

# ENG2200

## Electric Circuits

### Chapter 4

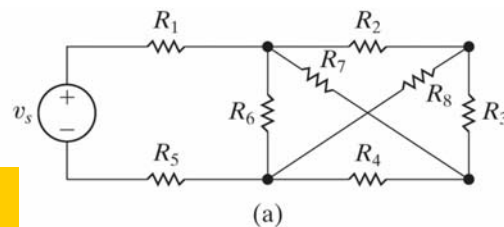
#### Techniques for Circuit Analysis

### Objectives

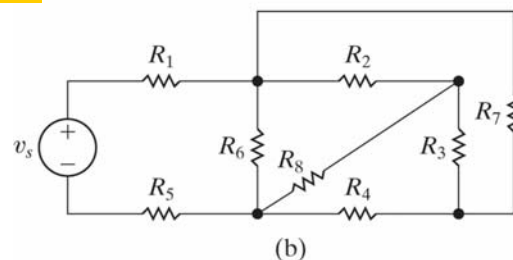
- Understand and being able to use node-voltage and mesh-current methods to solve a circuit.
- Understand source transformation and being able to use it to solve a circuit.
- Understand the concept of Thevenin and Norton equivalent circuits and being able to construct them for any circuit.
- Know the condition for maximum power transfer to a resistive load, and being able to calculate the value of the load resistor that achieves that.

## Terminology

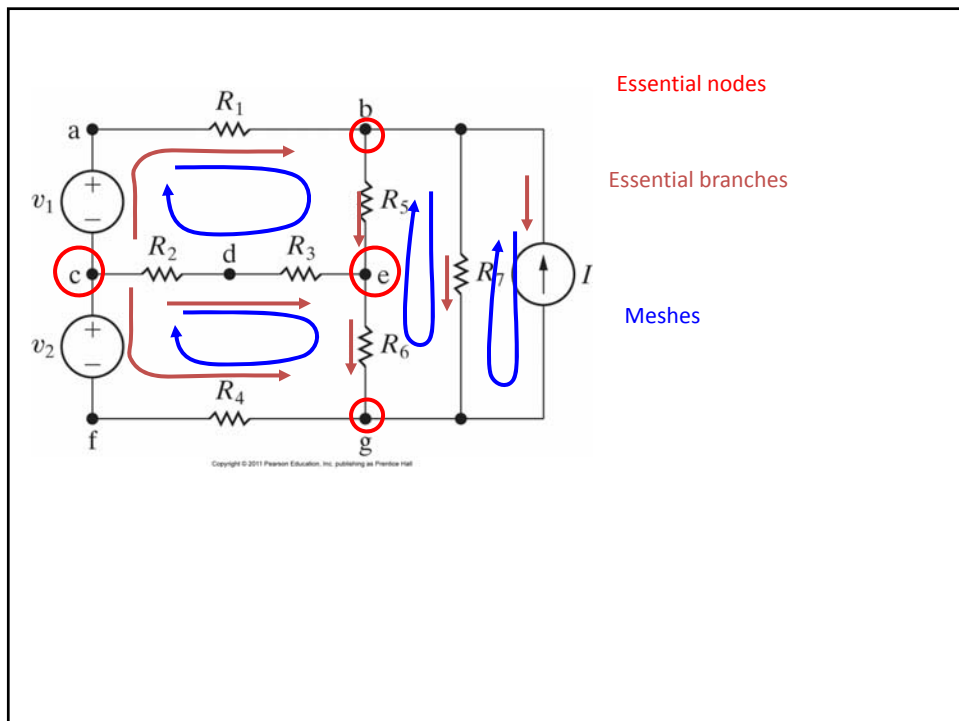
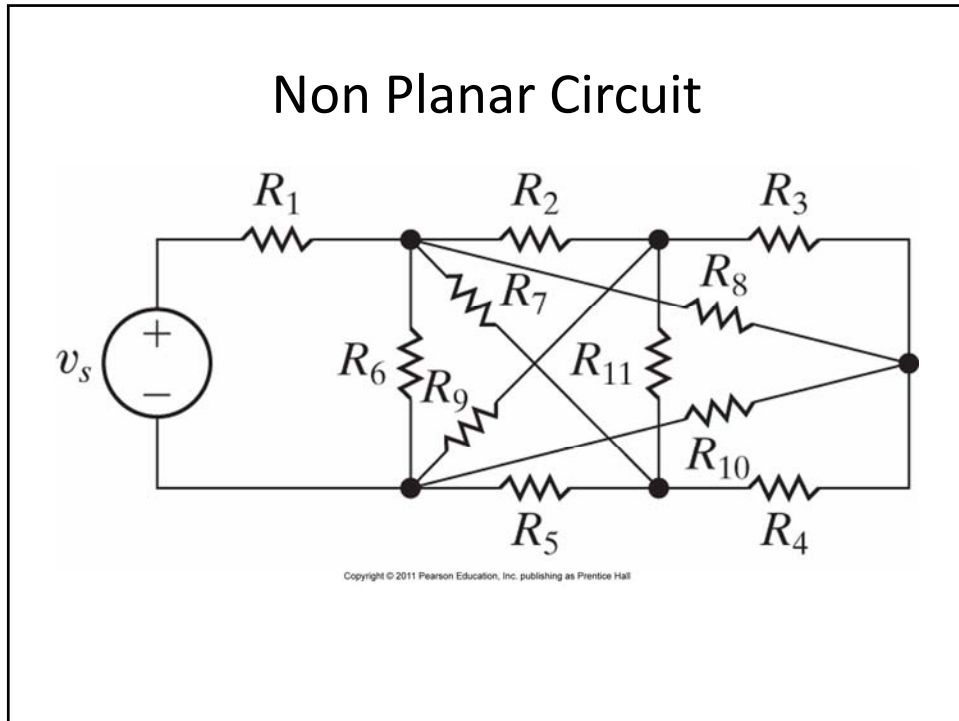
- **Node:** A point that connects two or more circuit elements.
- **Essential node:** A node where 3 or more circuit elements join.
- **Path:** A trace of adjoining basic elements with no element included twice.
- **Branch:** A path that connects two nodes.
- **Essential path:** A path which connects two essential nodes without passing through an essential node.
- **Loop:** A path whose last node is the same as the first
- **Mesh:** A loop that does not enclose any other loops
- **Planar circuit:** A circuit that could be drawn on a plane with no crossing branches.



