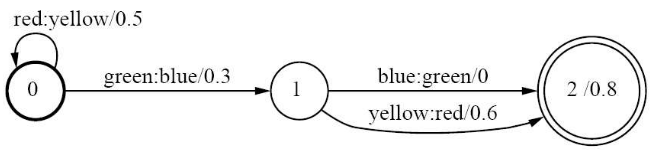
Weighted Finite State Transducer (WFST)

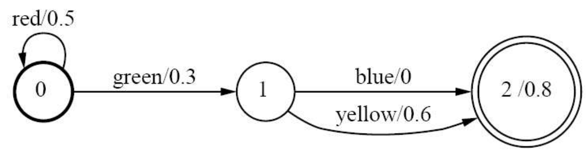
- Efficient algorithms for various operations.
- Weights
  - Handle uncertainty in text, handwritten text, speech, image, biological sequences.
- Applications:
  - Text: pattern-matching, indexation, compression.
  - Speech: speech recognition, speech synthesis.
  - Image: image compression, filters.

Weighted Finite State Transducer (WFST)

- Transducers:

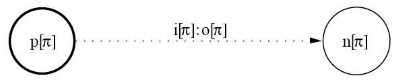


- Automata/Acceptors



WFST Definition (I)

- A path π: a sequence of transitions.
  - Original and destination states
  - Input and output labels



- A semiring ≡ a ring without negation
  - Number set K.
  - Sum $\oplus$ and Product $\otimes$.
- Semiring examples:
  - Probability semiring: *R, +, X.*
  - Tropical semiring: *R, min, +.*

WFST Definition (II)

- General Definitions
  - Alphabets: input *Σ*, output *Δ*
  - States: *Q*, initial *I*, final *F*.
  - Transitions: E → Q * (*Σ U є*) * (*Δ U є*) * *K* * *Q*
  - Initial/Final weights: *λ = I → K, ρ = F → K*
- WFST *T = (Σ, Q, I, F, E, λ, ρ):*

$$[T](x,y) = \bigoplus_{\pi \in P(I,x,y,F)} \lambda(p[\pi]) \otimes w[\pi] \otimes \rho(n[\pi])$$

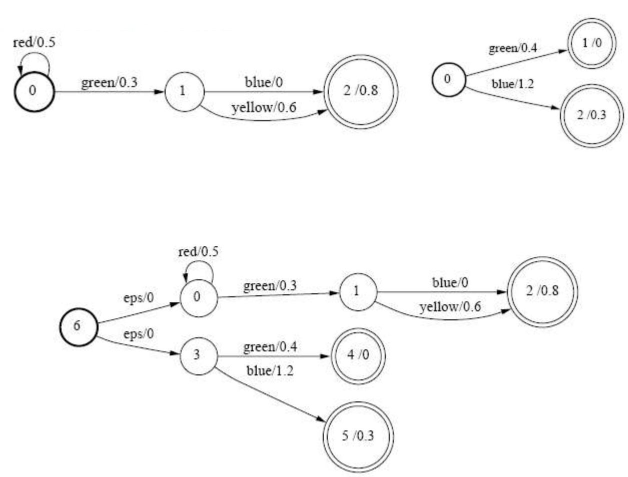
for all $x \in \Sigma^*$ and $y \in \Delta^*$.

WFST Operations

- Sum
- Product
- Closure
- Reversal
- Composition
- Determinization
- Weight pushing
- Minimization

