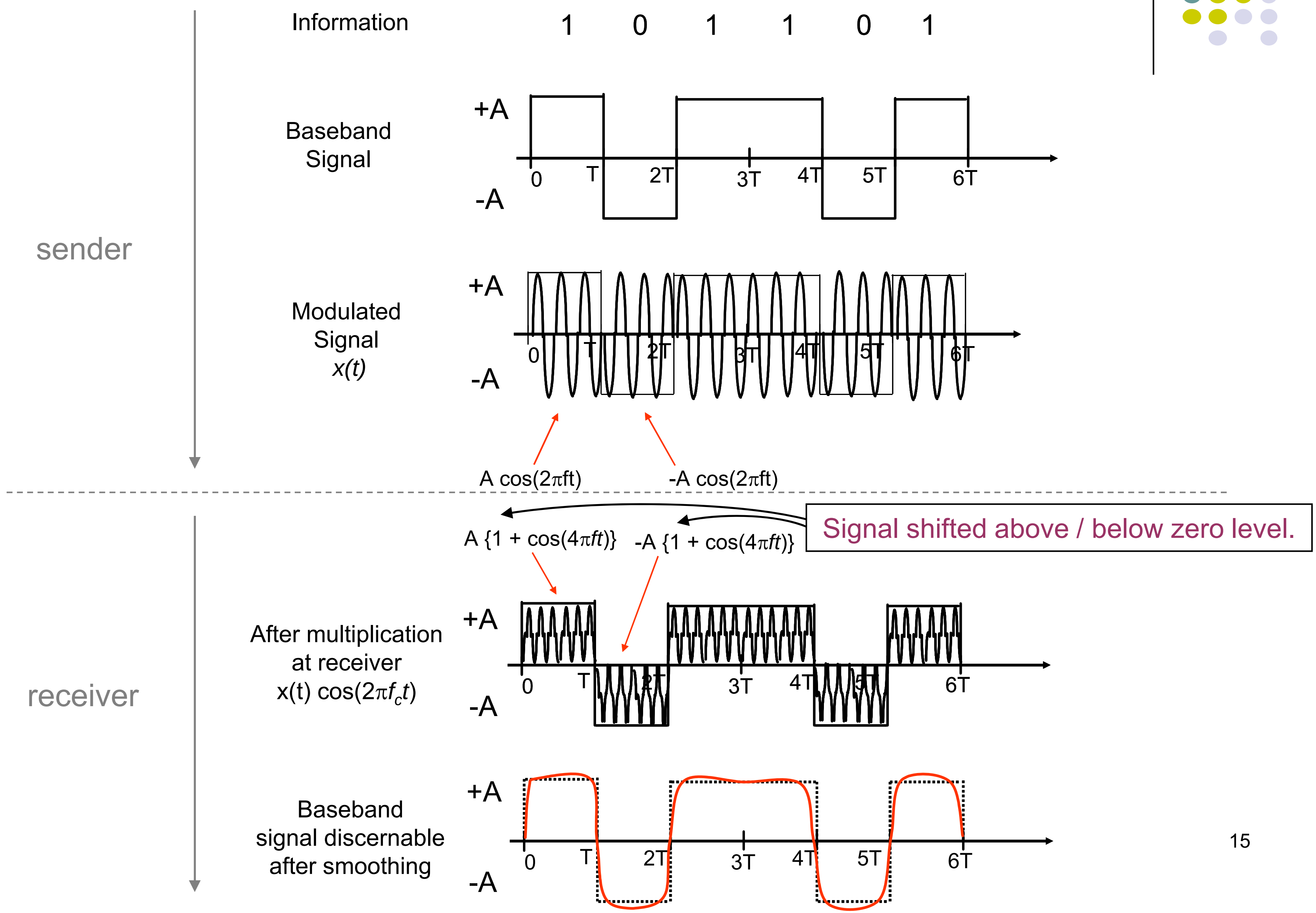
$$2A\cos^2(2\pi f_c t) = A[1 + \cos(4\pi f_c t)], \quad \text{binary } 1$$

$$-2A\cos^2(2\pi f_c t) = -A[1 + \cos(4\pi f_c t)], \quad \text{binary } 0$$

- by removing the oscillatory part with a low-pass filter, the original baseband signal (i.e. the original binary sequence) can be easily determined



Phase Modulation (Cont.)