Planar Circuit (same circuit).

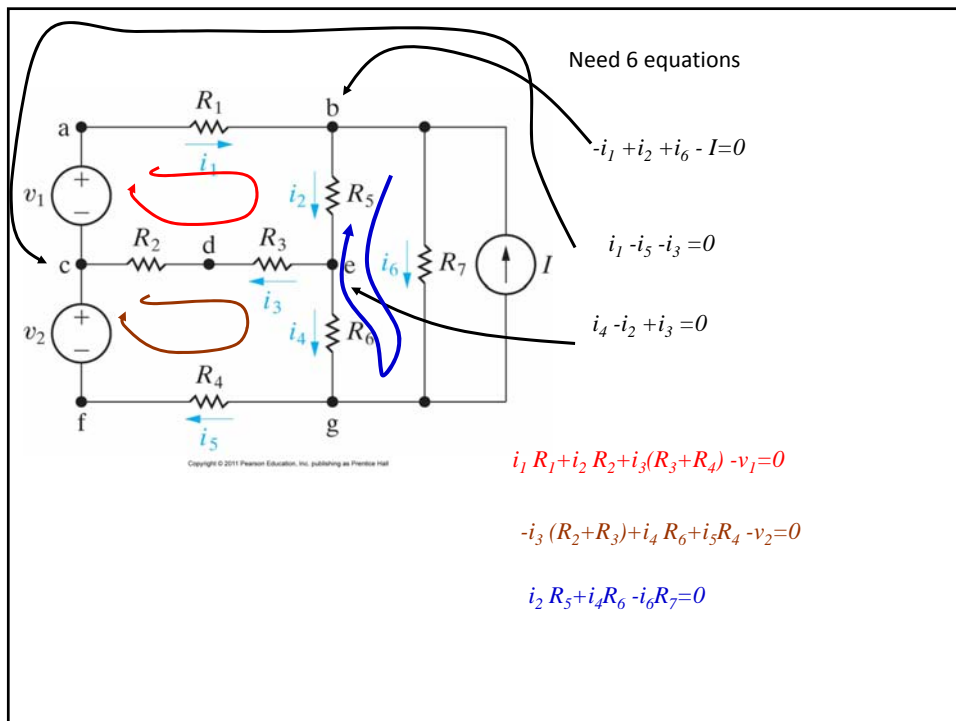


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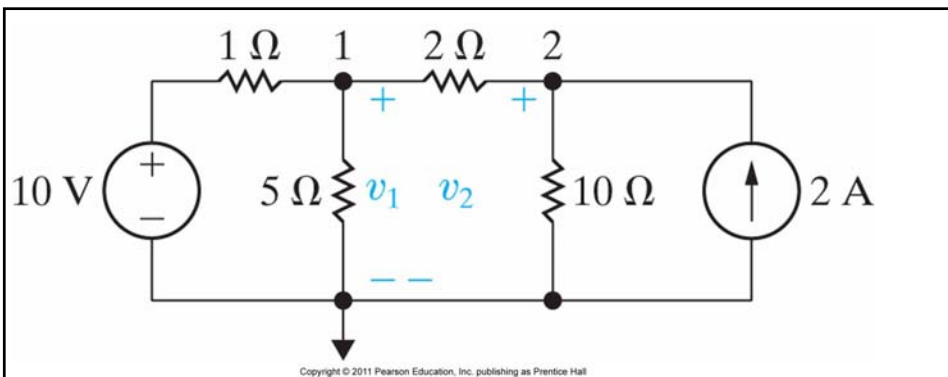
## Simultaneous Equations

- In a circuit with  $b$  essential branches, and  $n$  essential nodes
- Get  $n-1$  equations by applying KCL at  $n-1$  nodes (**why not  $n$ ?**)
- Still need  $b-n+1$  equations to solve for the currents in each essential branch.
- Use KVL on loops or meshes to get the remaining  $b-n+1$  equations
- Solve them.



