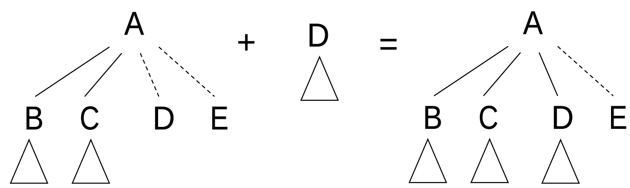
Earley's Algorithm (1970)

Nice combo of our parsing ideas so far:

- no restrictions on the form of the grammar:
  - A → B C spoon D x
- incremental parsing (left to right, like humans)
- left context constrains parsing of subsequent words
  - so waste less time building impossible things
  - makes it faster than $O(n^3)$ for many grammars

Overview of Earley's Algorithm

- Finds constituents and partial constituents in input
  - A → B C . D E is partial: only the first half of the A



A → B C . D E   A → B C D . E

Overview of Earley's Algorithm

- Proceeds incrementally, left-to-right
  - Before it reads word 5, it has already built all hypotheses that are consistent with first 4 words
  - Reads word 5 & attaches it to immediately preceding hypotheses. Might yield new constituents that are then attached to hypotheses immediately preceding *them* …
  - E.g., attaching D to A → B C . D E gives A → B C D . E
  - Attaching E to that gives A → B C D E .
  - Now we have a complete A that we can attach to hypotheses immediately preceding the A, etc.

Our Usual Example Grammar

ROOT → S			
S	→ NP VP	NP	→ Papa
NP	→ Det N	N	→ caviar
NP	→ NP PP	N	→ spoon
VP	→ VP PP	V	→ ate
VP	→ V NP	P	→ with
PP	→ P NP	Det	→ the
		Det	→ a

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → VP PP	NP → Papa	V → ate
S → NP VP	VP → V NP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0 "goal stack"
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . VP PP 1
        - 1 VP → . VP PP 1
          1 VP → . VP PP 1

            oops, stack overflowed
  - OK, let's pretend that didn't happen.
  - Let's suppose we didn't see VP → VP PP, and used VP → V NP instead.

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1 — after dot = nonterminal, so recursively look for it ("**predict**")
    - 1 VP → . V NP 1 — after dot = nonterminal, so recursively look for it ("**predict**")
      - 1 V → . ate 1 — after dot = terminal, so look for it in the input ("**scan**")
      - 1 V → ate . 2 — after dot = nothing, so parent's subgoal is completed ("**attach**")
    - 1 VP → V . NP 2 — **predict** (next subgoal)
      - 2 NP → . ... 2 — do some more parsing and eventually ...
      - 2 NP → ... . 7 — we complete the parent's NP subgoal, so **attach**
    - 1 VP → V NP . 7 — **attach** again
  - 0 S → NP VP . 7 — **attach** again

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . V NP 1
      - 1 V → . ate 1
      - 1 V → ate . 2
    - 1 VP → V . NP 2
      - 2 NP → 2
      - 2 NP → 7
    - 1 VP → V NP . 7
  - 0 S → NP VP . 7

implement by function calls:
S() calls NP() and VP(), which recurse

must backtrack to try predicting
a different VP rule here instead

But how about the other parse?

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . V NP 1
        - 1 V → . ate 1
        - 1 V → ate . 2
      - 1 VP → V . NP 2
        - 2 NP → 2
        - 2 NP → 4

we'd better backtrack here too!
(why?)

do some more parsing and eventually ...
... the correct NP is from 2 to 4 this time
(but might we find the one from 2 to 7 instead?)

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . VP PP 1
        - 1 VP → . VP PP 1
          - 1 VP → . VP PP 1
            - 1 VP → . VP PP 1

oops, stack overflowed
no fix after all
– must transform grammar to eliminate left-recursive rules

Use a Parse Table

- Earley's algorithm resembles recursive descent, but solves the left-recursion problem. No recursive function calls.
- Use a parse table as we did in CKY, so we can look up anything we've discovered so far. **"Dynamic programming."**
- Entries in column 5 look like (3, S → NP . VP)
  (but we'll omit the → etc. to save space)
  - Built while processing word 5
  - Means that the input substring from 3 to 5 matches the initial NP portion of a **S → NP VP** rule
  - Dot shows how much we've matched as of column 5
  - Perfectly fine to have entries like **(3, S → is it . true that S)**

Use a Parse Table

- Entries in column 5 look like (3, S → NP . VP)
- What does it mean if we have this entry?
  - *Unknown right context: Doesn't* mean we'll necessarily be able to find a VP starting at column 5 to complete the S.
  - *Known left context: Does* mean that some dotted rule back in column 3 is looking for an S that starts at 3.
    - So if we actually do find a VP starting at column 5, allowing us to complete the S, then we'll be able to attach the S to something.
    - And when that something is complete, it too will have a customer to *its* left ... just as in recursive descent!
    - In short, a top-down (i.e., goal-directed) parser: it chooses to start building a constituent not because of the input but because that's what the left context needs. In **the spoon**, won't build **spoon** as a verb because there's no way to use a verb there.
    - So any hypothesis in column 5 *could* get used in the correct parse, if words 1-5 are continued in just the right way by words 6-n.

