CSE 4214 :: Problem Set 6

1. Let $\mathbf{H}$ represent a parity check matrix of a linear block code, where

$$\mathbf{H}^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   Find the generator matrix $\mathbf{G}$.

2. Given $\mathbf{H}$ from question 1, explain how to correct a single error, and give a complete example (starting with a codeword that contains exactly one error).

3. For a linear block code, show that $d_{\min} = w_{\min}$, where $w_{\min}$ is the minimum Hamming weight of any codeword in the code (excluding the all-zero codeword).

4. Recall the definition of a single-error-correcting Hamming code. Show that for any codeword length n and message length k, a Hamming code exists with those values of n and k if $n = 2^{n-k} - 1$.