## Node-Voltage Method

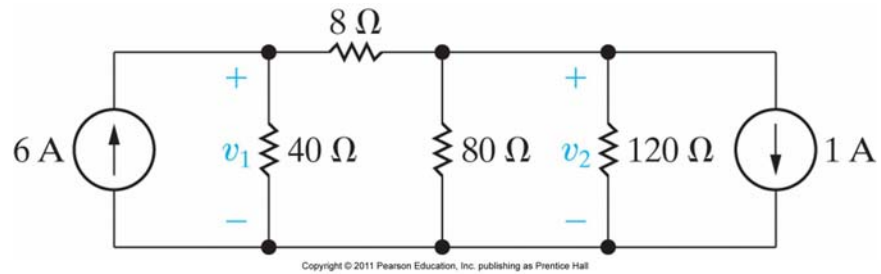
- Draw the circuit in a neat way
- If the circuit has  $n$  essential nodes, we need  $n-1$  equations.
- Select one of the  $n$  nodes as a reference node (usually the one with the most branches).
- Define the node voltages for the  $n-1$  nodes (voltage rise from reference to the node).
- Write KCL for the  $n-1$  nodes as a function of the node voltage.



$$\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0$$

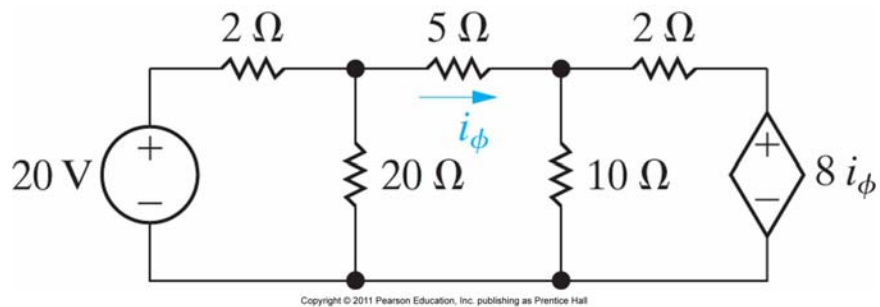
$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

## Example



## Node-Voltage and Dependent Sources.

- Another constraint is added by the dependent source



The circuit diagram shows a 20 V DC voltage source on the left. A 2 Ω resistor is in series with the positive terminal. Node 1 is the node after this resistor, connected to a 20 Ω resistor. A 5 Ω resistor is between node 1 and node 2. Node 2 is connected to a 10 Ω resistor and a 2 Ω resistor. The 2 Ω resistor is in series with a dependent current source of 8*i<sub>φ</sub>* pointing downwards. The current *i<sub>φ</sub>* is defined as the current flowing from node 1 to node 2 through the 5 Ω resistor. Node voltages are *v<sub>1</sub>* at node 1 and *v<sub>2</sub>* at node 2. A red handwritten 'V1' is next to the first equation.

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$$\frac{v_2 - 20}{2} + \frac{v_1 - v_2}{5} + \frac{v_1}{20} = 0$$

$$\frac{v_2 - 8i_\phi}{2} + \frac{v_2 - v_1}{5} + \frac{v_2}{10} = 0$$

$$i_\phi = \frac{v_1 - v_2}{5}$$

### Special cases

The circuit diagram shows a 100 V DC voltage source on the left. A 25 Ω resistor is connected between node 1 and the common ground. A 10 Ω resistor is between node 1 and node 2. A 50 Ω resistor is connected between node 2 and the common ground. A 5 A DC current source is connected between node 2 and the common ground, pointing upwards. Node voltages are *v<sub>1</sub>* at node 1 and *v<sub>2</sub>* at node 2. Red annotations show current expressions:  $(v_2 - v_1)/50$  flowing from node 2 to node 1 through the 10 Ω resistor, and  $v_2/50$  flowing downwards through the 50 Ω resistor. A red '5' is also shown near the current source.

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$$i + \frac{v_2 - 50}{5} + \frac{v_2}{50} = 0$$

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 Add these 2 equations. To get rid of  $i$

$$\frac{v_3}{100} - 4 - i = 0$$

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$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

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When we have a voltage source between 2 nodes, we can use the concept of a supernode to derive the previous equation

$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

Need one more equation in  $v_2$  and  $v_3$  to be able to solve the circuit

$$v_3 = 10i_\phi + v_2$$



Using the concept of a super node

$$\frac{v_b - V_{CC}}{R_1} + \frac{v_b}{R_2} + \frac{v_c}{R_E} - \beta i_B = 0$$

Need 2 more equation to get rid of  $v_c$  and  $i_B$

$$v_c = v_b - V_0$$

$$v_C = (i_B + \beta i_B) R_E$$

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### Example

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## Concept of reference node

2 different reference nodes

What is the voltages at a, b, c are they the same?

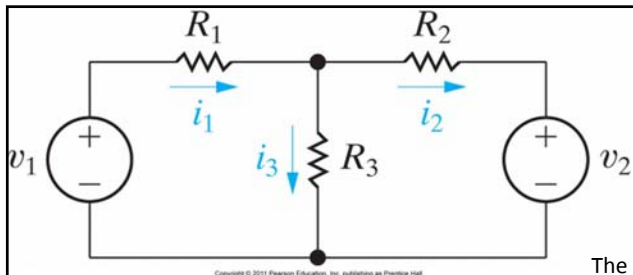
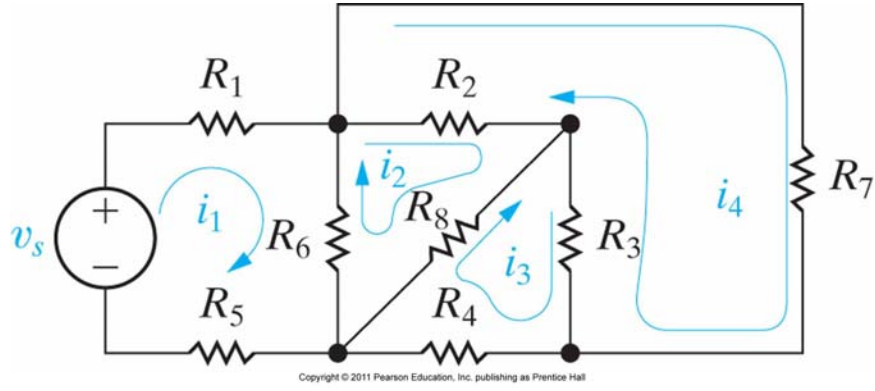
What about voltage differences?