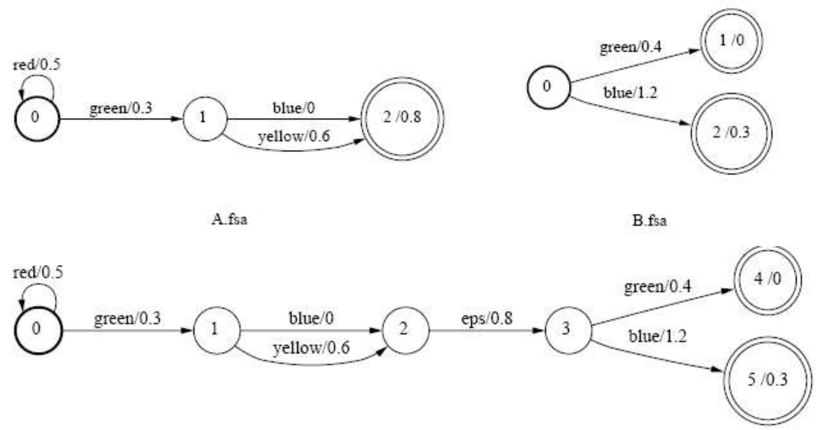
WFST Sum

- Sum: $[T_1 \oplus T_2](x, y) = [T_1](x, y) \oplus [T_2](x, y)$



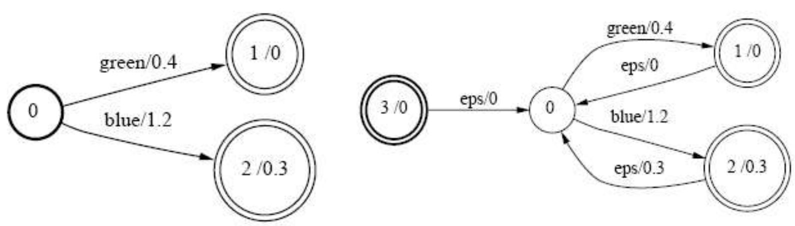
WFST Product

- **Product:** $[T_1 \otimes T_2](x,y) = \bigoplus_{x=x_1x_2,\, y=y_1y_2} [T_1](x_1,y_1) \otimes [T_2](x_2,y_2)$



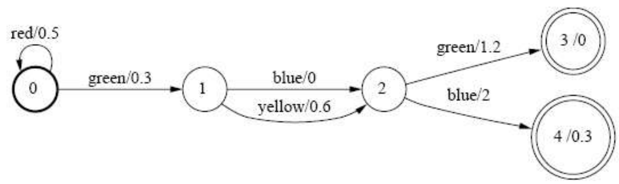
WFST Closure

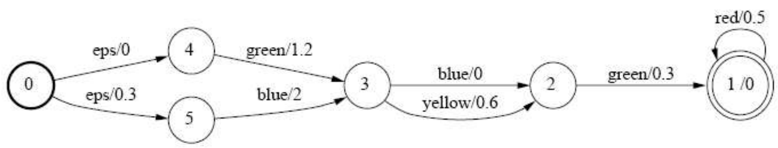
- **Closure:** $[T^*](x,y) = \bigoplus_{n=0}^{\infty} [T]^n(x,y)$



WFST Reversal

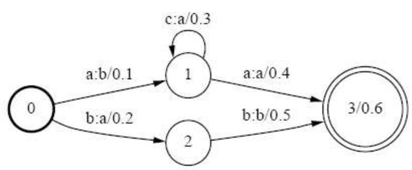
- **Reversal:** $\left[\widetilde{T}\right](x,y) = \left[T\right](\widetilde{x},\widetilde{y})$

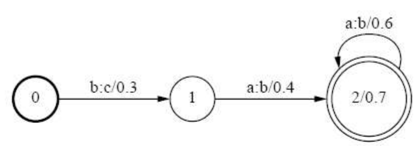


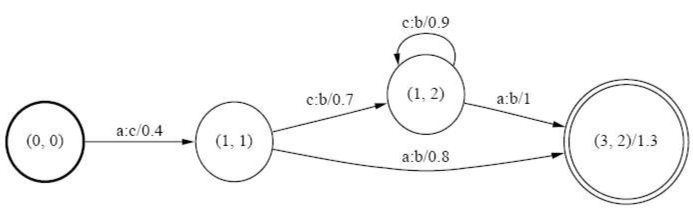


WFST Composition

- **Composition:** $\left[T_1 \circ T_2\right](x,y) = \bigoplus_{z} \left[T_1\right](x,z) \otimes [T_2](z,y)$







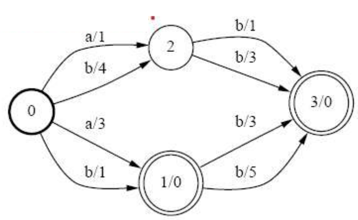
WFST Composition Algorithm

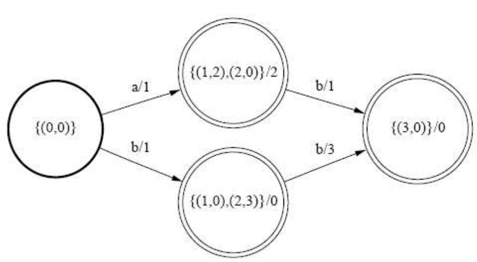
WEIGHTED-COMPOSITION(T_1, T_2)