Signaling rate and Transmission Bandwidth

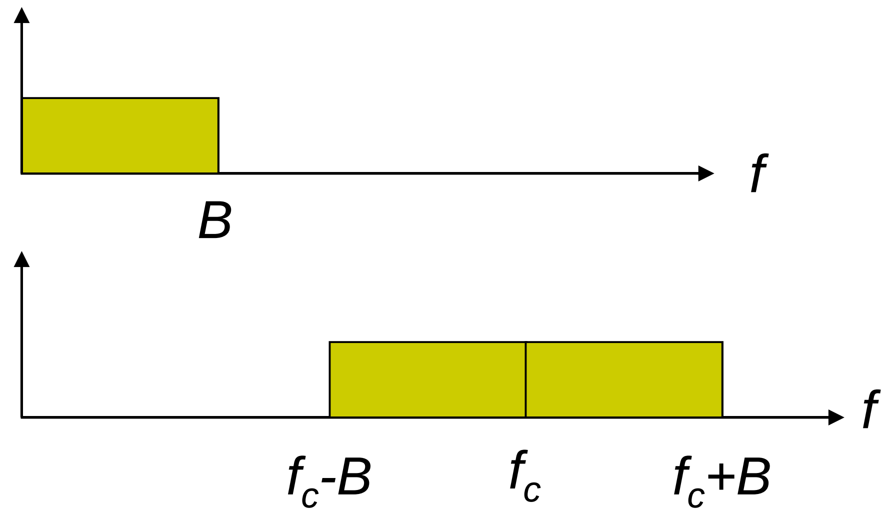
- Fact from modulation theory:

If

Baseband signal *x(t)* with bandwidth *B* Hz

then

Modulated signal $x(t)\cos(2\pi f_c t)$ has bandwidth 2*B* Hz



- If bandpass channel has bandwidth W_c Hz,
  - Then baseband channel has $W_c/2$ Hz available, so
  - modulation system supports $W_c/2 \times 2 = W_c$ pulses/second

Recall **baseband transmission system of bandwidth W_c [Hz] can theoretically support 2 W_c pulses/sec**

Phase Modulation (Cont.)

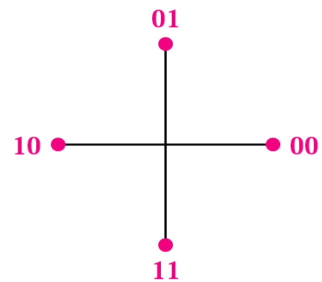
QPSK = 4-PSK

– PSK that uses phase shifts of 90º=π/2 rad ⇒ 4 different signals are generated, each representing 2 bits

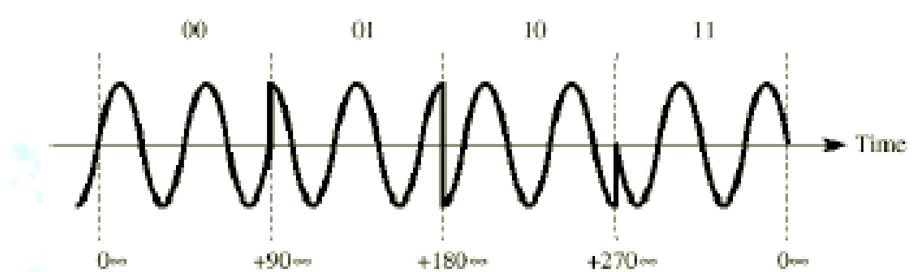
$$s(t) = \begin{cases} A\cos(2\pi f_c t), & \text{binary } 00 \\ A\cos(2\pi f_c t + \frac{\pi}{2}), & \text{binary } 01 \\ A\cos(2\pi f_c t + \pi), & \text{binary } 10 \\ A\cos(2\pi f_c t + \frac{3\pi}{2}), & \text{binary } 11 \end{cases}$$