## Mesh-Current Method

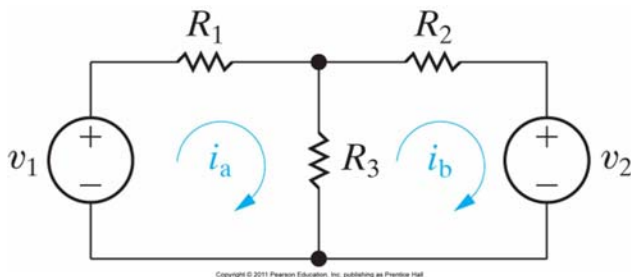
- For planar circuit
- A mesh current is a current that exists only in the perimeter in the mesh
- Mesh current may or may not be an actual current in a circuit element ( $i_1$  and  $i_2$ )

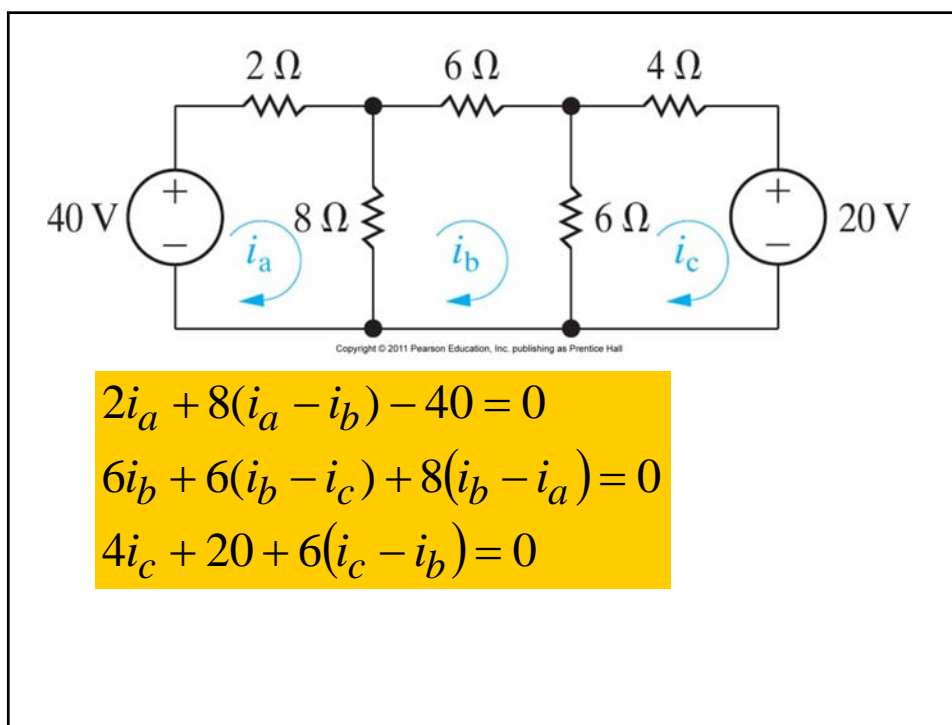
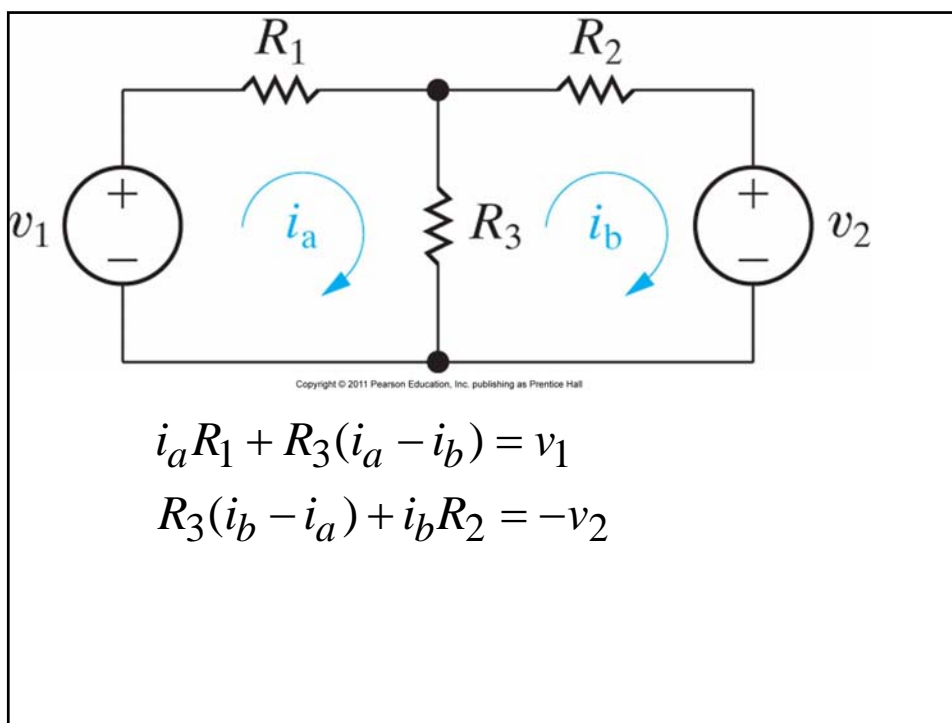
7 branches vs. 4 meshes (the number of equations you need to solve the circuit)

Is  $i_7$  an actual current in a circuit element? What about  $i_2$ ?