$Q \leftarrow I_1 \times I_2$
$S \leftarrow I_1 \times I_2$
while $S \neq \emptyset$ **do**
    $(q_1, q_2) \leftarrow$ HEAD(S)
    DEQUEUE(S)
    if $(q_1, q_2) \in I_1 \times I_2$ **then**
        $I \leftarrow I \cup \{(q_1, q_2)\}$
        $\lambda(q_1, q_2) \leftarrow \lambda_1(q_1) \otimes \lambda_2(q_2)$
    if $(q_1, q_2) \in F_1 \times F_2$ **then**
        $F \leftarrow F \cup \{(q_1, q_2)\}$
        $\rho(q_1, q_2) \leftarrow \rho_1(q_1) \otimes \rho_2(q_2)$
    for each $(e_1, e_2) \in E[q_1] \times E[q_2]$ such that $o[e_1] = i[e_2]$ **do**
        if $(n[e_1], n[e_2]) \notin Q$ **then**
            $Q \leftarrow Q \cup \{(n[e_1], n[e_2])\}$
            ENQUEUE$(S, (n[e_1], n[e_2]))$
        $E \leftarrow E \cup \{((q_1, q_2), i[e_1], o[e_2], w[e_1] \otimes w[e_2], (n[e_1], n[e_2]))\}$
return T

WFST Determinization

- Deterministic WFST: no common input label for all outgoing transitions from any state.
- Determinimization: determinizable WFST → deterministic W.





WFST Determinization Algorithm

WEIGHTED-DETERMINIZATION(A)

$i' \leftarrow \{(i, \lambda(i)) : i \in I\}$

$\lambda'(i') \leftarrow \overline{1}$

$S \leftarrow \{i'\}$

while $S \neq \emptyset$ **do**

  $p' \leftarrow$ HEAD(S)

  DEQUEUE(S)

  for each $x \in i[E[Q[p']]]$ **do**

    $w' \leftarrow \bigoplus \{v \otimes w : (p, v) \in p', (p, x, w, q) \in E\}$

    $q' \leftarrow \{(q, \bigoplus \{w'^{-1} \otimes (v \otimes w) : (p, v) \in p', (p, x, w, q) \in E\}) : q = n[e], i[e] = x, e \in E[Q[p']]\}$

    $E' \leftarrow E' \cup \{(p', x, w', q')\}$

    if $q' \notin Q'$ **then**

      $Q' \leftarrow Q' \cup \{q'\}$

      if $Q[q'] \cap F \neq \emptyset$ **then**

        $F' \leftarrow F' \cup \{q'\}$

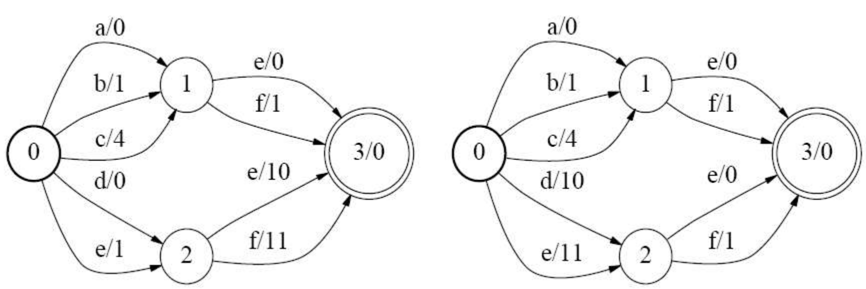
        $\rho'(q') \leftarrow \bigoplus \{v \otimes \rho(q) : (q, v) \in q', q \in F\}$

      ENQUEUE(S, q')

return T'