Dibit	Phase
00	0
01	90
10	180
11	270

Dibit
(2 bits)



Constellation diagram



- advantage: higher data rate than in PSK (2 bits per bit interval), while bandwidth occupancy remains the same
- 4-PSK can easily be extended to 8-PSK, i.e. n-PSK
- however, higher rate PSK schemes are limited by the ability of equipment to distinguish small differences in phase

Quadrature Amplitude Modulation (QAM)

– uses “two-dimensional” signalling

- original information stream is <u>split into two sequences</u> that consist of odd and even symbols, e.g. B_k and A_k

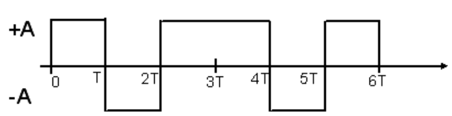
1 0 1 1 0 1 …

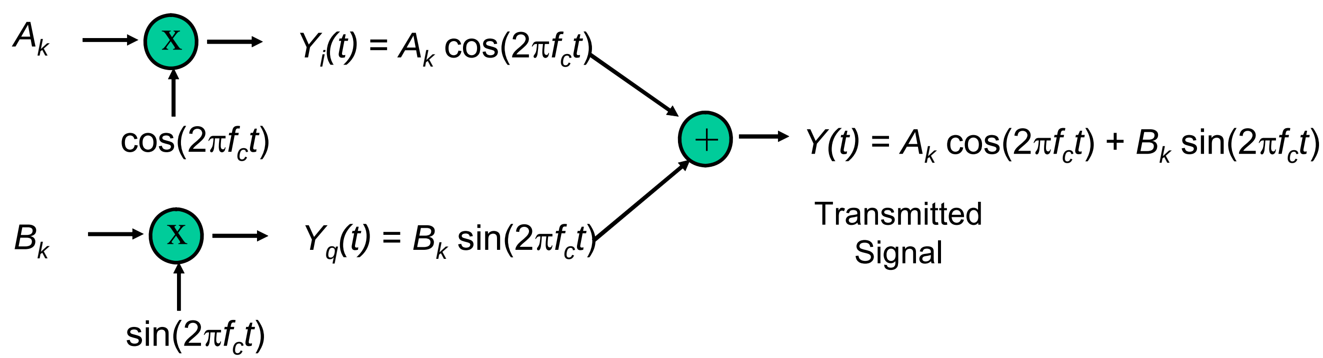
1 -1 1 1 -1 1 …

B_1 A_1 B_2 A_2 B_3 A_3 …

- A_k sequence (in-phase component) is modulated by $\cos(2\pi f_c t)$, while B_k sequence (quadrature-phase component) is modulated by $\sin(2\pi f_c t)$

- composite signal $A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t)$ is sent through the channel

