

# Concurrent Implementation of Skip Trees

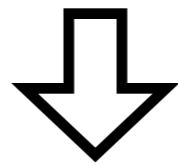
Vladimir Magdin  
Feb 7 2011

# Introduction

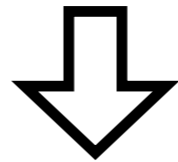
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**topic:** alternate balanced tree data structure

**outline:** balanced search trees



skip lists



skip trees

# Balanced Search Trees

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- can represent ordered sets and dictionaries
- $O(\log(n))$  for search, insert, delete operations
- two major types:
  - rotations of nodes (e.g. AVL tree)
  - splitting/merging of nodes (e.g. B-tree)

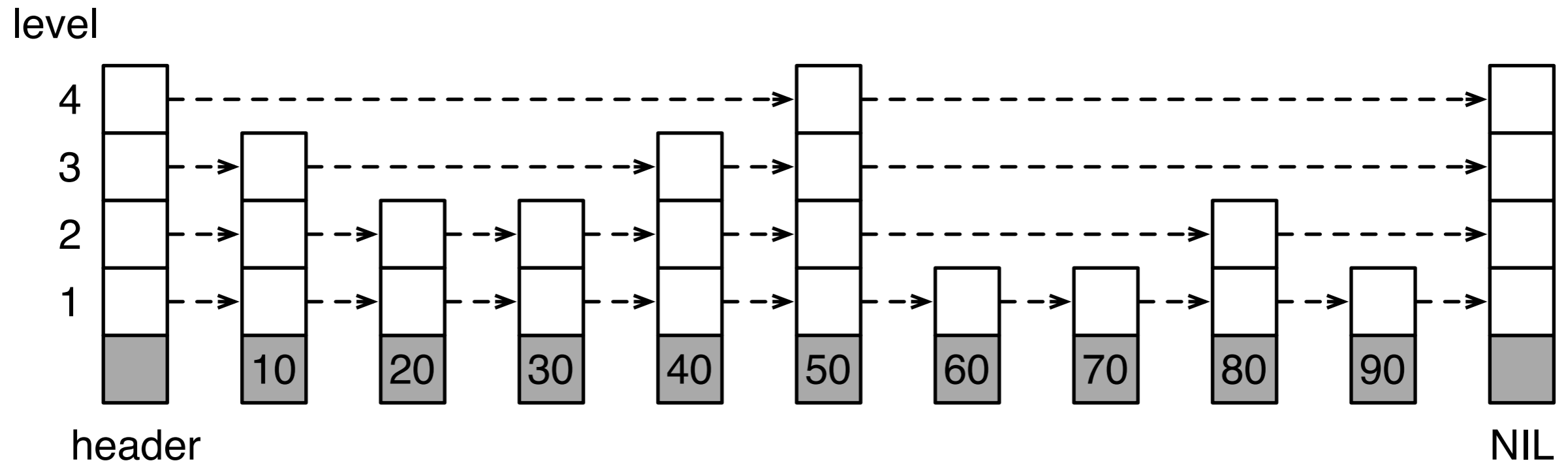
# Balanced Search Trees

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## disadvantages

- rotations are complicated
- implementation
- constant factor:  $O(c \log(n))$
- order of insertions might matter

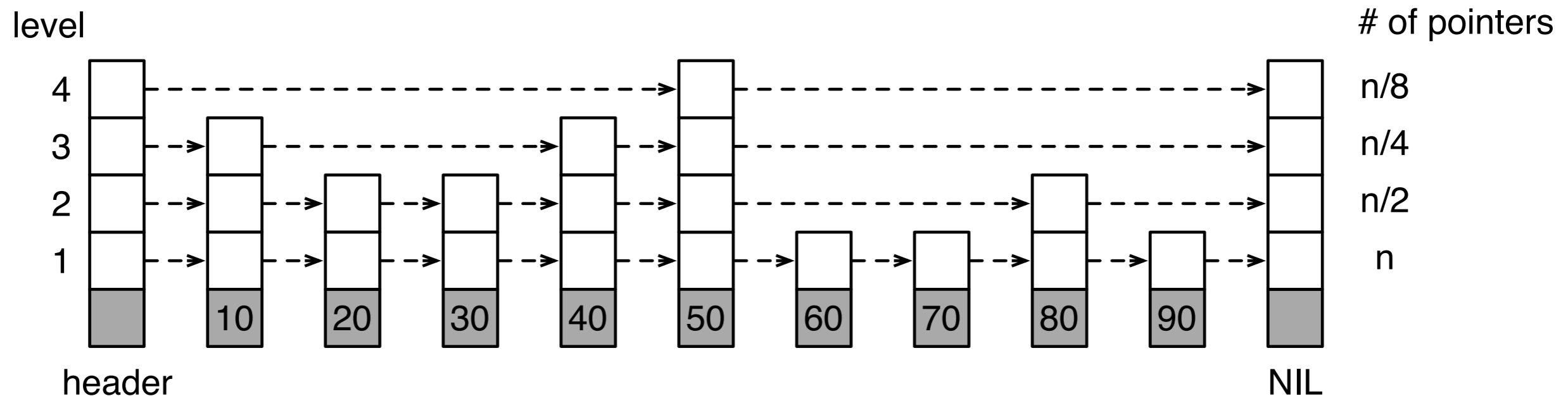
# Skip Lists



- described by William Pugh in 1990
- hierarchy of lists with different degrees of connectivity
- levels of newly-inserted nodes are chosen randomly

# Node Level Selection

- fraction  $p$  (e.g.  $1/2$ ) of the nodes at level( $i$ ) also have pointers at level( $i+1$ )

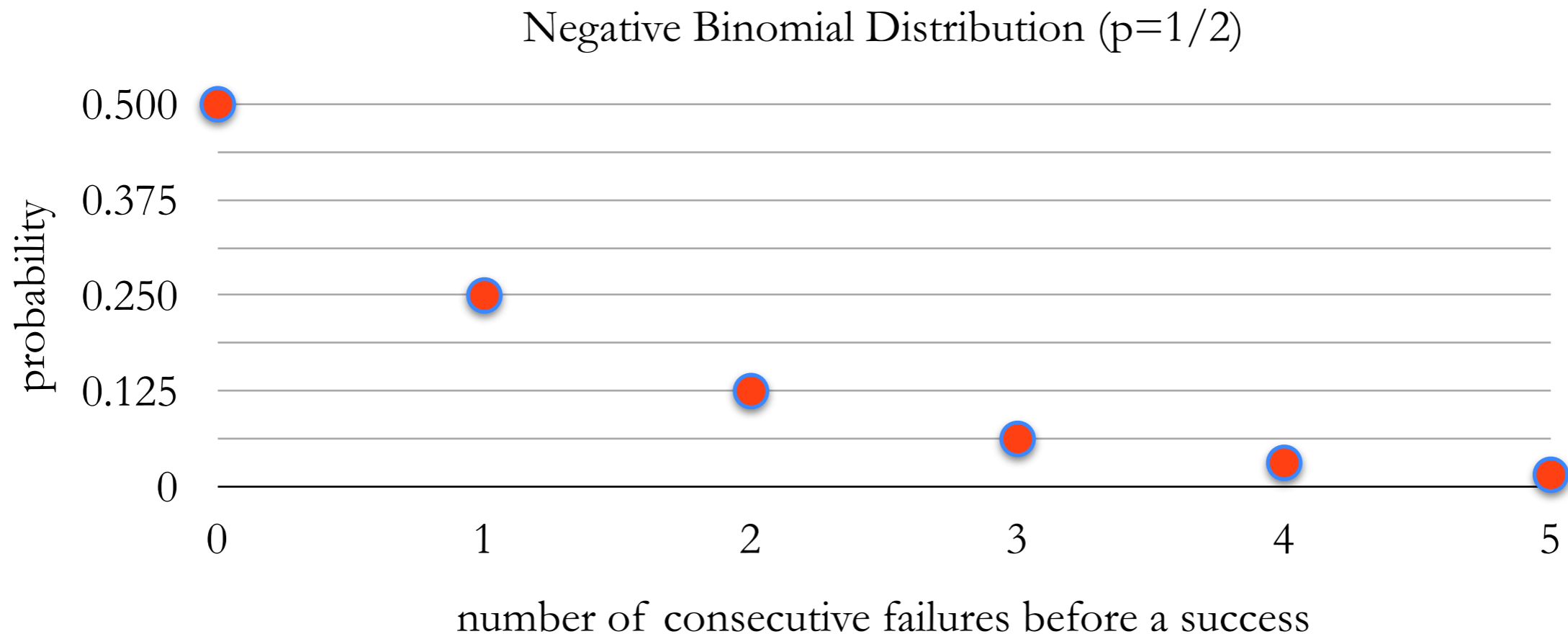


# Node Level Selection

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- drawing level from the negative binomial distribution

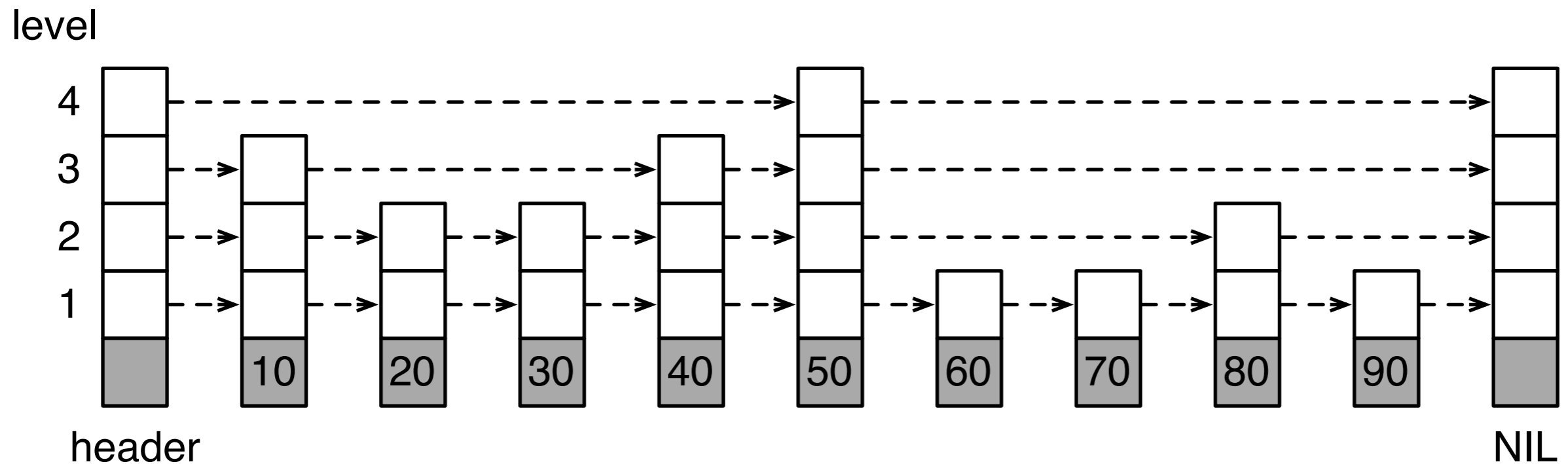
NB(1,p) leads to  $O(\log_{1/p}(n))$  search



# Skip List - Insertion

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insert: 85

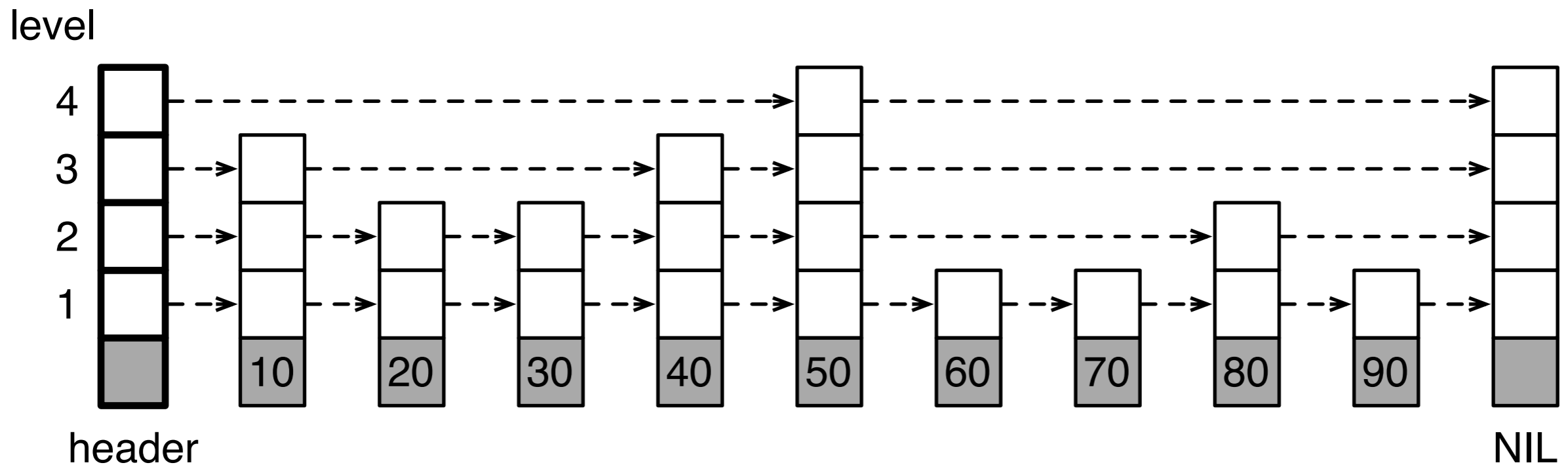




# Skip List - Insertion

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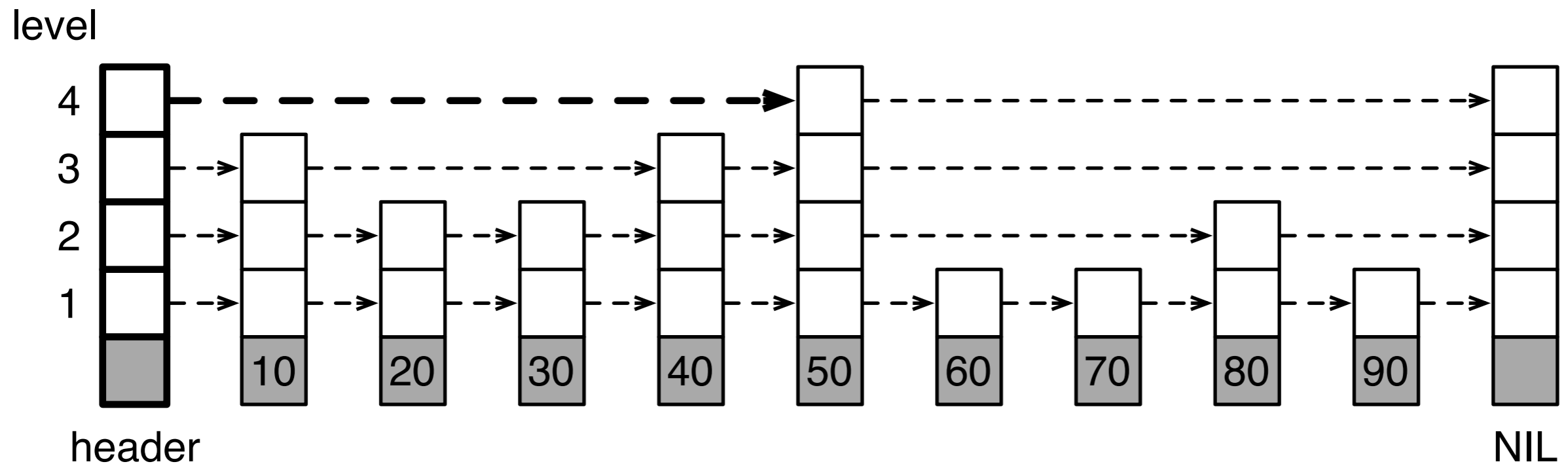
insert: 85



# Skip List - Insertion

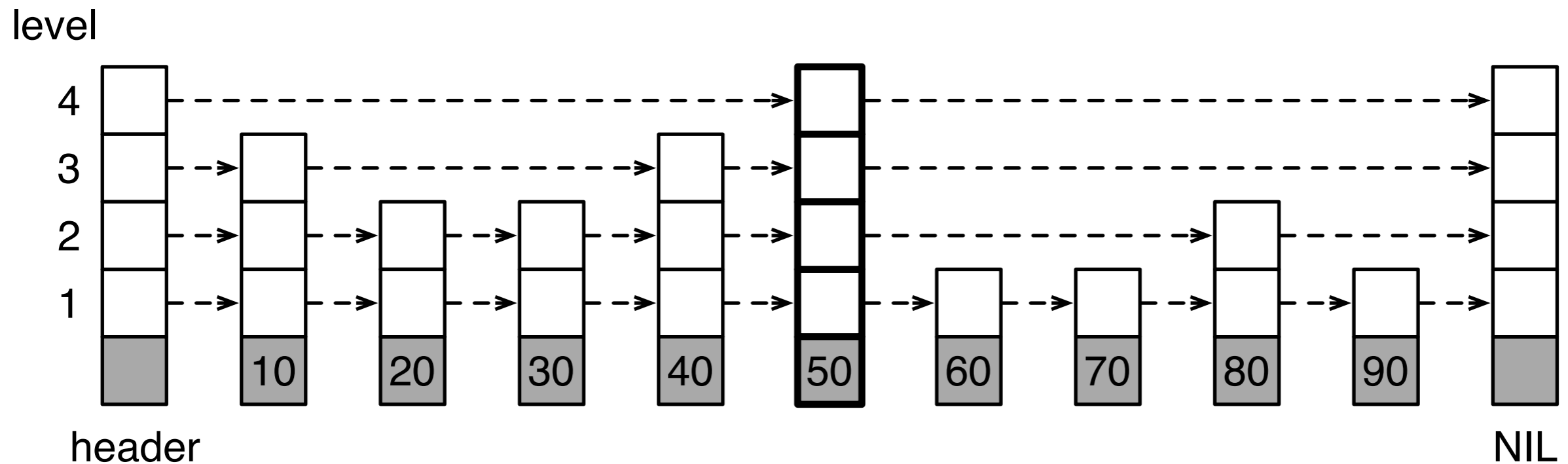
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insert: 85