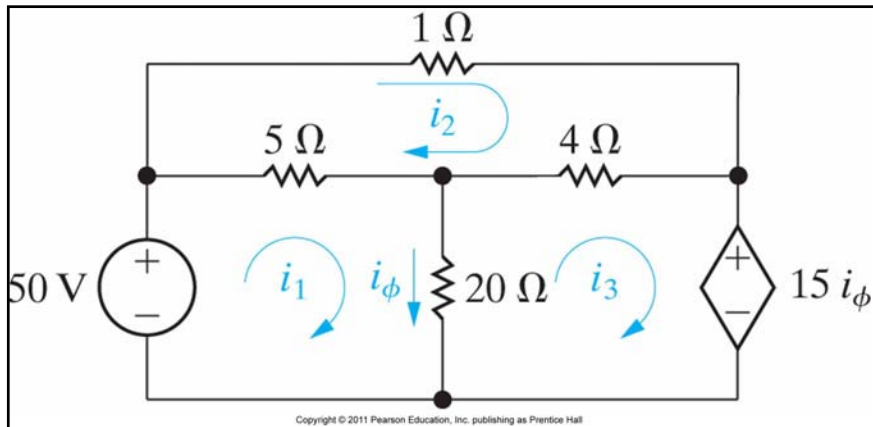
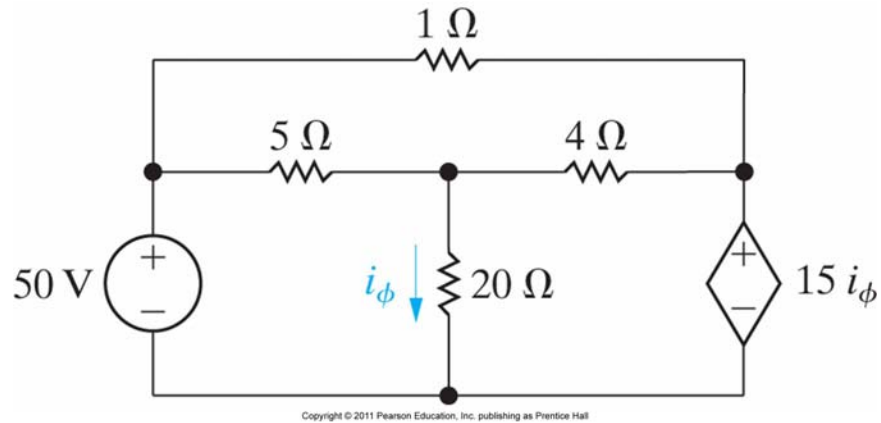


The relation between mesh currents and currents in a circuit element.