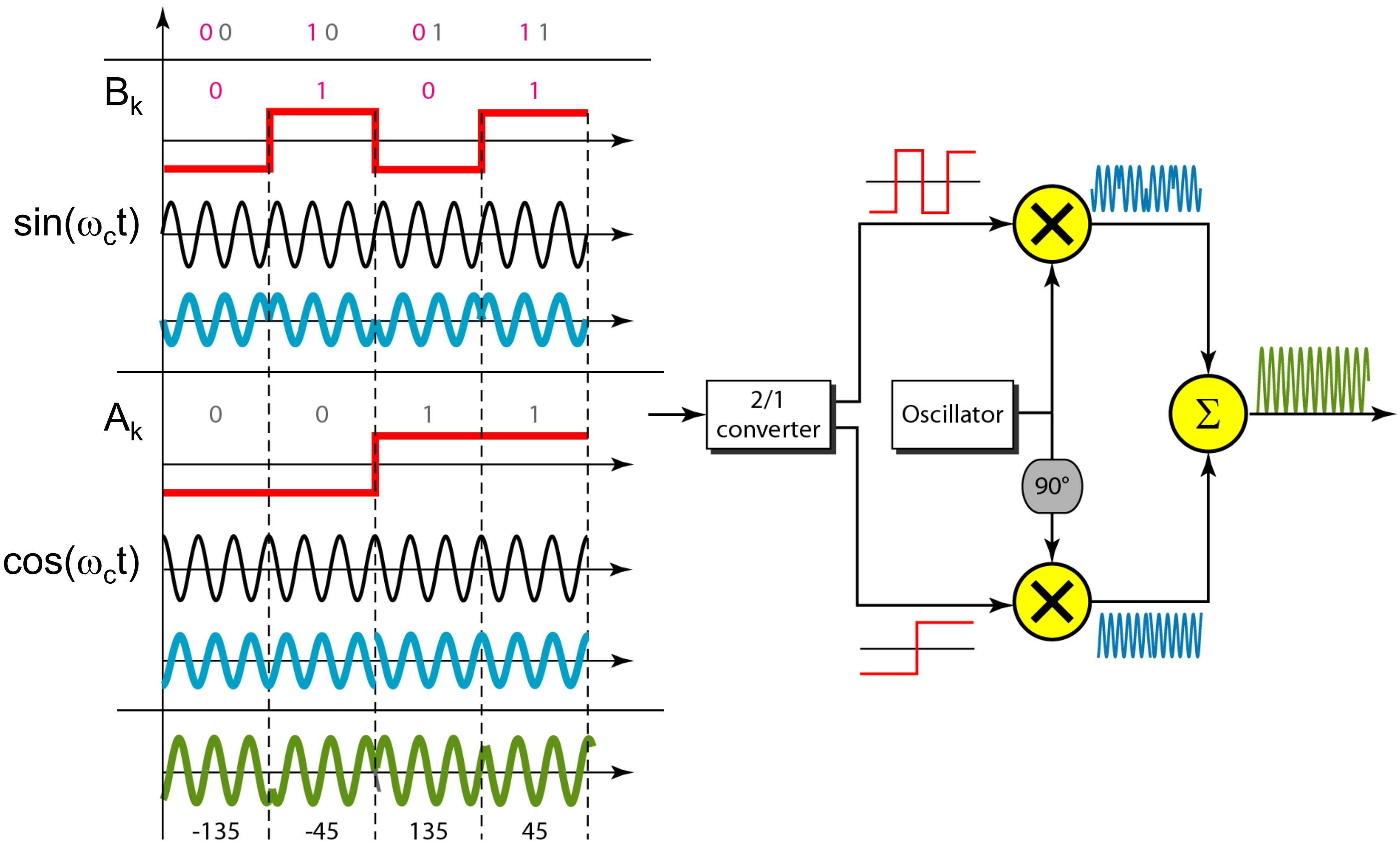




- advantage: data rate = 2 bits per bit-interval!

QAM (Cont.)