# Skip List - Insertion

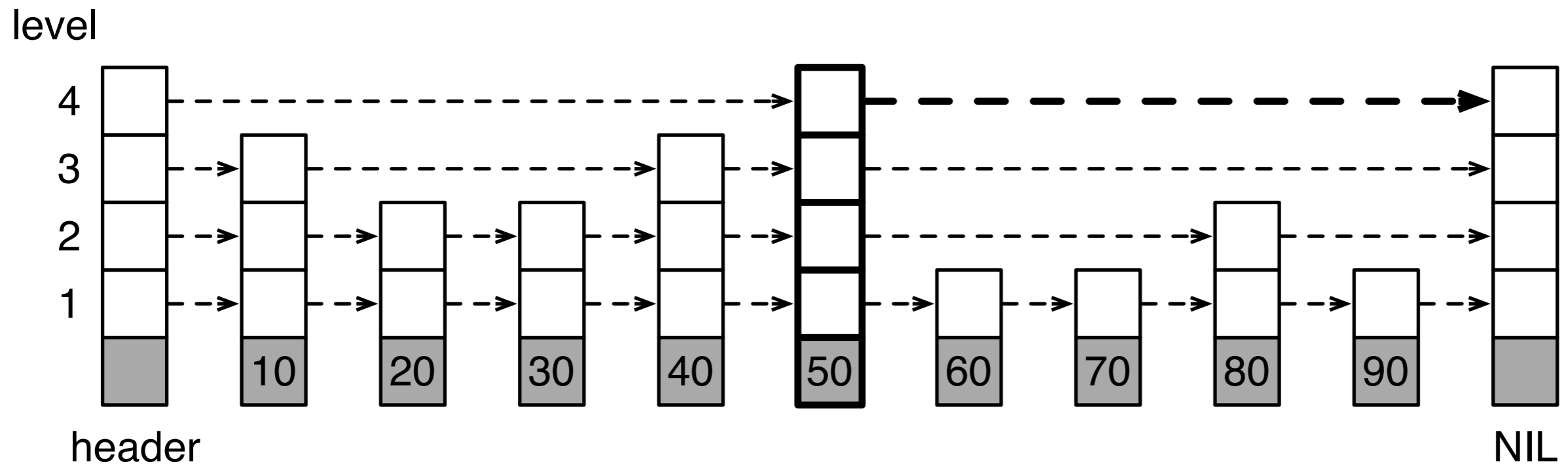
insert: 85



# Skip List - Insertion

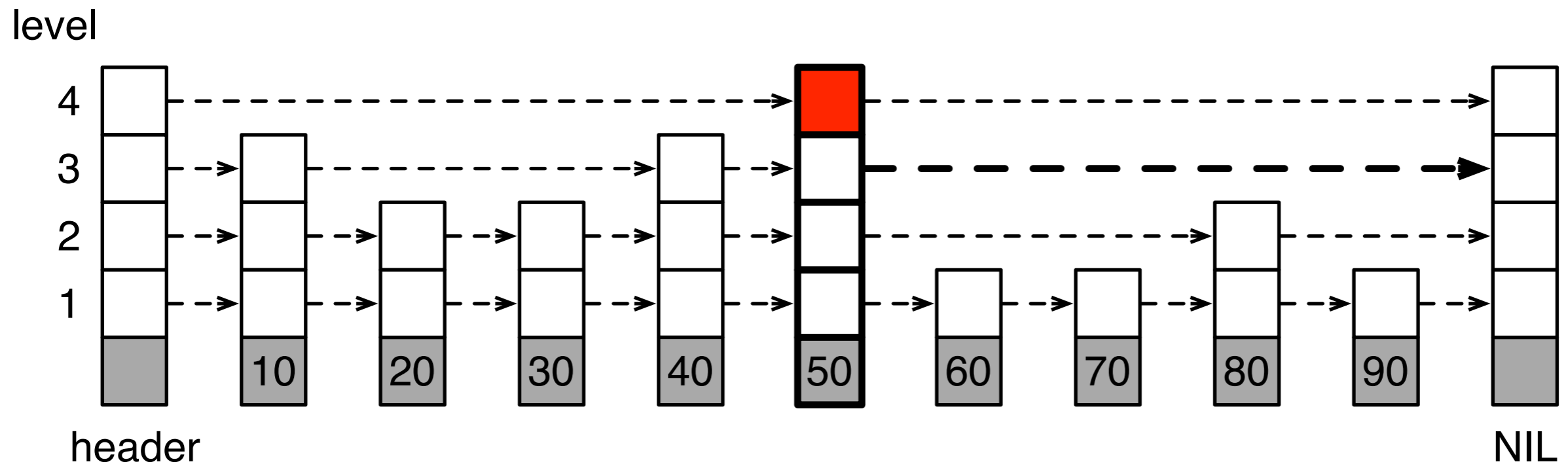
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insert: 85



# Skip List - Insertion

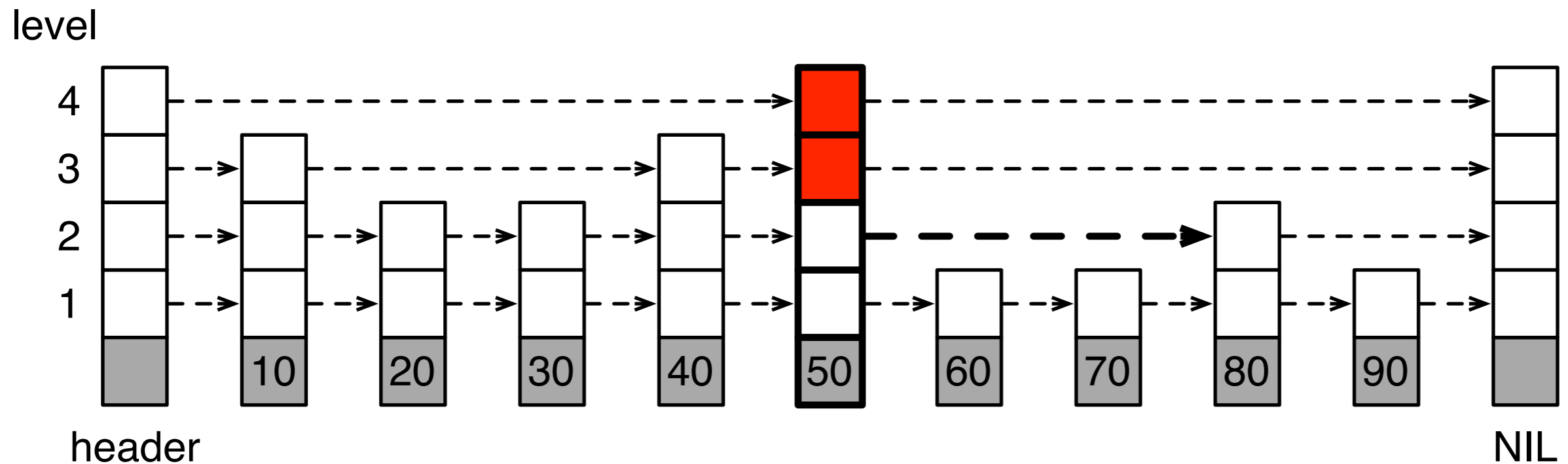
insert: 85



# Skip List - Insertion

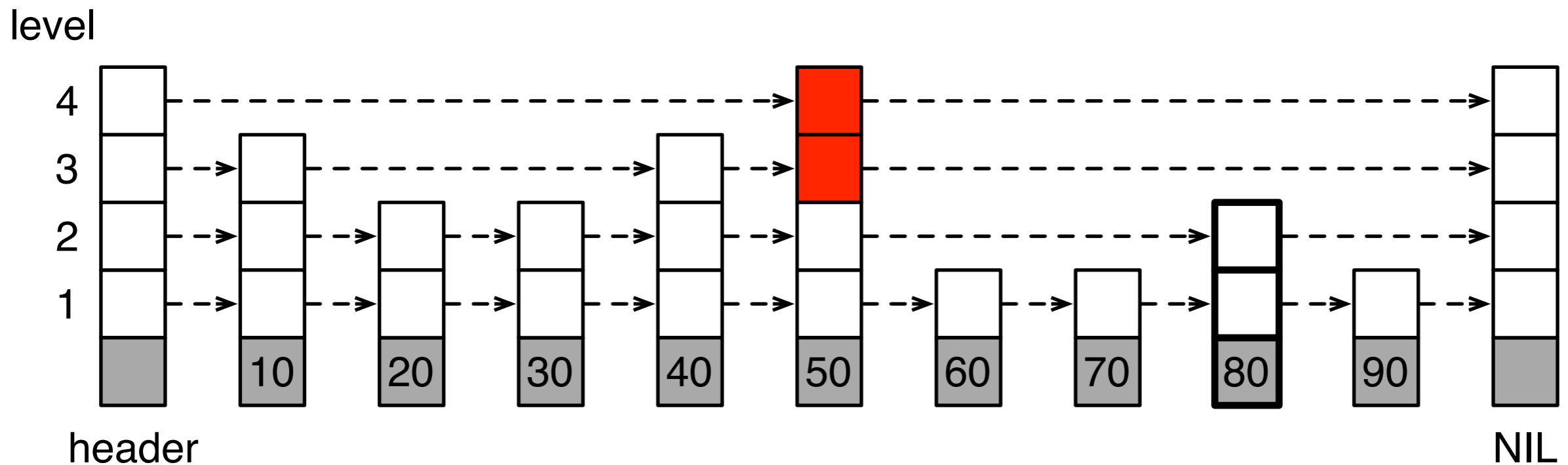
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insert: 85



# Skip List - Insertion

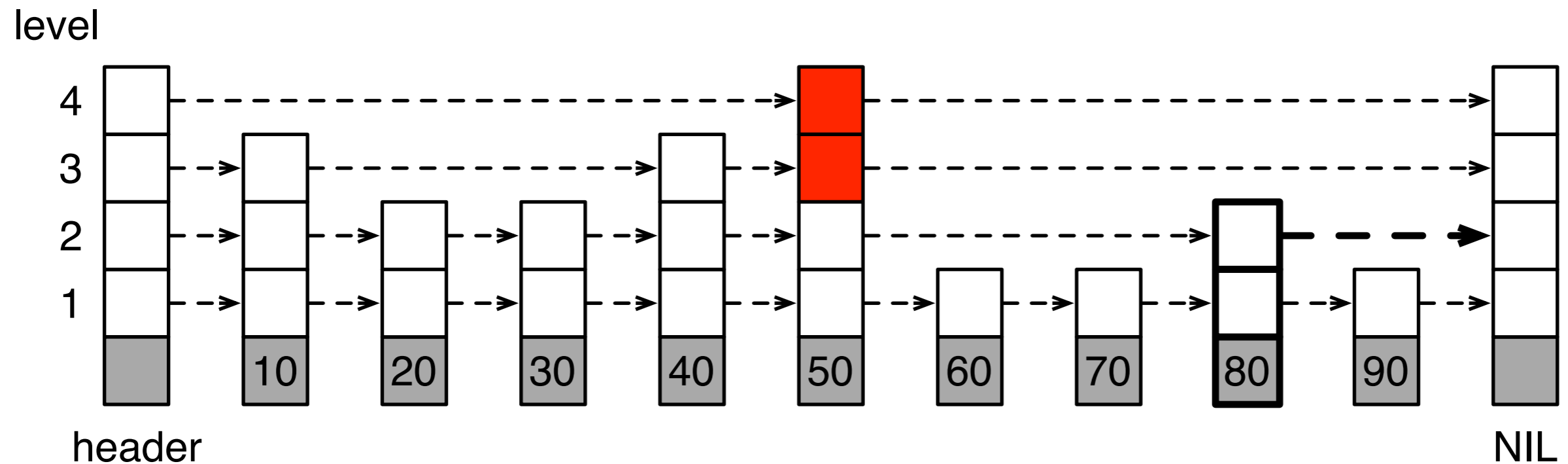
insert: 85



# Skip List - Insertion

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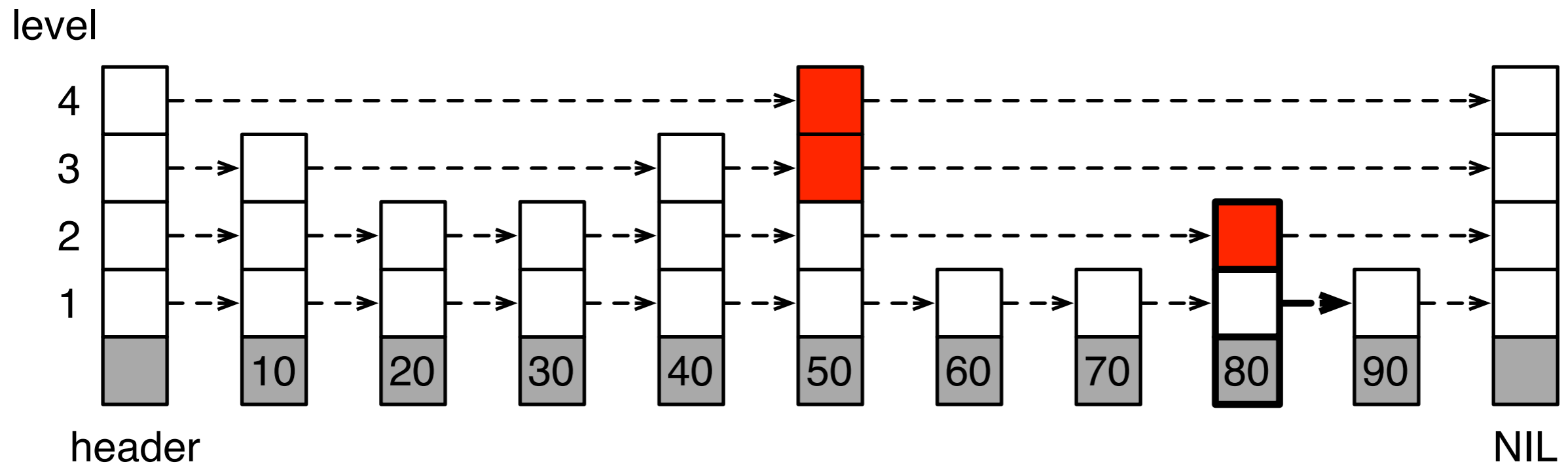
insert: 85