## Mesh-Current with Dependent Sources

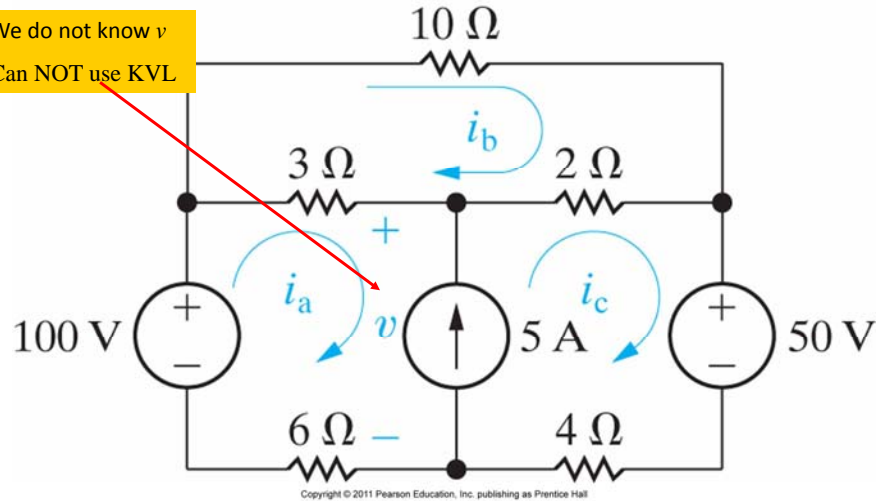


You can easily write 3 mesh-current equations. But what about  $i_\phi$

$$i_\phi = i_1 - i_2$$

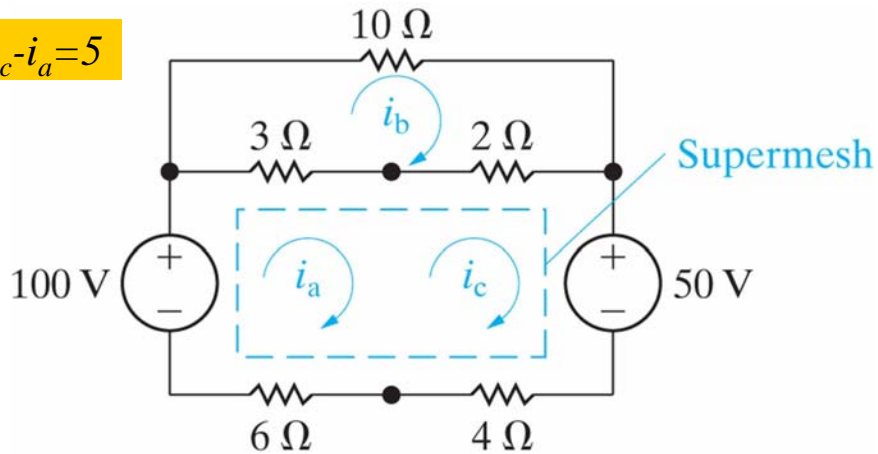
### Current Source between two nodes

We do not know  $v$   
Can NOT use KVL

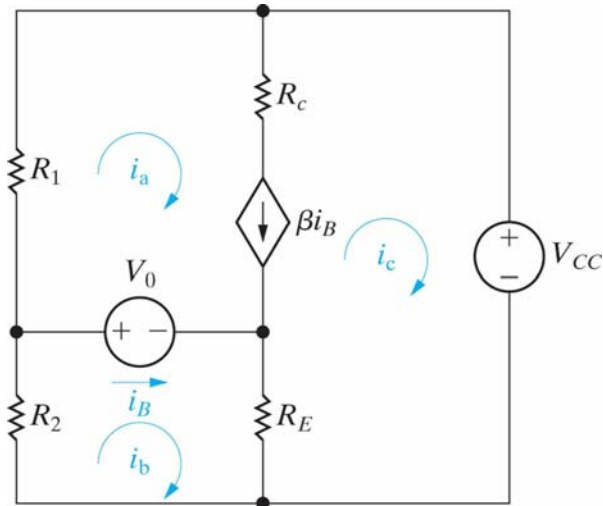


### 2 equations in 3 unknown

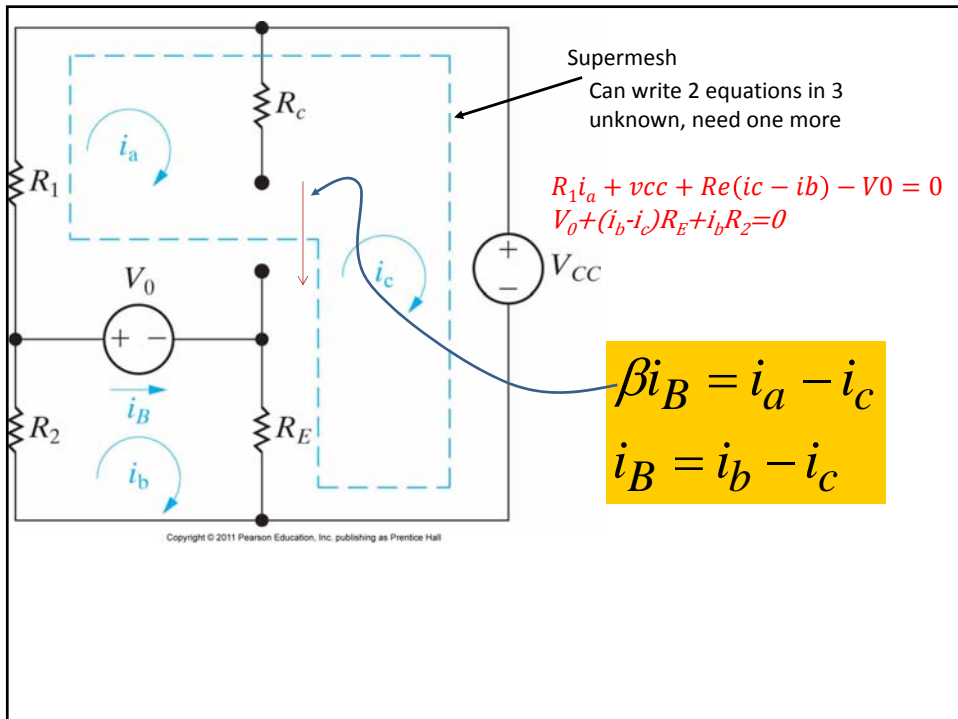
$$i_c - i_a = 5$$



We do not know the voltage drop across the current source.  
 Can not write KVL for mesh a and c without introducing another variable



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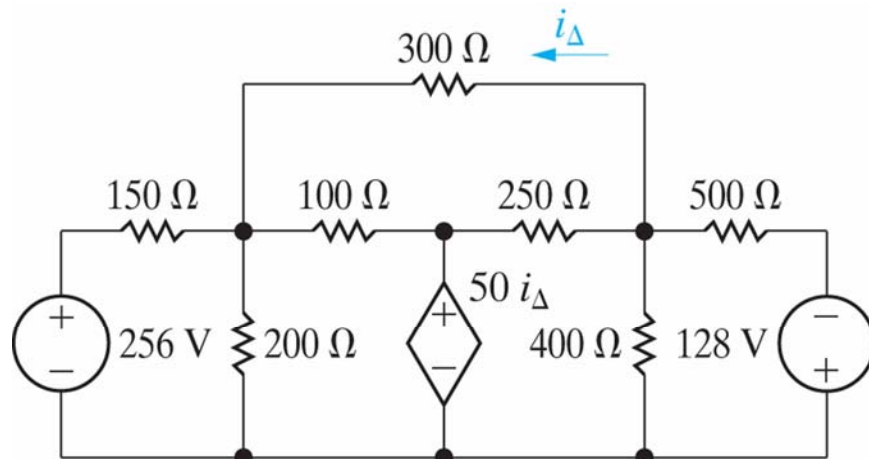


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## Node-Voltage vs. Mesh Current

- Which is better (less equations)
- Depends
  - Number of equations formed for each
  - Any supernode (node voltage)
  - Supermesh? (mesh-current)
  - Do we have to solve for the entire circuit?