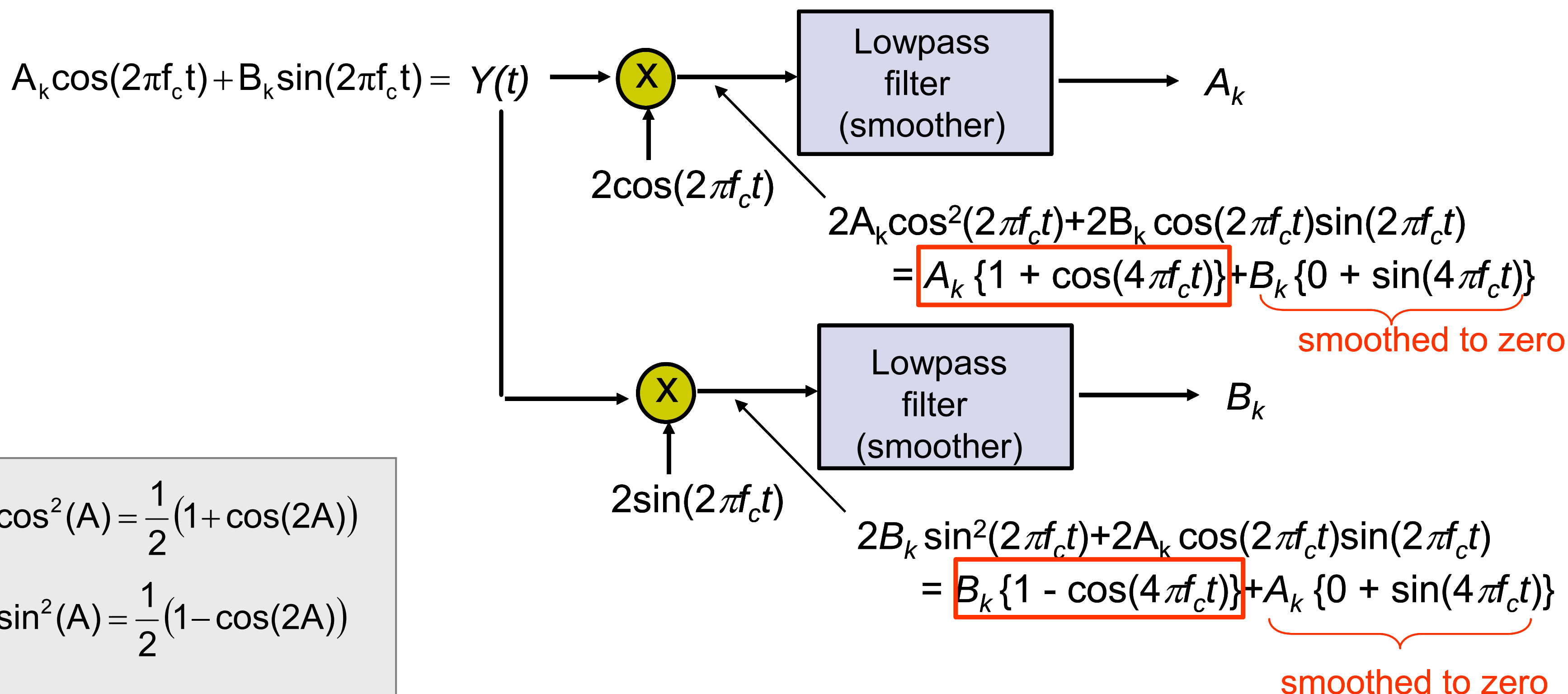
Example [QAM]



QAM (Cont.)

QAM Demodulation

- by multiplying Y(t) by $2\cdot\cos(2\pi f_c t)$ and then low-pass filtering the resultant signal, sequence A_k is obtained
- by multiplying Y(t) by $2\cdot\sin(2\pi f_c t)$ and then low-pass filtering the resultant signal, sequence B_k is obtained