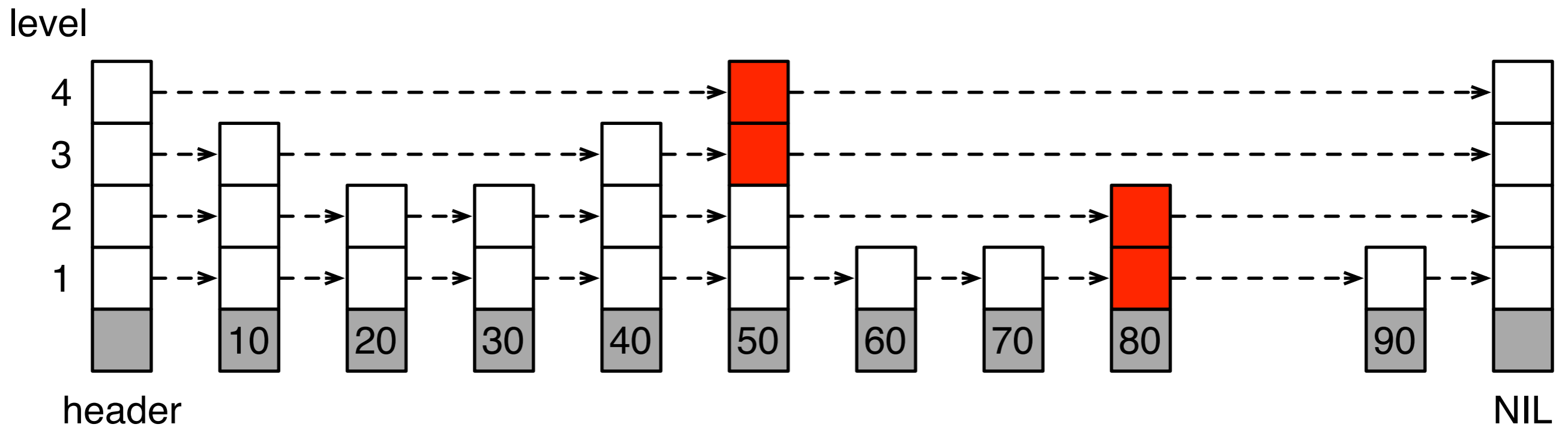
# Skip List - Insertion

insert: 85



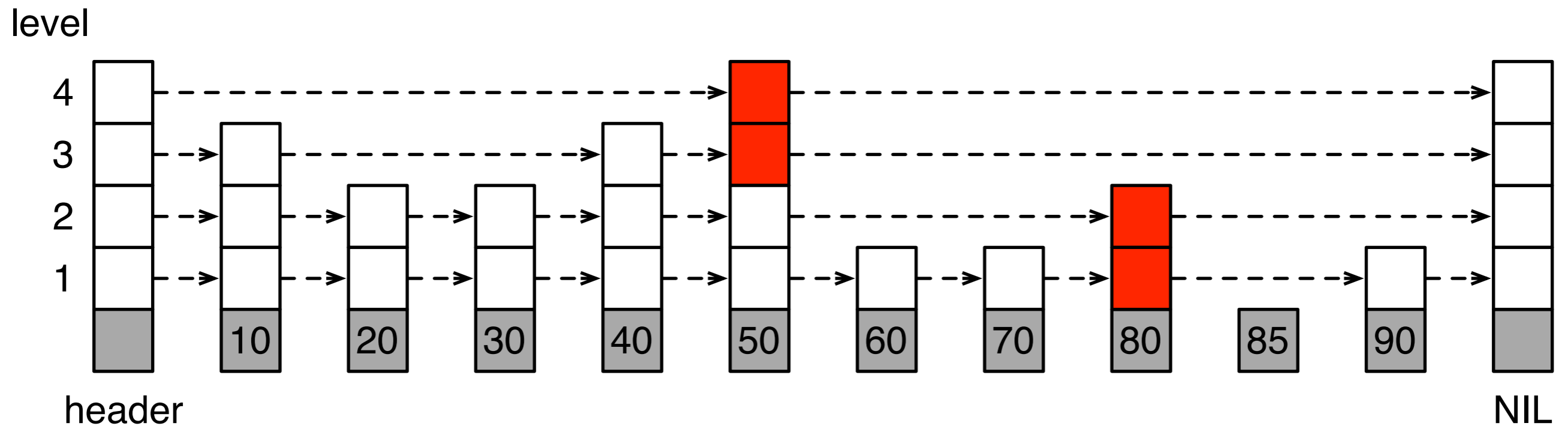
# Skip List - Insertion

insert: 85



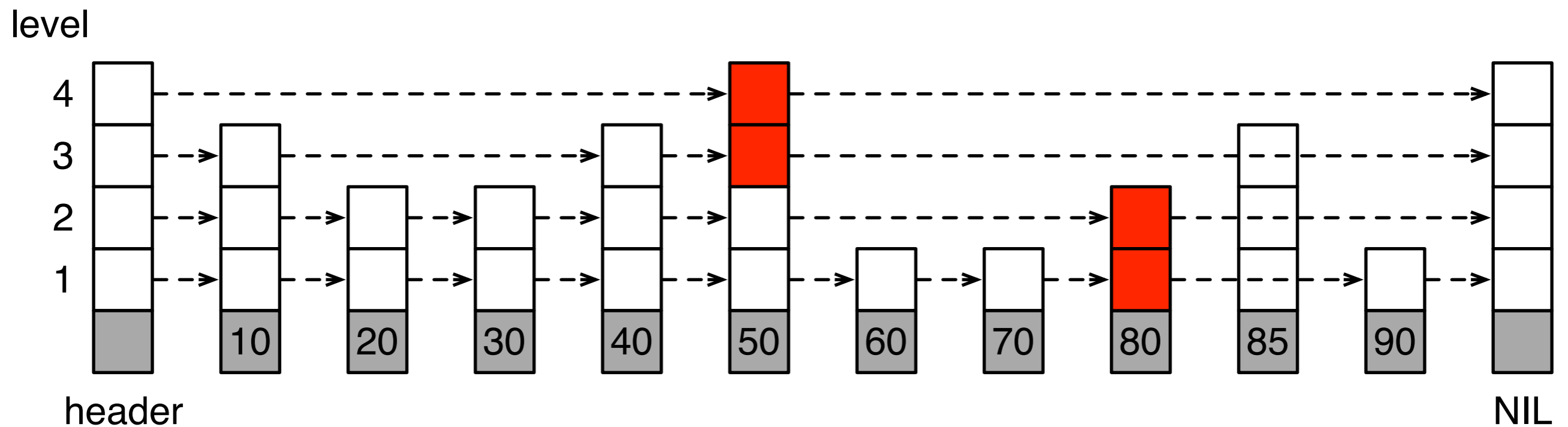
# Skip List - Insertion

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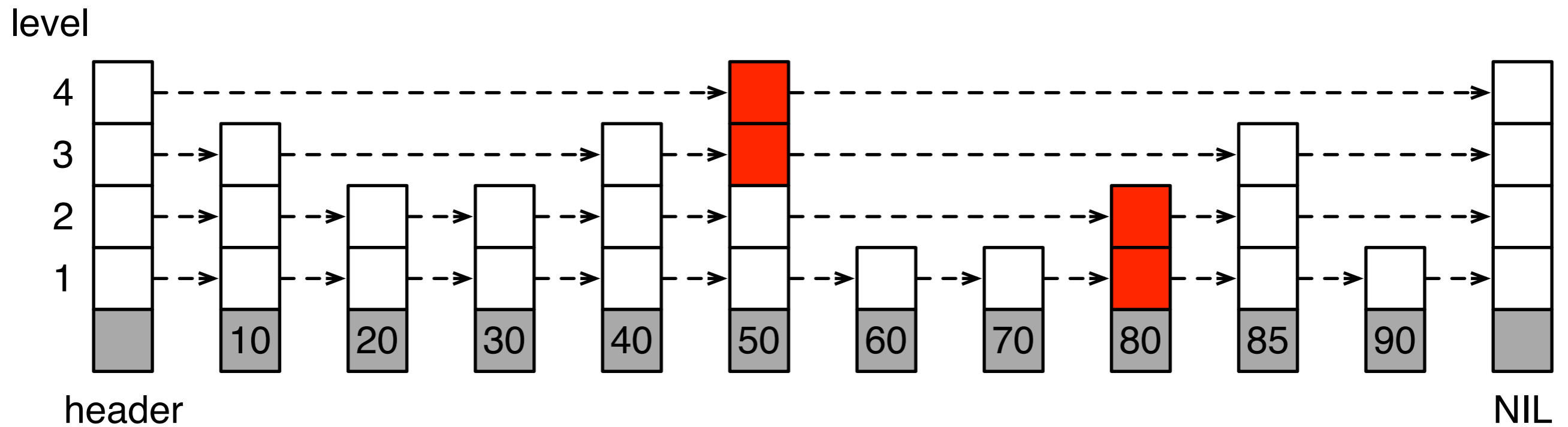
# Skip List - Insertion

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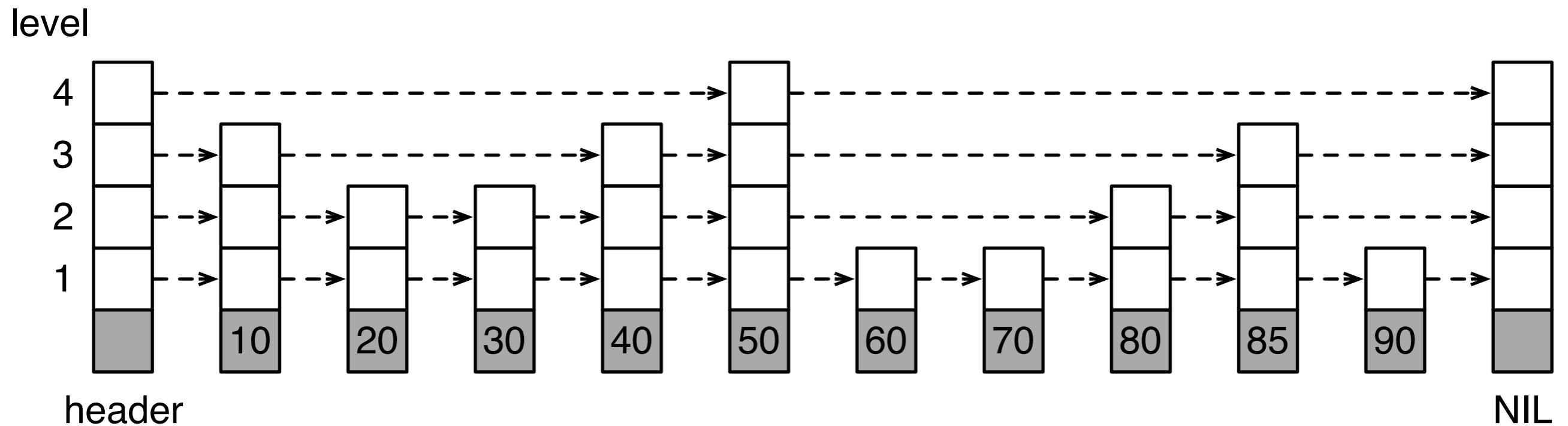
# Skip List - Insertion

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# Skip List - Insertion

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# Skip Lists

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## ADVANTAGES

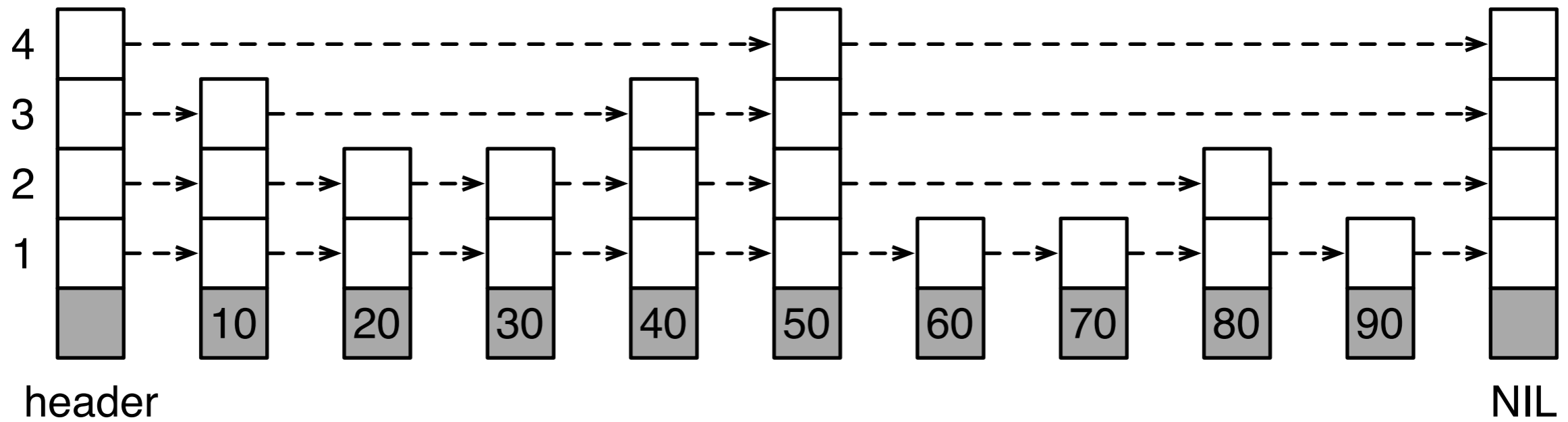
- order of insertion does not matter
- easy to implement
- constant factor performance advantage over trees
- space efficient

## DISADVANTAGES

- cannot be paginated efficiently
- insertion/deletion requires global search

skip list

level

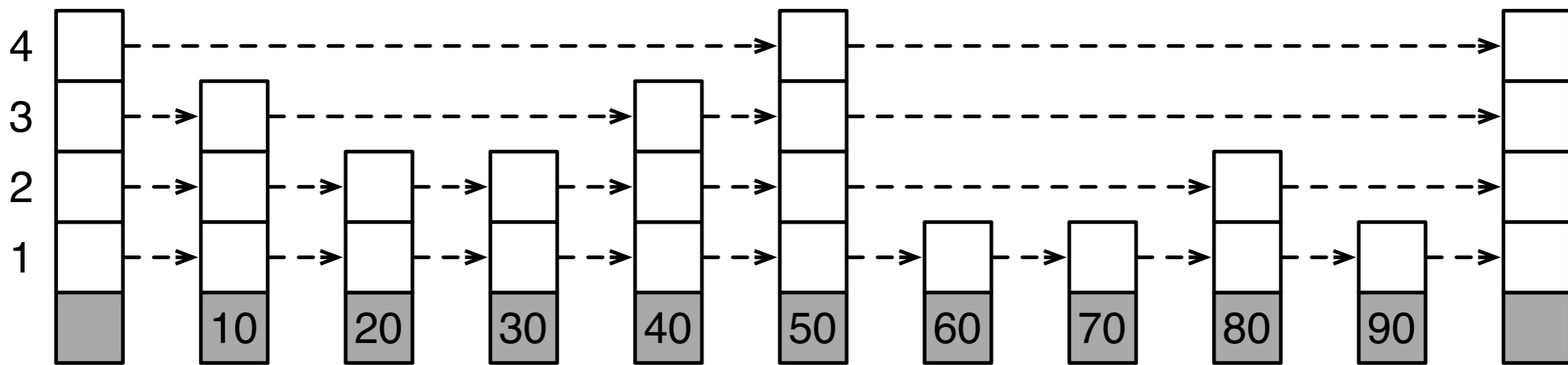


skip lists can be re-envisioned as trees  
(Xavier Messeguer, 1997)