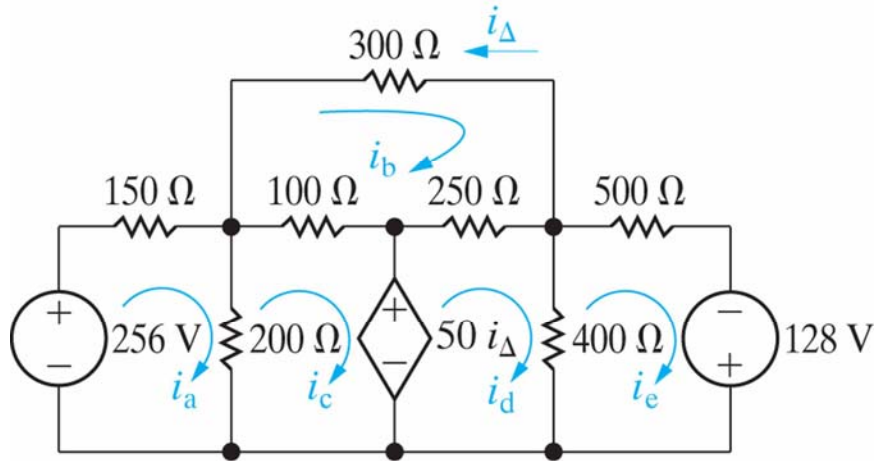
Find power in the 300 ohm R,  
which is better



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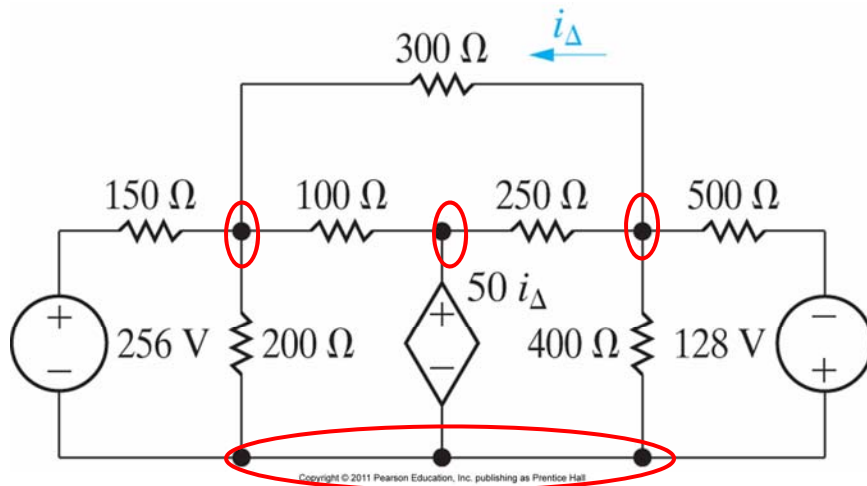


Mesh current: 5 equations (note that  $i_{\Delta} = -i_b$ )



Node voltages: 4 nodes, need only 3 equations

Which node to choose a reference node?



reference

Can not write KCL at v1 or v3 WHY?

$$\frac{v_1}{100} + \frac{v_3}{200} + \frac{v_3 + 256}{150} + \frac{v_1 - v_2}{250} + \frac{v_3 - v_2}{400} + \frac{v_3 - 128 - v_2}{500} = 0$$

$$\frac{v_2 - v_1}{250} + \frac{v_2 + 128 - v_3}{500} + \frac{v_2 - v_3}{400} + \frac{v_2}{300} = 0 \quad \text{At node 2}$$

$$v_1 - v_3 = 50i_\Delta = 50 \frac{v_2}{300} \quad \text{Constraint at supernode}$$

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$$\frac{v_a}{200} + \frac{v_a - 256}{150} + \frac{v_a - v_b}{100} + \frac{v_a - v_c}{300} = 0 \quad \text{Node a}$$