Operation of the Algorithm

- Process all hypotheses one at a time in order. (Current hypothesis is shown in blue.)
- This may add new hypotheses to the end of the to-do list, or try to add old hypotheses again.
- Process a hypothesis according to what follows the dot – just as in recursive descent:
  - If a word, **scan** input and see if it matches
  - If a nonterminal, **predict** ways to match it
    (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible with this limited *right context*)
  - If nothing, then we have a complete constituent, so **attach** it to all its customers

0
0 ROOT . S

initialize

Remember this stands for (0, ROOT → . S)

0
0 ROOT . S
0 S . NP VP

predict the kind of S we are looking for

Remember this stands for (0, S → . NP VP)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa

predict the kind of NP we are looking for
(actually we'll look for 3 kinds: any of the 3 will do)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of Det we are looking for *(2 kinds)*

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of NP we're looking for
but we were already looking for these so don't add duplicate goals! Note that this happened when we were processing a left-recursive rule.

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: the desired word is in the input!

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach the newly created NP (which starts at 0) to its customers (incomplete constituents that *end* at 0 and have NP after the dot)

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	
0 Det . a	

predict

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	

predict

0	Papa 1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 P . with

predict

0 Papa 1	ate 2	
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: success!

0 Papa 1	ate 2	
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: failure

0 Papa 1	ate 2	
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

attach

0 Papa 1	ate 2	
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict *(these next few steps should look familiar)*

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

scan *(this time we fail since Papa is not the next word)*

0 Papa 1 ate 2 the 3			
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

scan: success!

0 Papa 1 ate 2 the 3			
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa 1 ate 2 the 3			
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa	1 ate	2 the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach
(again!)

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach
(again!)

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

attach *(again!)*

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

ate	2 the	3 caviar	4 with	5 a	6
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa	
P NP	2 Det . the		1 VP VP . PP	5 Det . the	
ate	2 Det . a		4 PP . P NP	5 Det . a	
with			0 ROOT S .		
			4 P . with		

ate	2 the	3 caviar	4 with	5 a	6
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa	
P NP	2 Det . the		1 VP VP . PP	5 Det . the	
ate	2 Det . a		4 PP . P NP	5 Det . a	
with			0 ROOT S .		
			4 P . with		

ate	2 the	3 caviar	4 with	5 a	6
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa	
P NP	2 Det . the		1 VP VP . PP	5 Det . the	
ate	2 Det . a		4 PP . P NP	5 Det . a	
with			0 ROOT S .		
			4 P . with		

ate	2 the	3 caviar	4 with	5 a	6
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa	
P NP	2 Det . the		1 VP VP . PP	5 Det . the	
ate	2 Det . a		4 PP . P NP	5 Det . a	
with			0 ROOT S .		
			4 P . with		

ate	2 the	3 caviar	4 with	5 a	6 spoon	7
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa		
P NP	2 Det . the		1 VP VP . PP	5 Det . the		
ate	2 Det . a		4 PP . P NP	5 Det . a		
with			0 ROOT S .			
			4 P . with			

ate	2 the	3 caviar	4 with	5 a	6 spoon	7
Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa		
P NP	2 Det . the		1 VP VP . PP	5 Det . the		
ate	2 Det . a		4 PP . P NP	5 Det . a		
with			0 ROOT S .			
			4 P . with			

ate 2 the 3 caviar 4 with 5 a 6 spoon 7

Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
. VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	4 PP P NP .
V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	5 NP NP . PP
VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa		
P NP	2 Det . the		1 VP VP . PP	5 Det . the		
ate	2 Det . a		4 PP . P NP	5 Det . a		
with			0 ROOT S .			
			4 P . with			

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

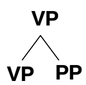
0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa 1	ate 2	the 3	caviar 4	... with a spoon 7	
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	7 PP . P NP
	1 P . with			0 ROOT S .	1 VP V NP .
				4 P . with	2 NP NP . PP
					0 S NP VP .
					1 VP VP . PP
					7 P . with
					0 ROOT S .

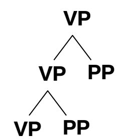
Left Recursion Kills Pure Top-Down Parsing …

VP

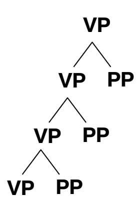
Left Recursion Kills Pure Top-Down Parsing …



Left Recursion Kills Pure Top-Down Parsing …



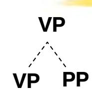
Left Recursion Kills Pure Top-Down Parsing …



makes new hypotheses ad infinitum before we've seen the PPs at all

hypotheses try to predict in advance how many PP's will arrive in input

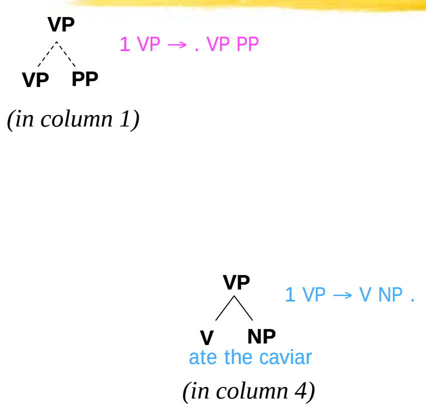
… but Earley's Alg is Okay!