skip list

level

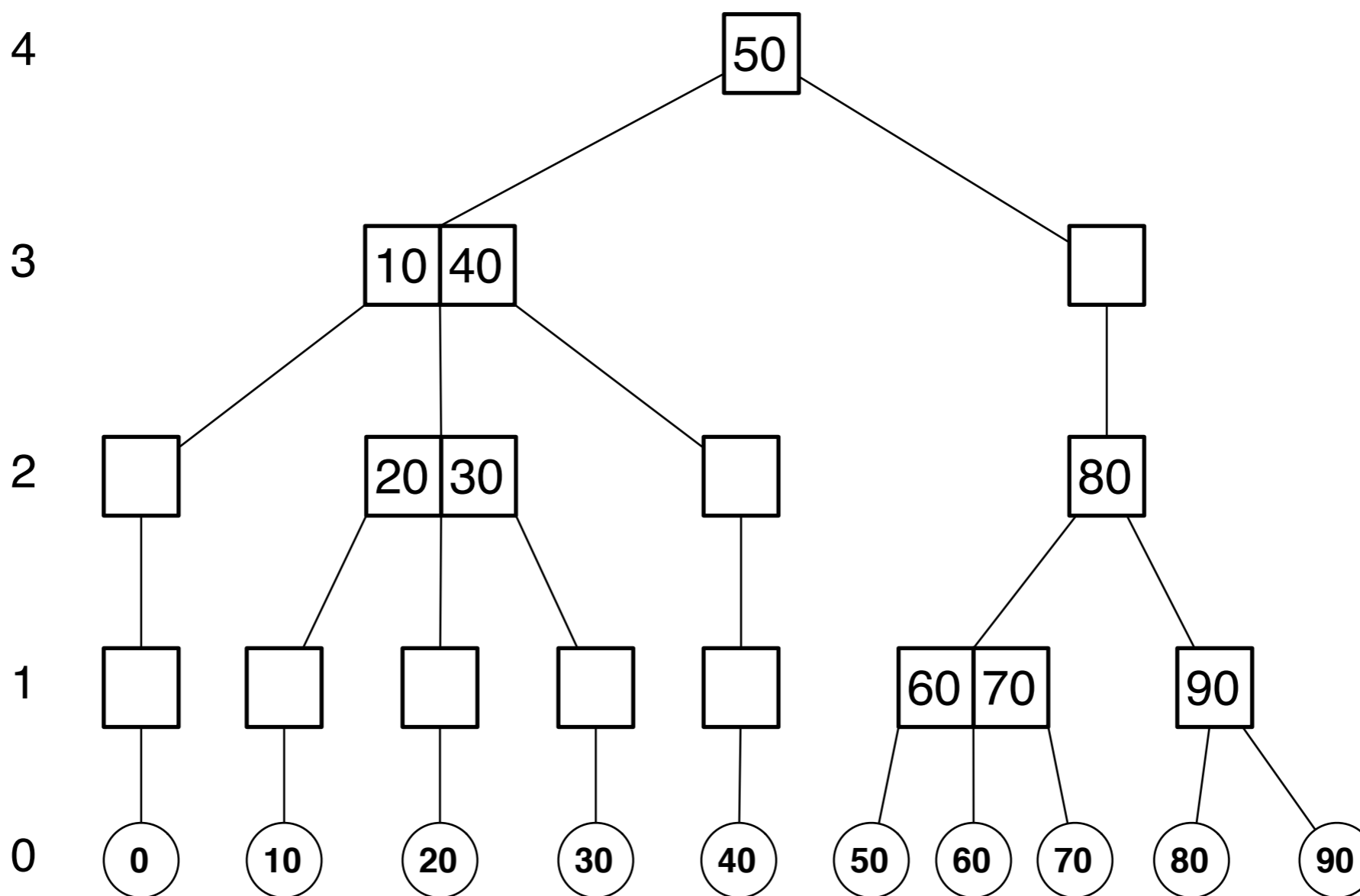


header

NIL

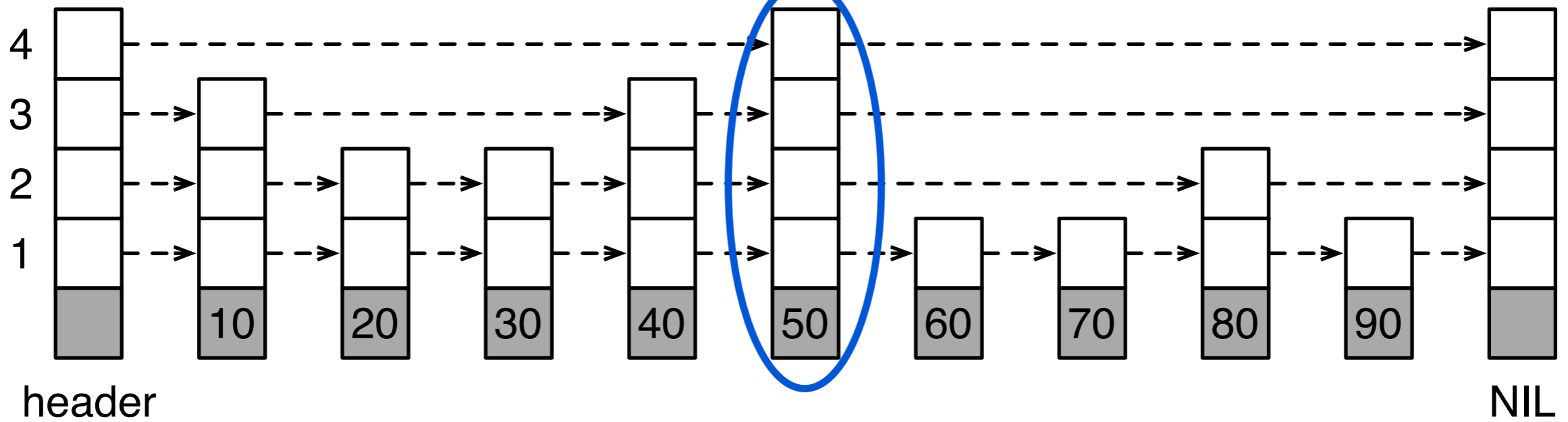
skip tree

Level



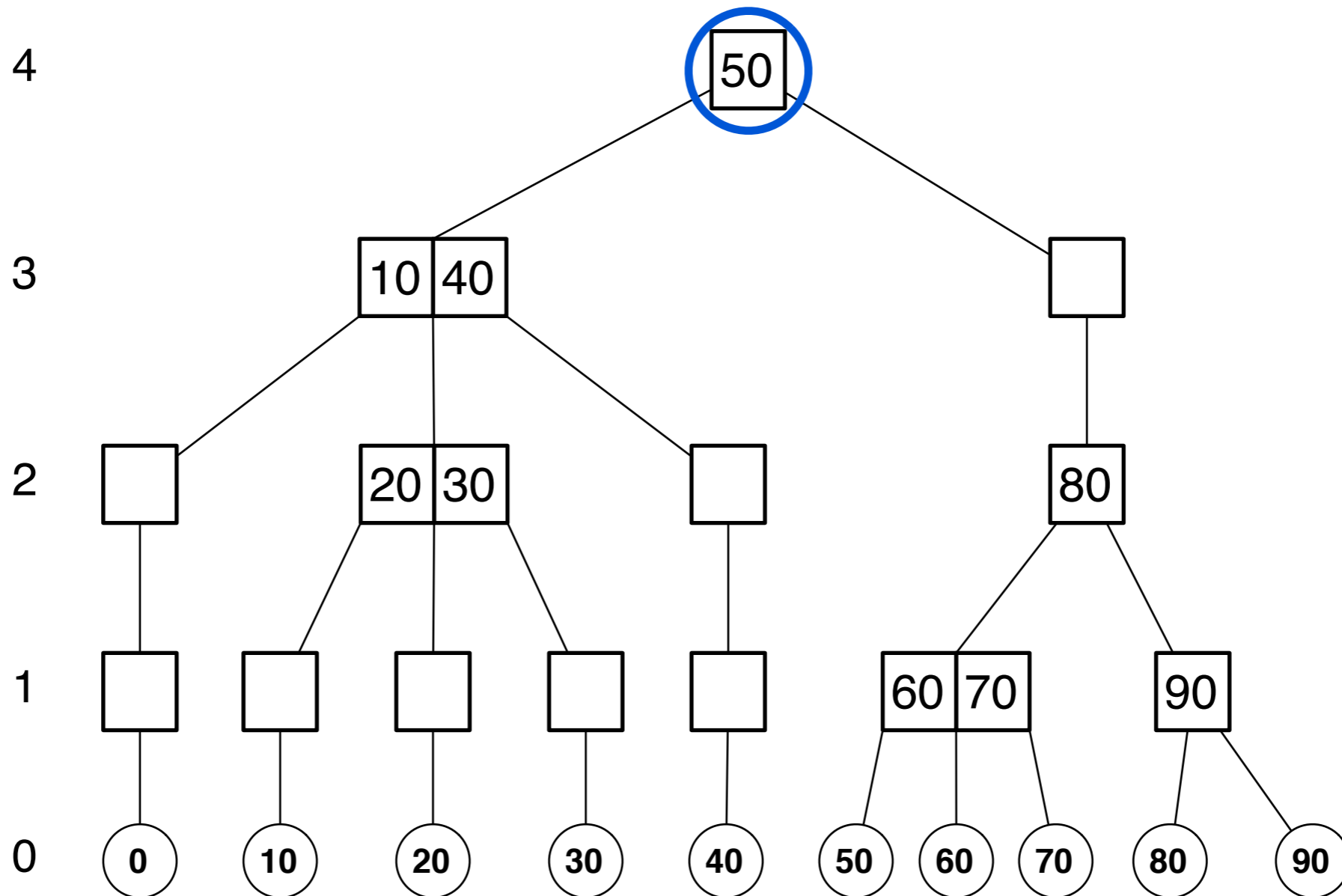
skip list

level



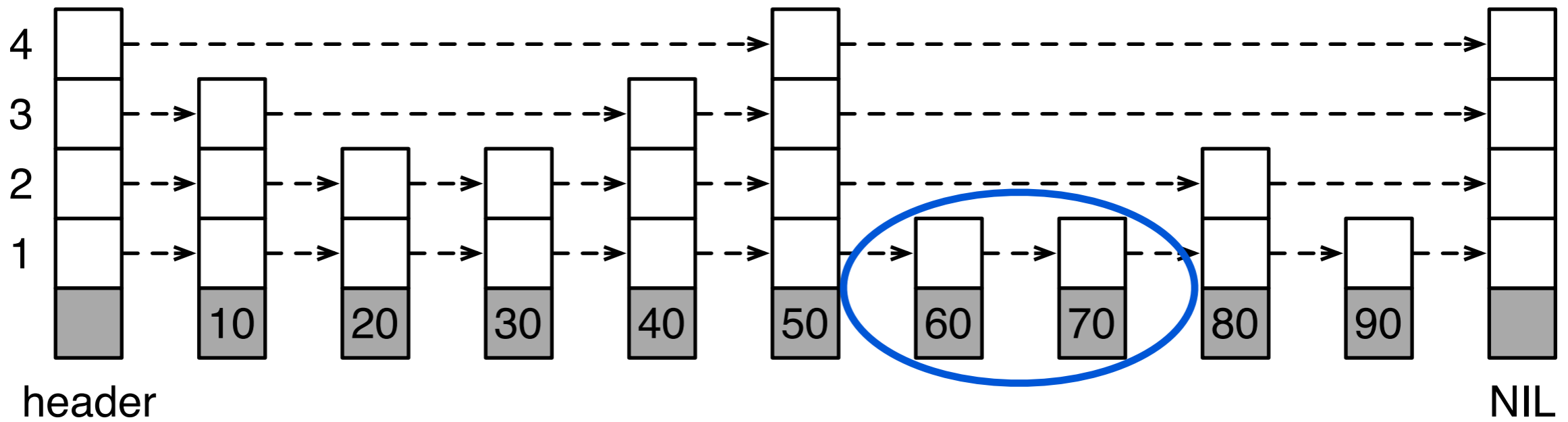
skip tree

Level



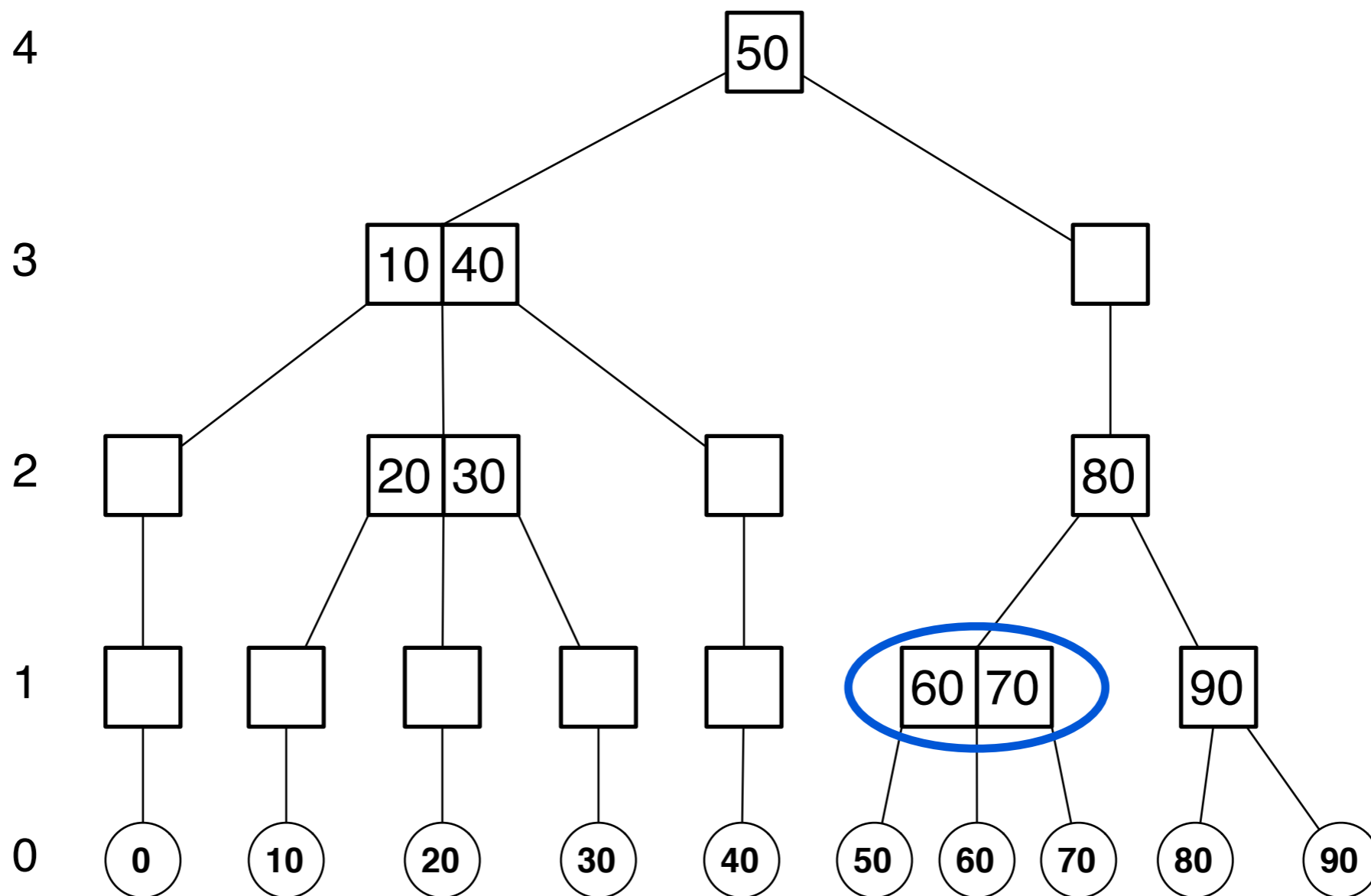
skip list

level



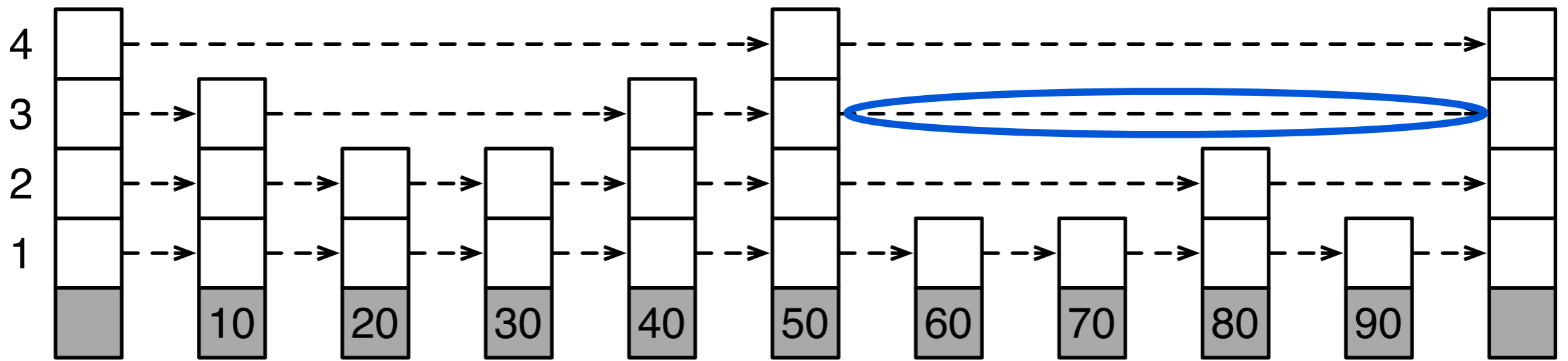
skip tree

Level