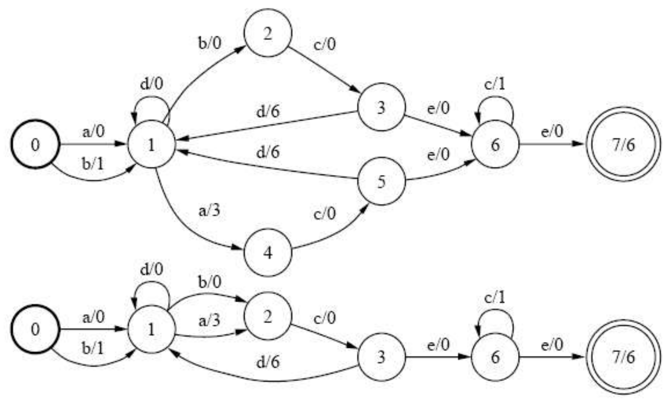
WFST Weights Pushing

- **Weight pushing: re-distribute all weights along paths.**



WFST Minimization

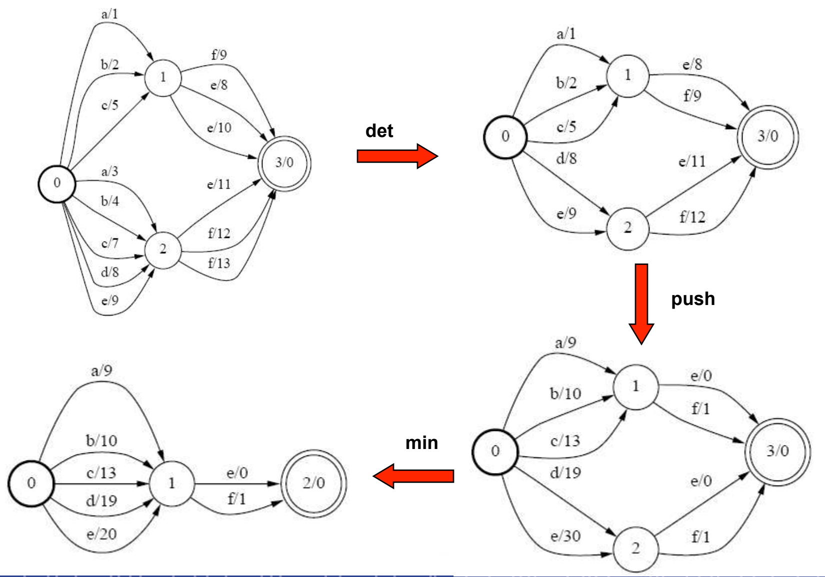
- Minimize number of states and transitions of a deterministic WFST.



WFST Operations

- Composition: *C = A ○ B*
- Determinization: *D = det(C)*
  - *deterministic automaton: every state has at most one out-going transition with any given label.*
- Re-weighting (Weight pushing): *E = push(D)*
- Minimization: *F = min(E)*

WFST Operations: Examples

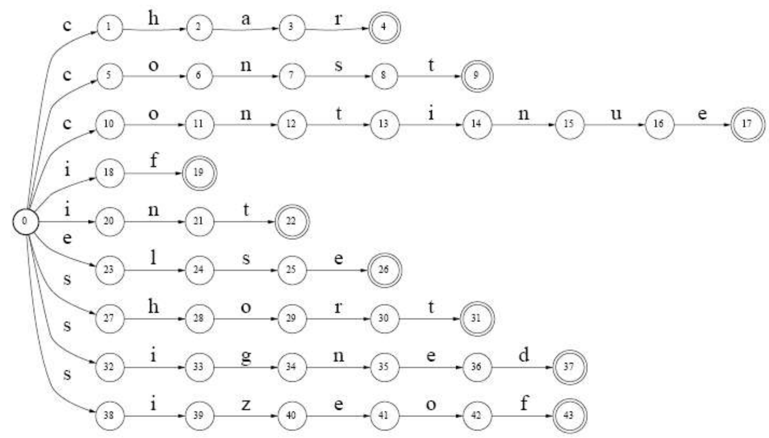


Example I:
keyword detection

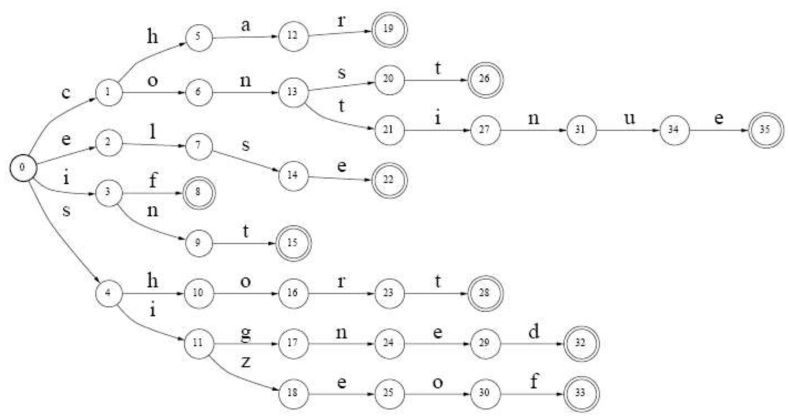
- **C identifiers:**
  $\{char, const, continue, if, int, else, short, signed, sizeof\}$
- **Brute-force search:**

```
if(strcmp(token, "char") == 0) return 1;
if(strcmp(token, "const") == 0) return 1;
if(strcmp(token, "oontinue") == 0) return 1;
if(strcmp(token, "if") == 0) return 1;
if(strcmp(token, "int") == 0) return 1;
if(strcmp(token, "else") == 0) return 1;
if(strcmp(token, "short") == 0) return 1;
if(strcmp(token, "signed") == 0) return 1;
if(strcmp(token, "sizeof") == 0) return 1;
else return 0;
```

Example I: keyword detection: Automata Search



Example I: keyword detection: Deterministic Search



Example I: keyword detection: Minimal Deterministic Search