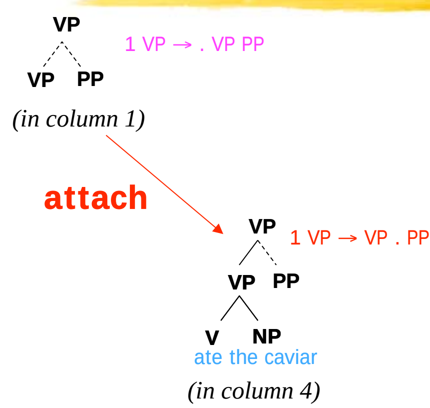
1 VP → . VP PP

(in column 1)

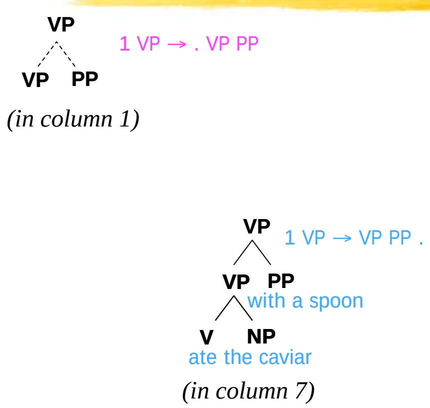
… but Earley's Alg is Okay!



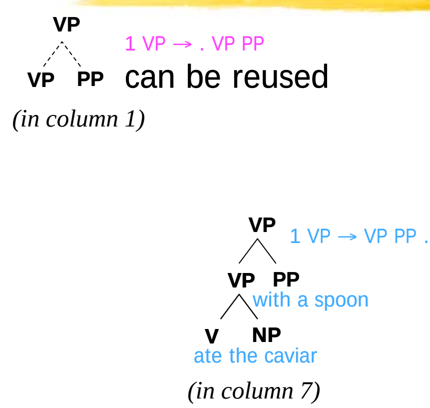
… but Earley's Alg is Okay!



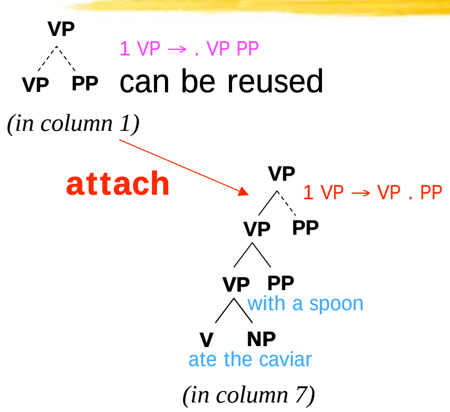
… but Earley's Alg is Okay!



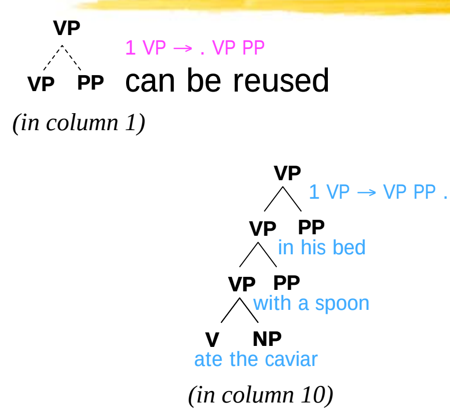
… but Earley's Alg is Okay!



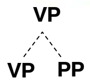
… but Earley's Alg is Okay!



… but Earley's Alg is Okay!

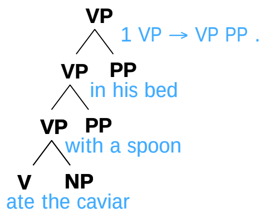


… but Earley's Alg is Okay!



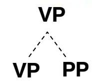
1 VP → . VP PP can be reused again

(in column 1)



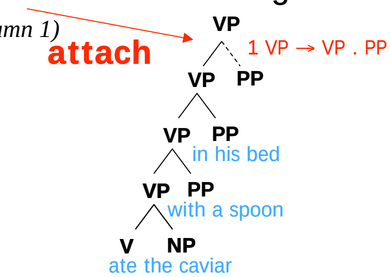
(in column 10)

… but Earley's Alg is Okay!



1 VP → . VP PP can be reused again

(in column 1)



(in column 10)

0 Papa	1 ate	2 the	3 caviar	4 with a spoon	...	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		**1 VP VP . PP**		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed a VP in col 4
col 1 lets us use it in a VP PP structure

0 Papa	1 ate	2 the	3 caviar	4 with a spoon	...	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed that VP = VP PP in col 7
col 1 would let us use *it* in a VP PP structure
can reuse col 1 as often as we need

What's the Complexity?