$$\cos^2(A)=\frac{1}{2}(1+\cos(2A))$$

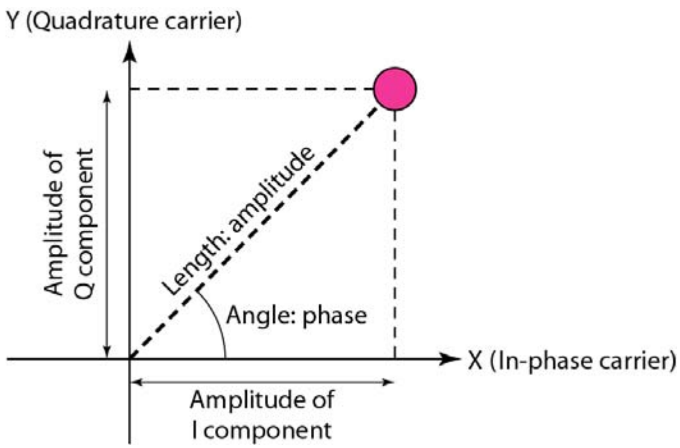
$$\sin^2(A)=\frac{1}{2}(1-\cos(2A))$$

$$\sin(2A)=2\sin(A)\cos(A)$$

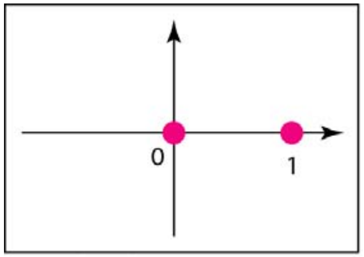
Signal Constellations

Constellation Diagram

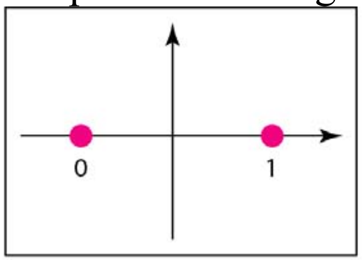
– used to represents possible symbols that may be selected by a given modulation scheme as points in 2-D plane



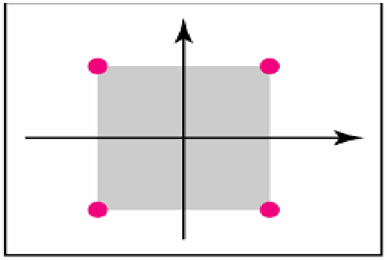
- X-axis is related to in-phase carrier: $\cos(\omega_c t)$
  - the projection of the point on the X-axis defines the peak amplitude of the in-phase component
- Y-axis is related to the quadrature carrier: $\sin(\omega_c t)$
  - the projection of the point on the Y-axis defines the peak amplitude of the quadrature component
- the length of the line that connects the point to the origin is the peak amplitude of the signal element (combination of X and Y components)
- the angle the line makes with the X-axis is the phase of the signal element



a. ASK (OOK)



b. BPSK



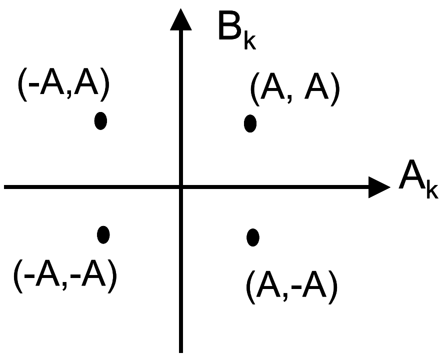
b. 4-QAM

QAM (Cont.)

QAM cont. – QAM can also be seen as a combination of ASK and PSK

$$Y(t) = A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t) = \left(A_k^{\;2} + B_k^{\;2}\right)^{\frac{1}{2}} \cos(2\pi f_c t + \tan^{-1}\frac{B_k}{A_k})$$

4-level QAM



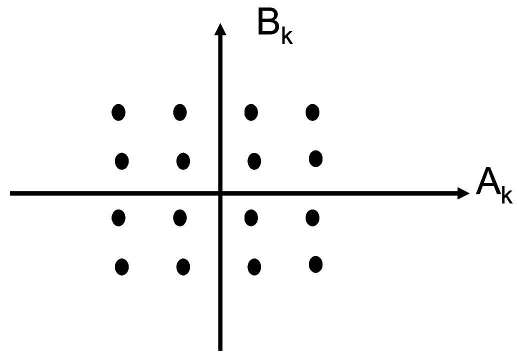
QAM (Cont.)

16-level QAM – the number of bits transmitted per T [sec] interval can be further increased by increasing the number of levels used

- in case of 16-level QAM, A_k and B_k individually can assume 4 different levels: -1, -1/3, 1/3, 1

- data rate: 4 bits/pulse $\Rightarrow$ 4W bits/second

$$Y(t) = A_k\cos(2\pi f_c t) + B_k\sin(2\pi f_c t) = \left(A_k^{\,2} + B_k^{\,2}\right)^{\frac{1}{2}}\cos(2\pi f_c t + \tan^{-1}\frac{B_k}{A_k})$$



A_k and B_k individually can take on 4 different values; the resultant signal can take on (only) 3 different values!!!

In QAM various combinations of amplitude and phase are employed to achieve higher digital data rates.

Amplitude changes are susceptible to noise $\Rightarrow$ the number of phase shifts used by a QAM system is always greater than the number of amplitude shifts.