skip list

level

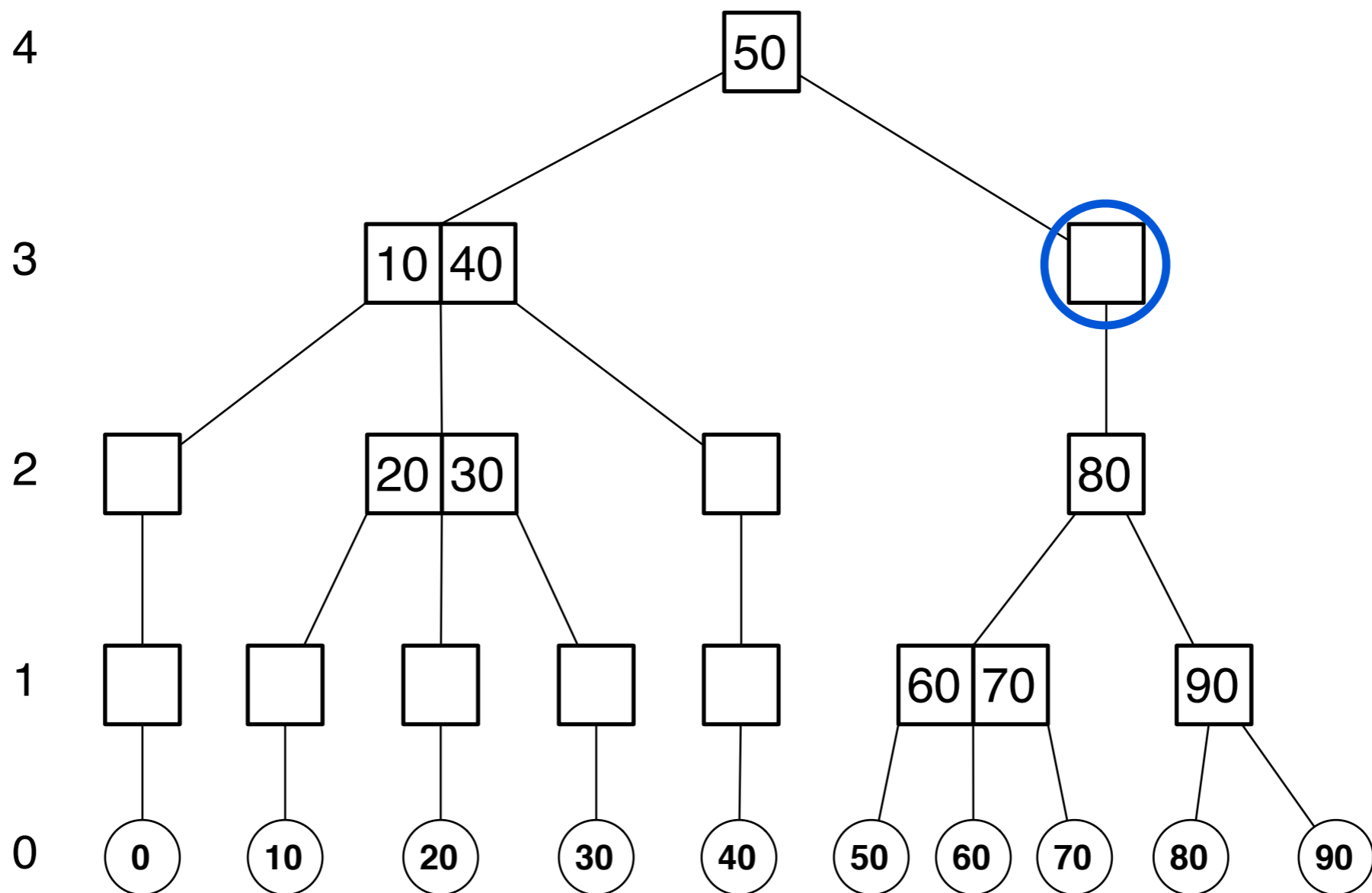


header

NIL

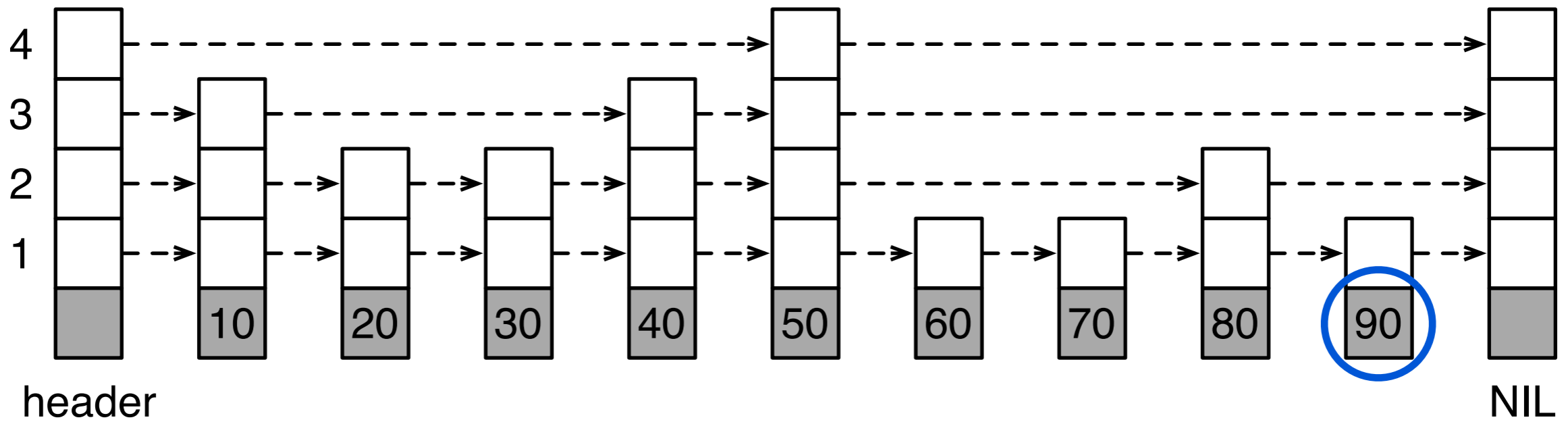
skip tree

Level



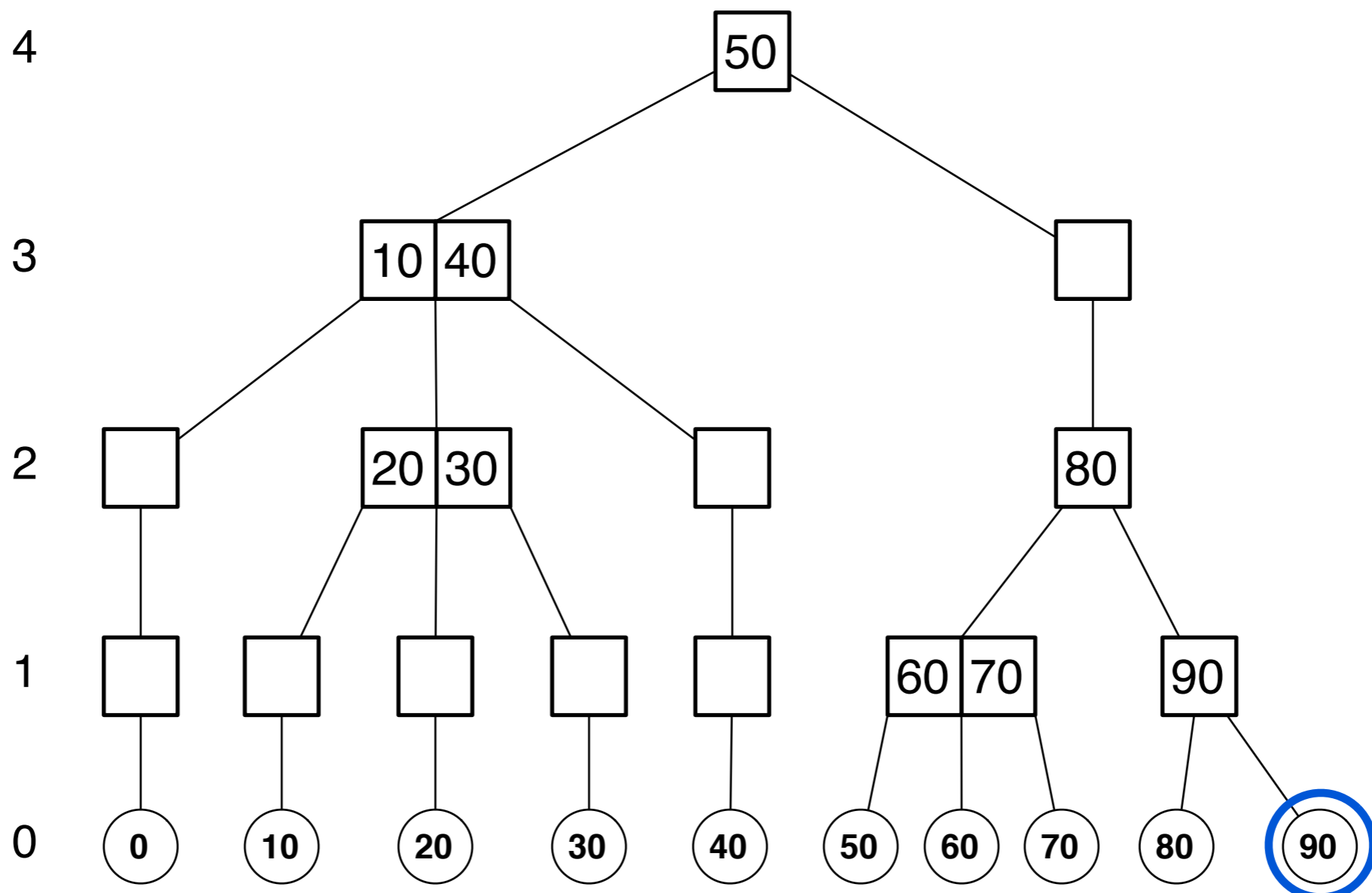
skip list

level



skip tree

Level



# Skip Trees - Insertion

insert: 85

Level

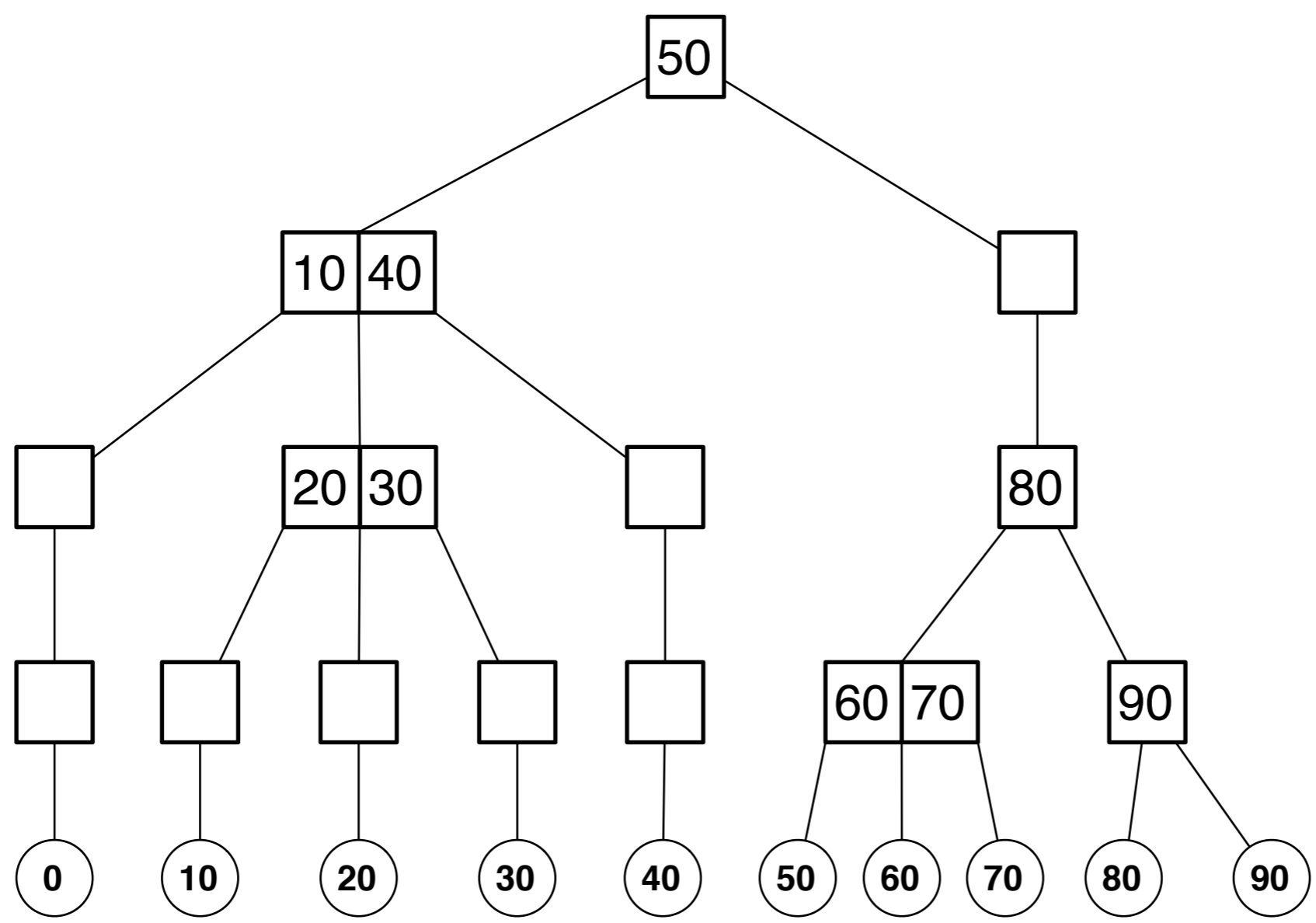
4

3

2

1

0



# Skip Trees - Insertion

## 1. Percolate(1)

Level

4

50

insert:

85

3

10

40

2

20

30

80

1

60

70

90

0

0

10

20

30

40

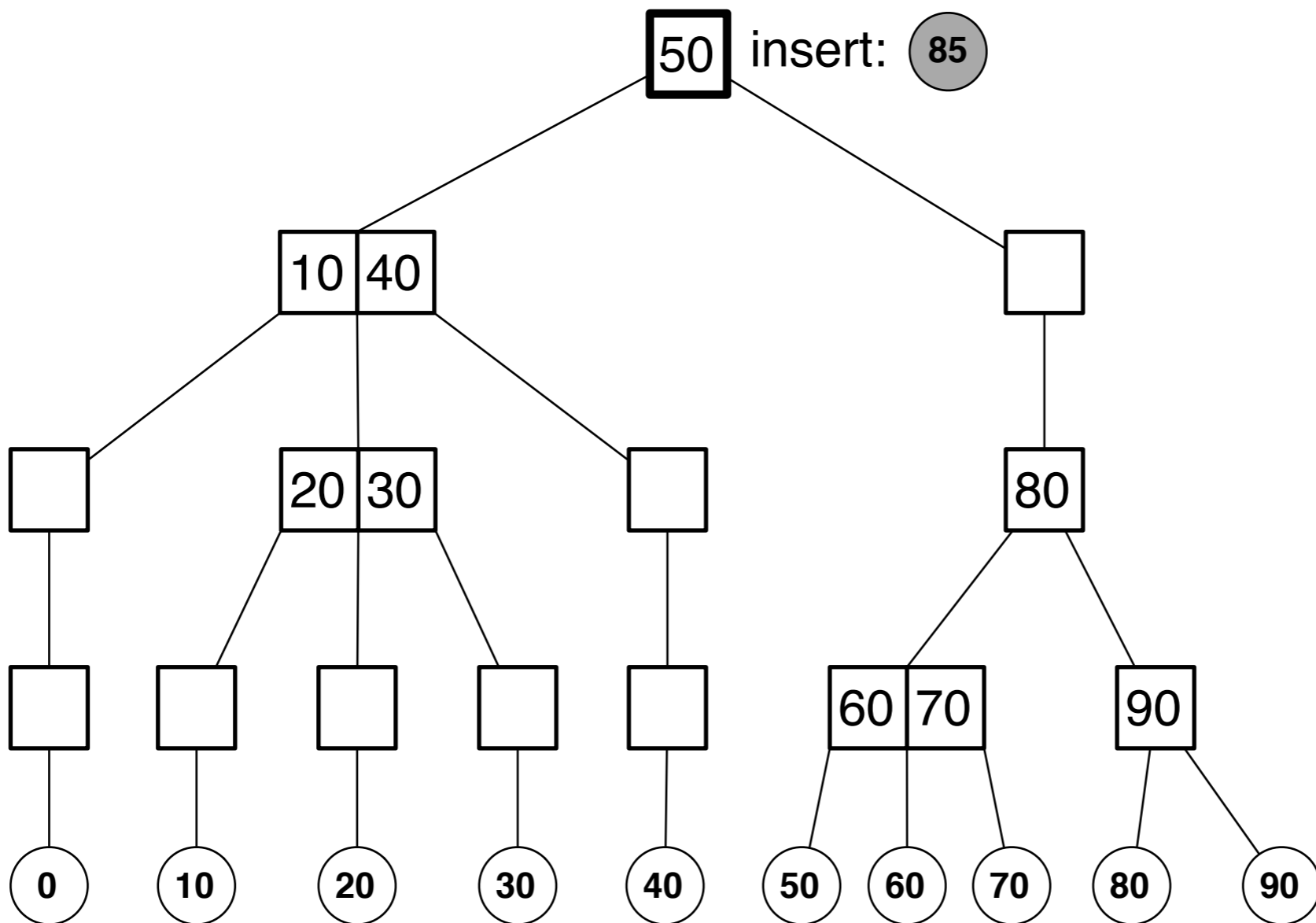
50

60

70

80

90



# Skip Trees - Insertion

## 1. Percolate(2)

Level

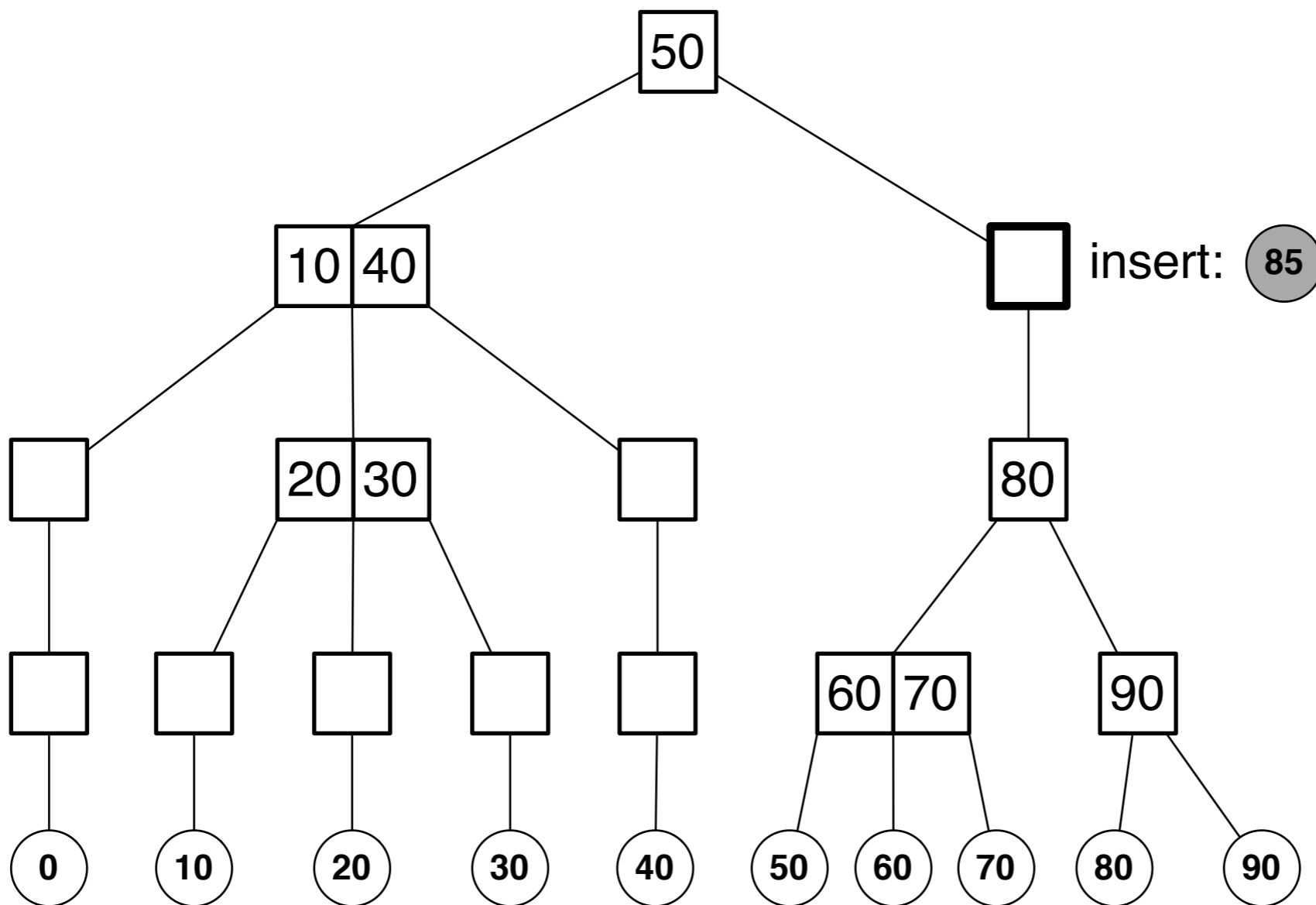
4

3

2

1

0





# Skip Trees - Insertion

## 1. Percolate(3)

Level

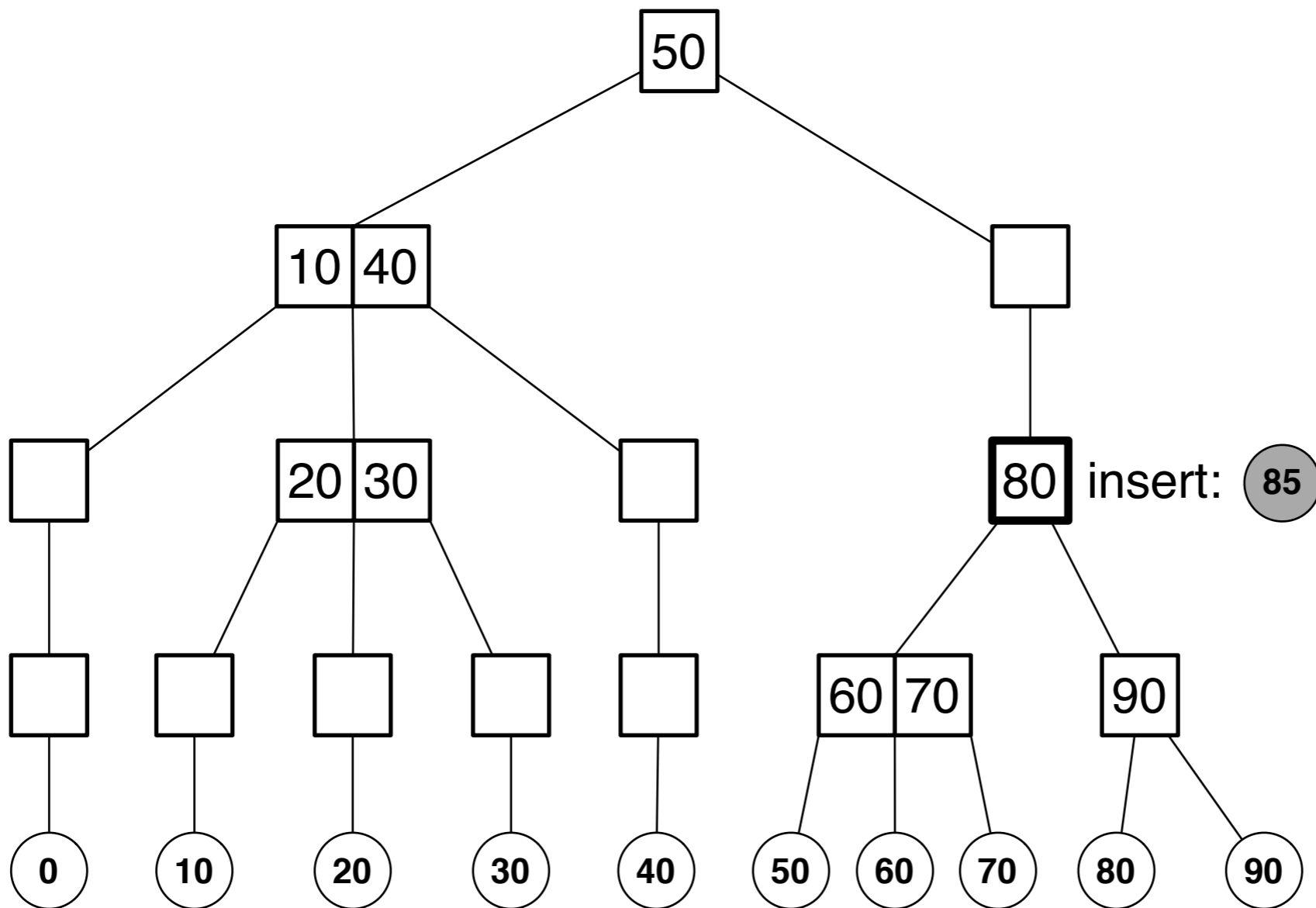
4

3

2

1

0



# Skip Trees - Insertion

## 1. Percolate(4)

Level

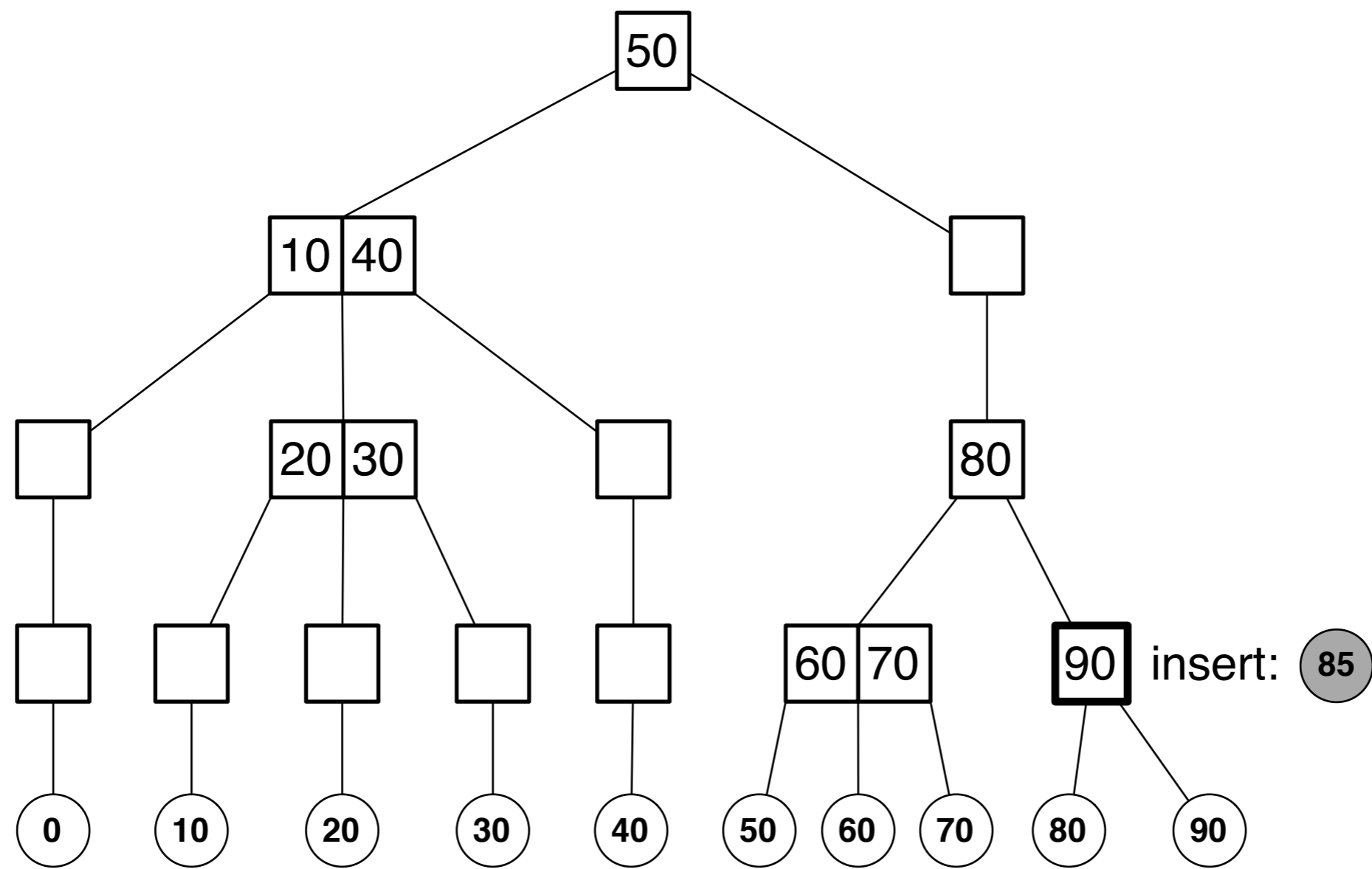
4

3

2

1

0



# Skip Trees - Insertion

## 1. Percolate(5)

Level

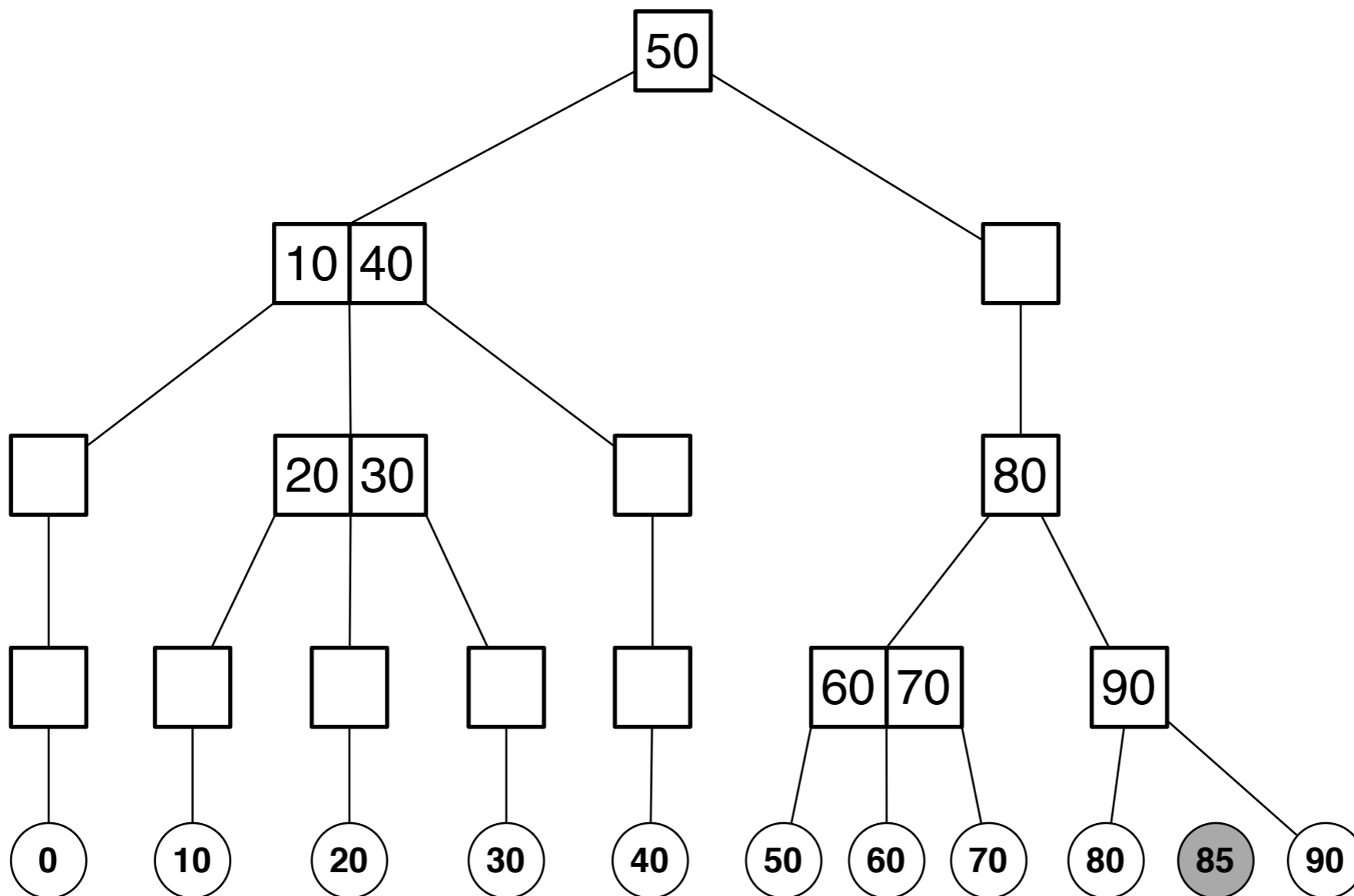
4

3

2

1

0



# Skip Trees - Insertion

## 2. Attach

Level

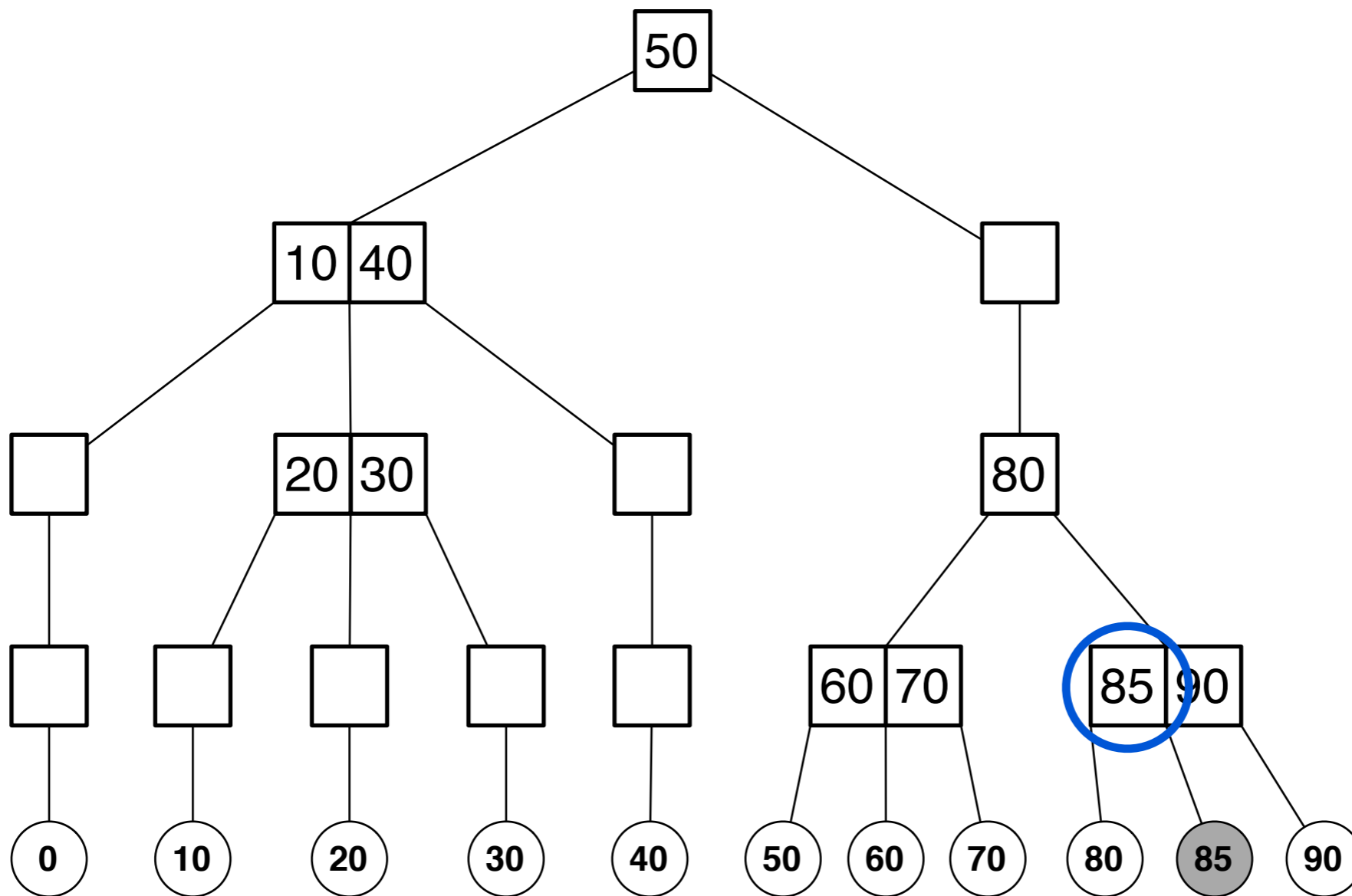
4

3

2

1

0



# Skip Trees - Insertion

Level

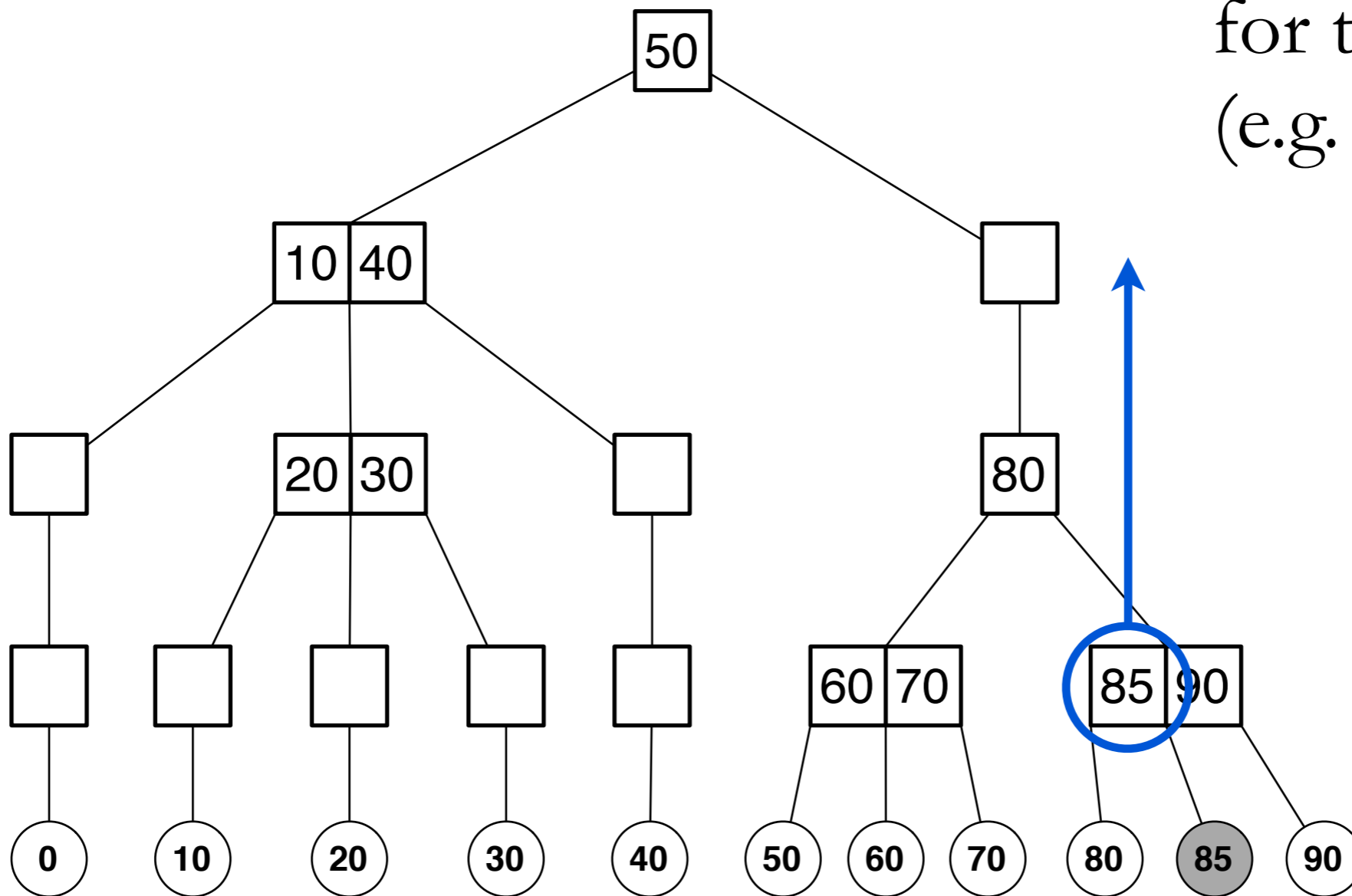
4

3

2

1

0



3. Choose a level for the new key (e.g. 3)

# Skip Trees - Insertion

## 3. Split(1)

Level

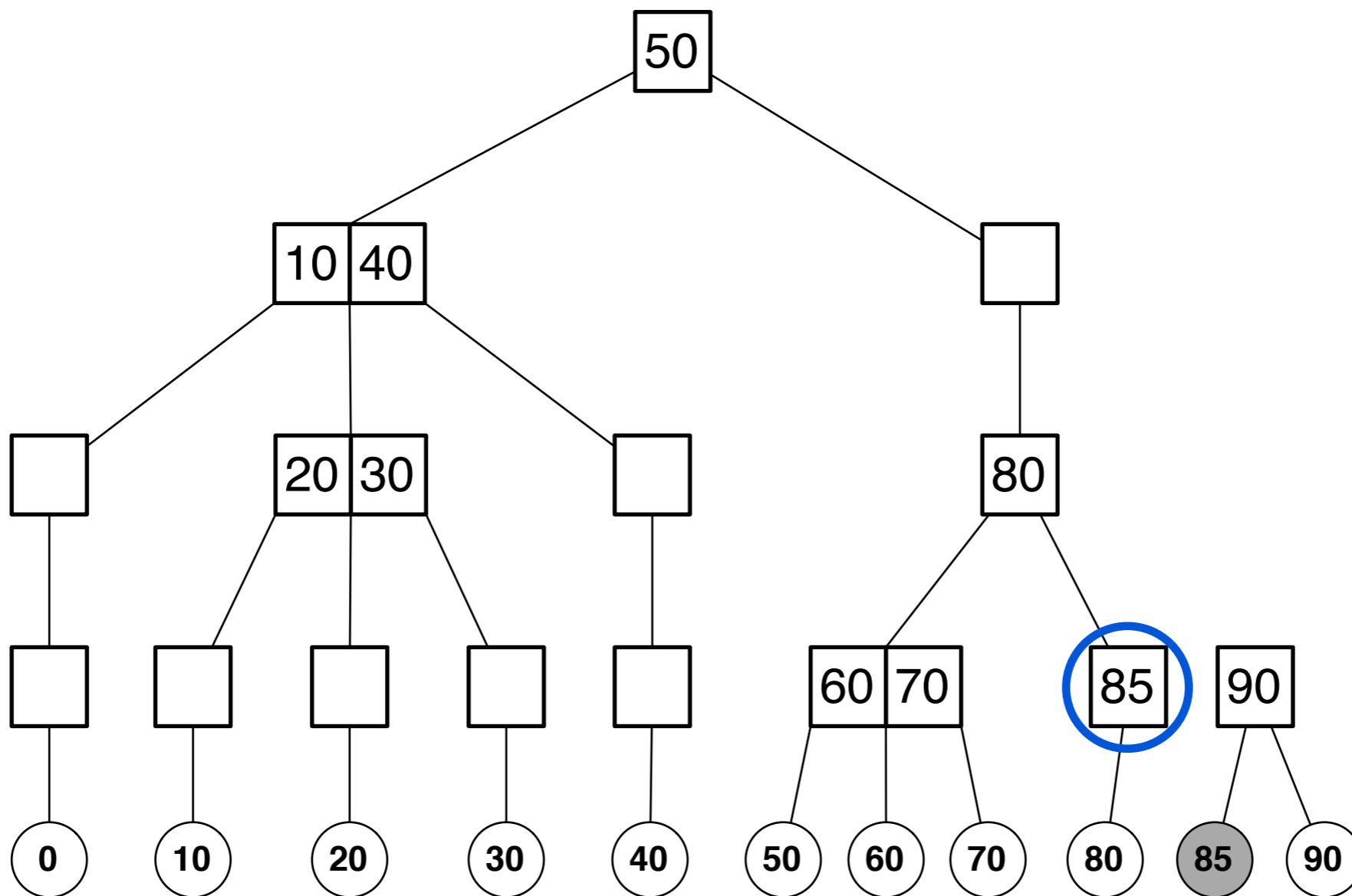
4

3

2

1

0



# Skip Trees - Insertion

## 3. Split(1)

Level

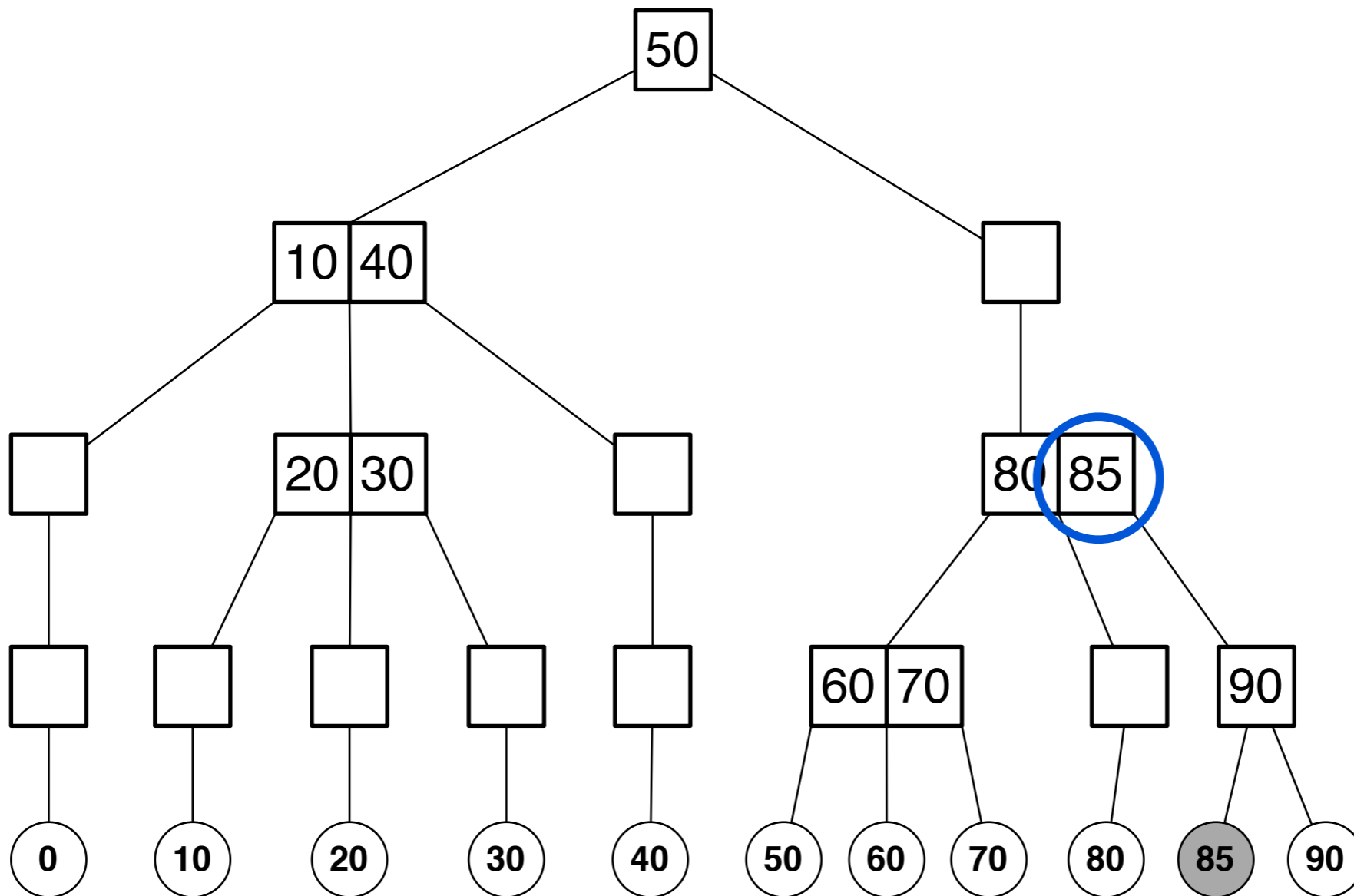
4

3

2

1

0



# Skip Trees - Insertion

## 3. Split(2)

Level

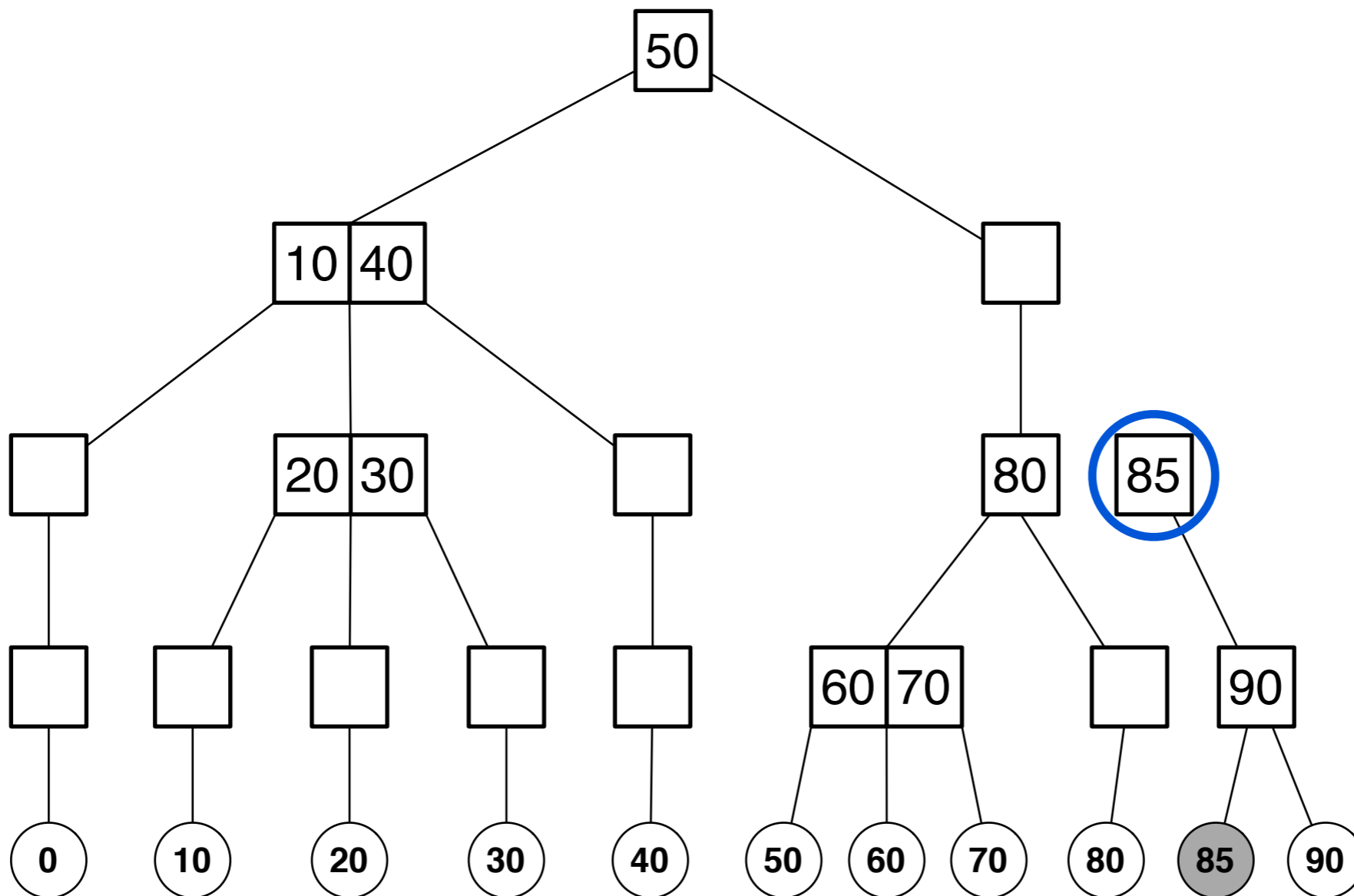
4

3

2

1

0





# Skip Trees - Insertion

## 3. Split(2)

Level

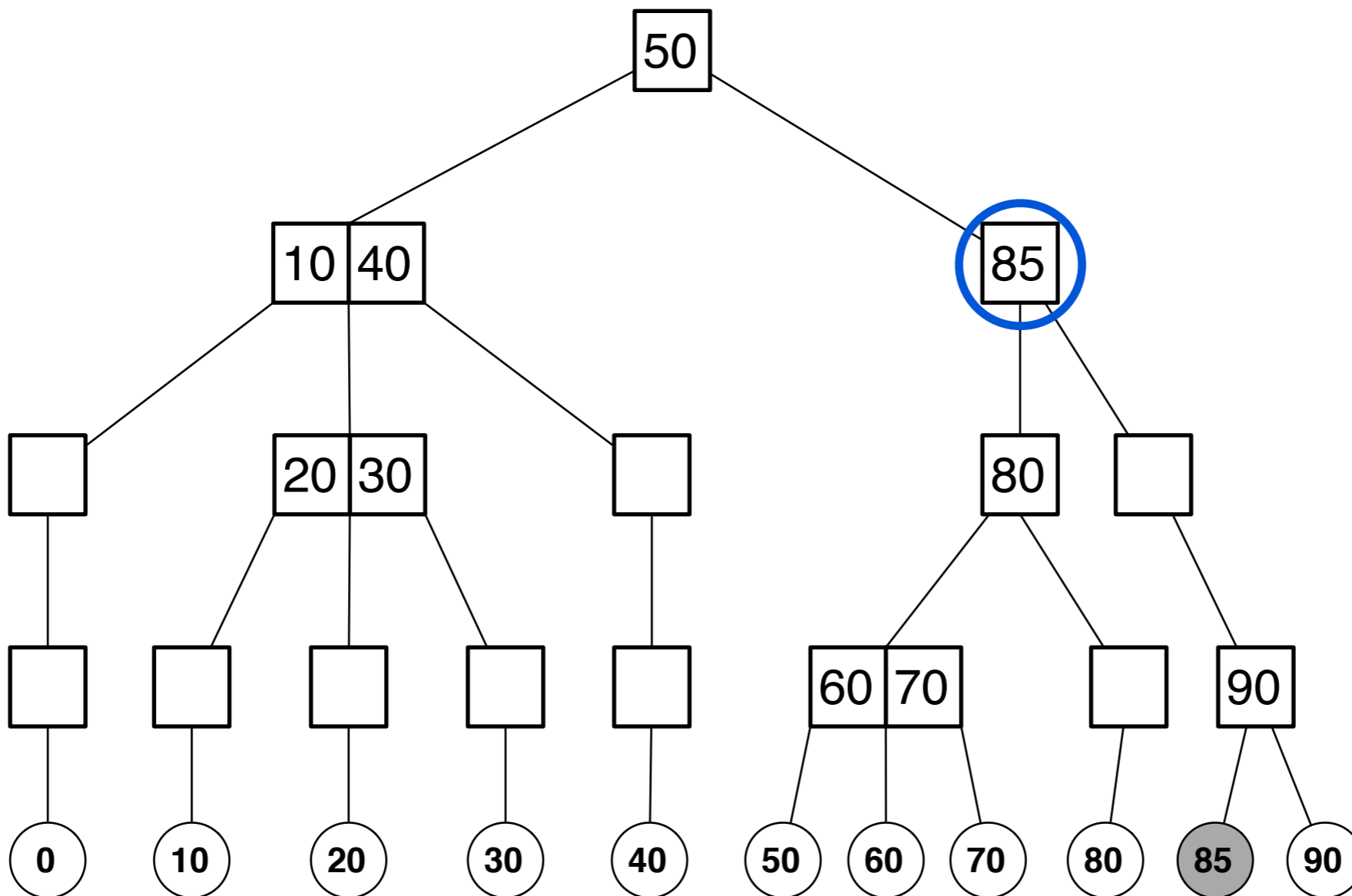
4

3

2

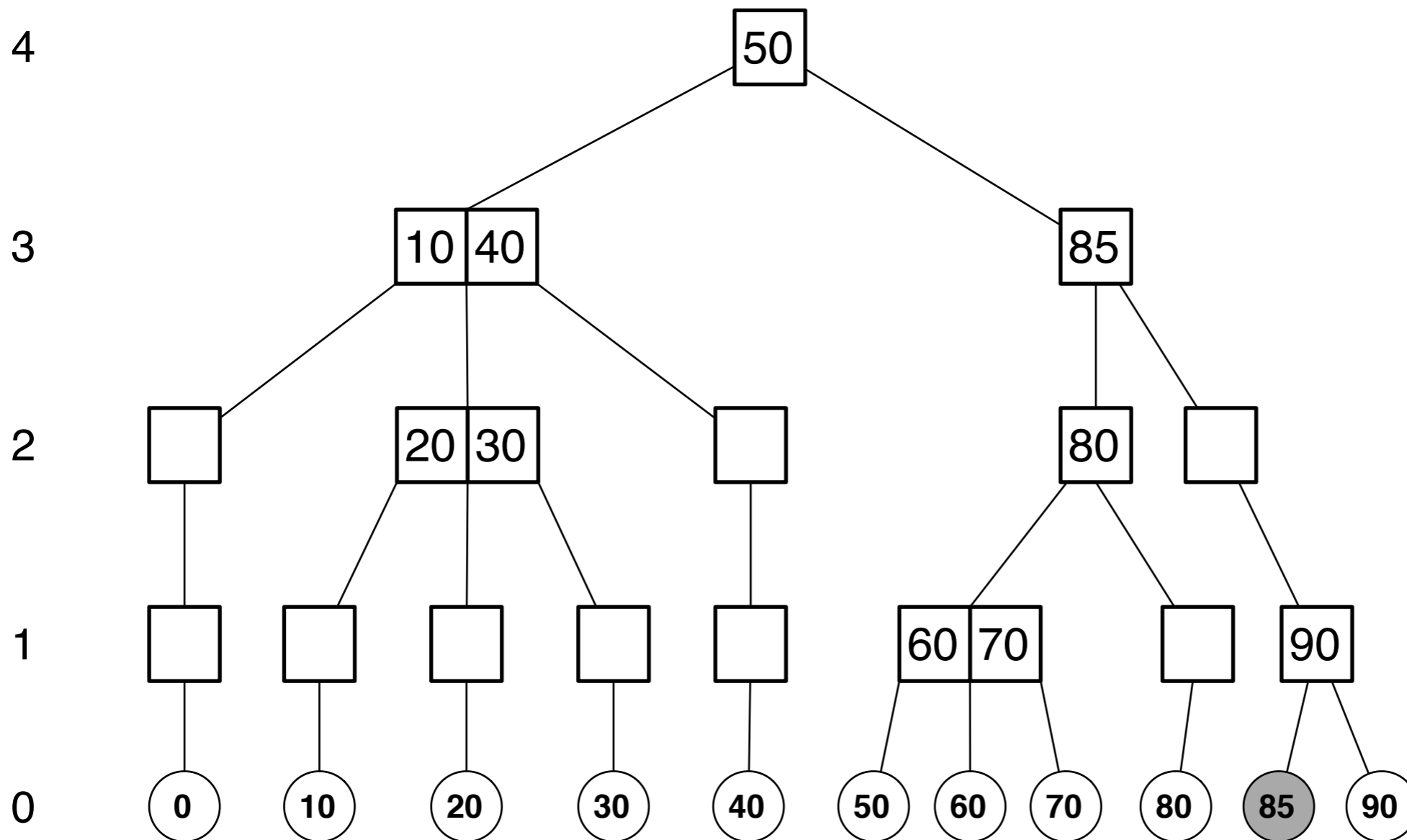
1

0



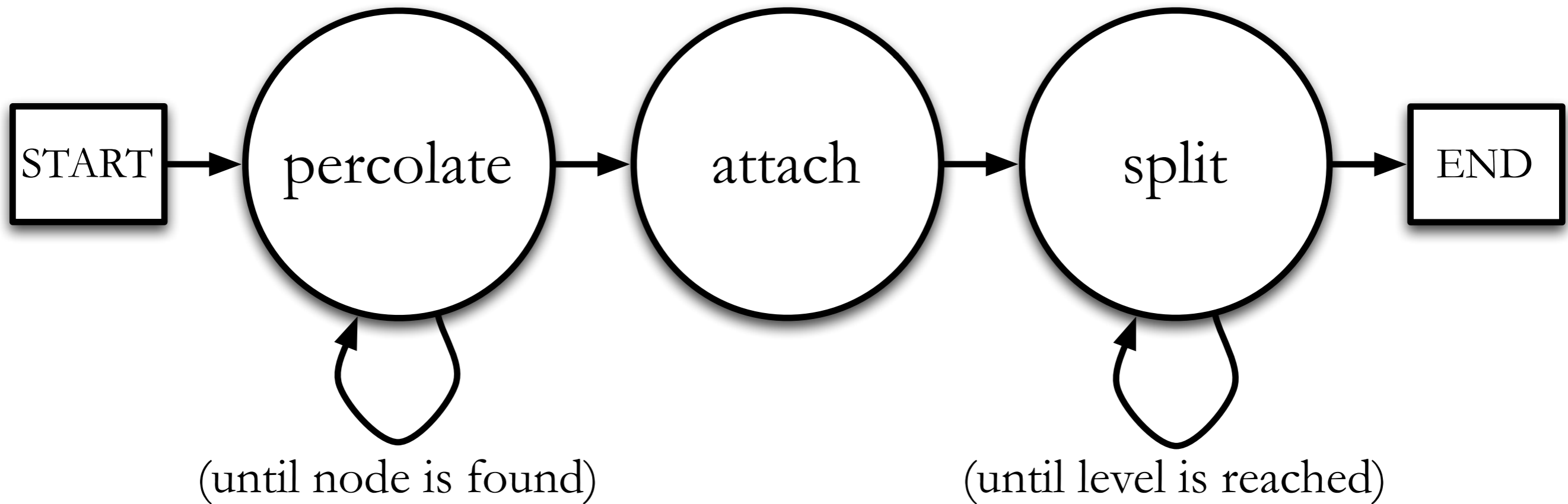
# Skip Trees - Insertion

Level



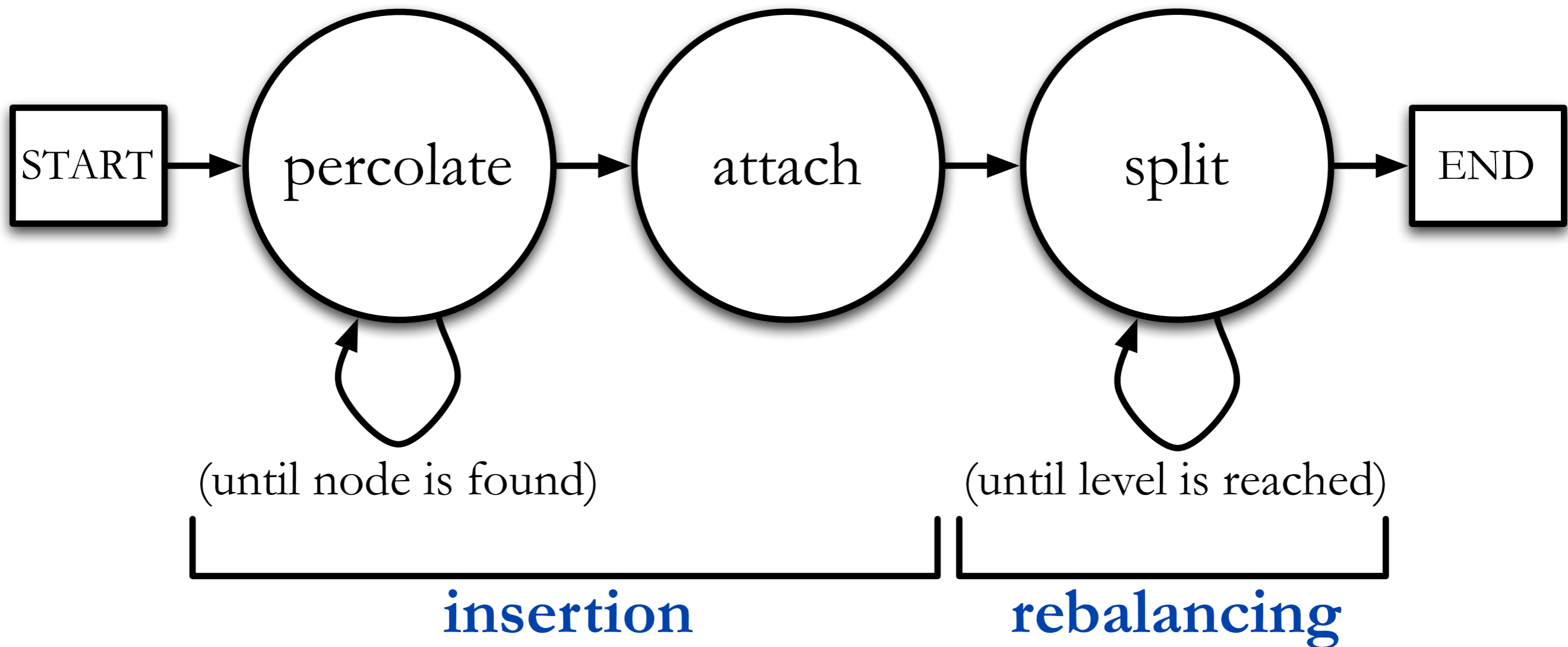
# Concurrent Insertion

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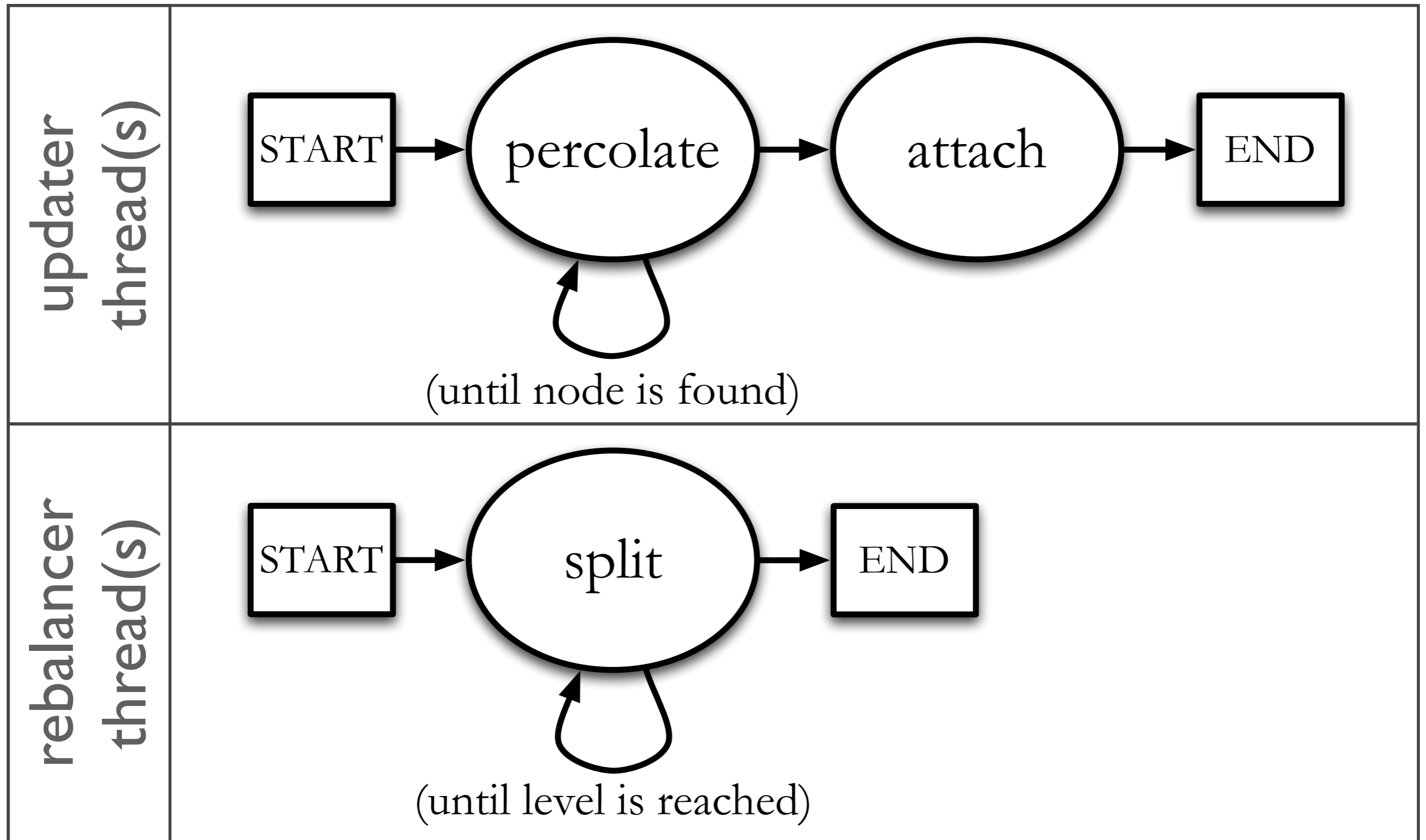


# Concurrent Insertion

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# Concurrent Insertion



# Concurrent Insertion

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## **approach summary**

- updater and rebalancer threads run in parallel
- several rebalancer threads can coexist
- percolate and split have local scope
- concurrent deletion is similar to insertion

# Project Plan

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- study the effects of
  - # of reader threads
  - # of updater threads
  - # of rebalancer threads

# References

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1. Xavier Messeguer. Skip trees, an alternative data structure to skip lists in a concurrent approach. *RAIRO Theoretical Informatics and Applications*, 31(3):251–269, May 1997.
2. Otto Nurmi and Eljas Soisalon-Soininen. Chromatic binary search trees: a structure for concurrent rebalancing. *Acta Informatica*, 31(6):547–557, September 1996.
3. William Pugh. Skip lists: A probabilistic alternative to balanced trees. *Communications of the ACM*, 33(6):668–676, June 1990.