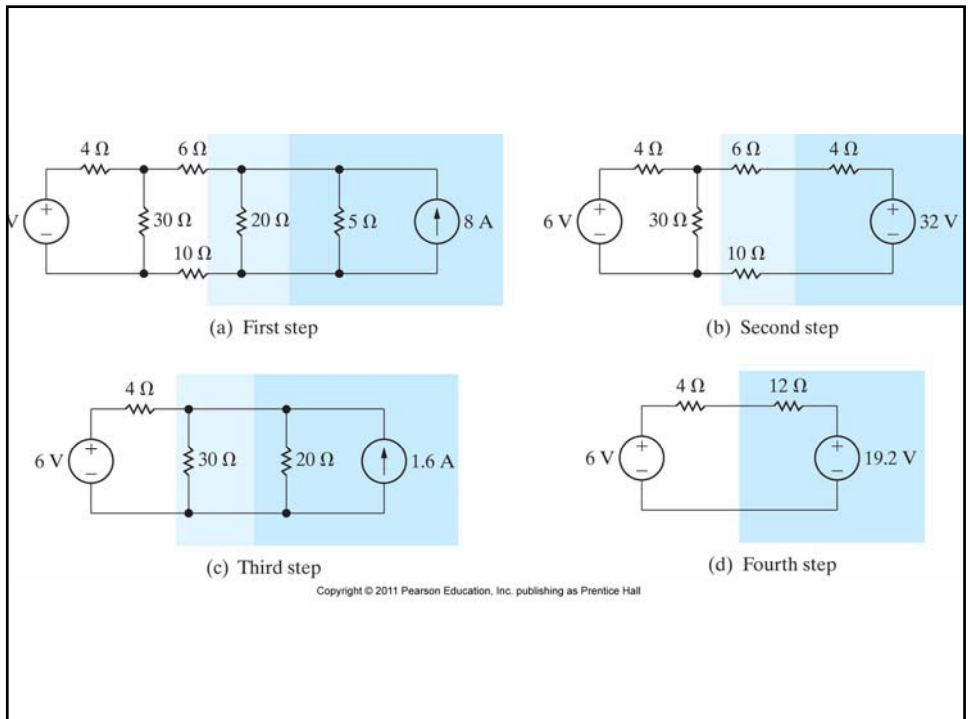
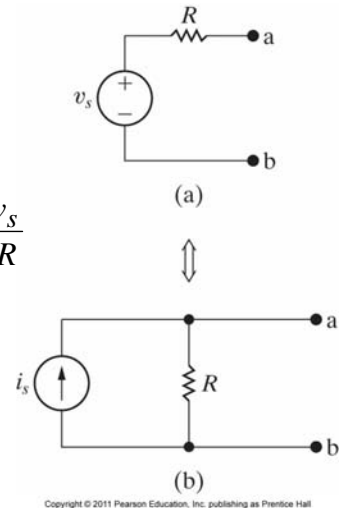
$$\frac{v_c - v_b}{250} + \frac{v_c}{400} + \frac{v_c + 128}{500} + \frac{v_c - v_a}{300} = 0 \quad \text{Node c}$$

$$v_b = 50i_\Delta = 50 \frac{v_c - v_b}{300} \quad \text{Constraint at supernode}$$

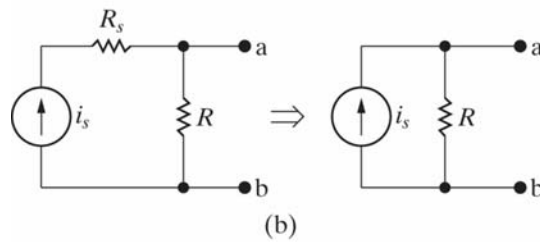
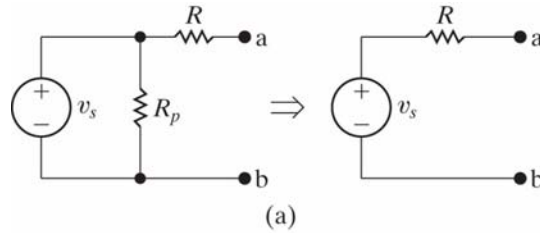
# Source Transformation

- Some times it helps in reducing the circuit complexity

Why?  $i_s = \frac{v_s}{R}$

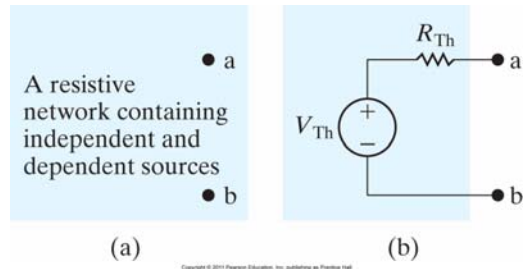


Are these 2 circuits equivalent? Power lost in  $R_p$ ?



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## Thevenin Equivalent Circuit

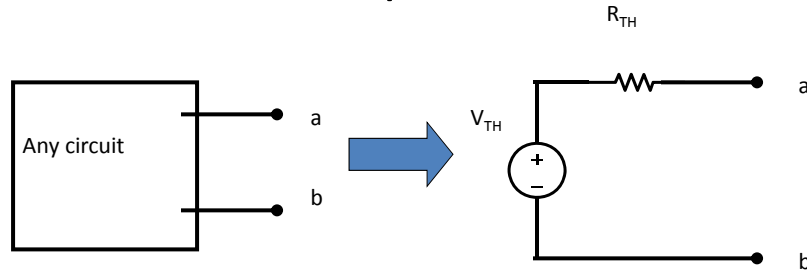


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Especially useful when we are interested in the behavior of the circuit between two terminals.

Any resistive network with dependent or independent sources could be represented as a dependent voltage source  $V_{TH}$  in series with a resistance  $R_{TH}$ .

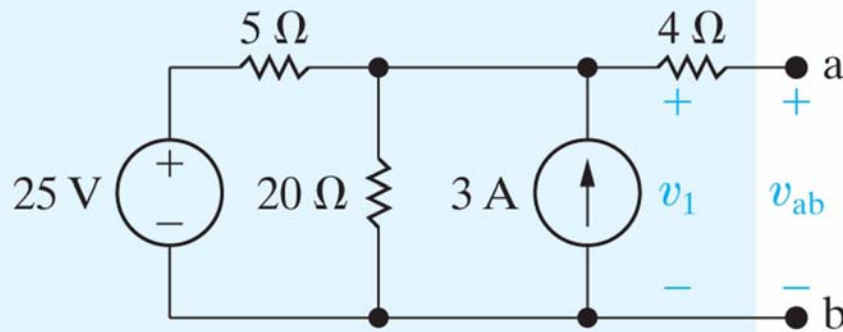
## Thevenin equivalent Circuit



$V_{TH}$  = the open circuit voltage between a and b

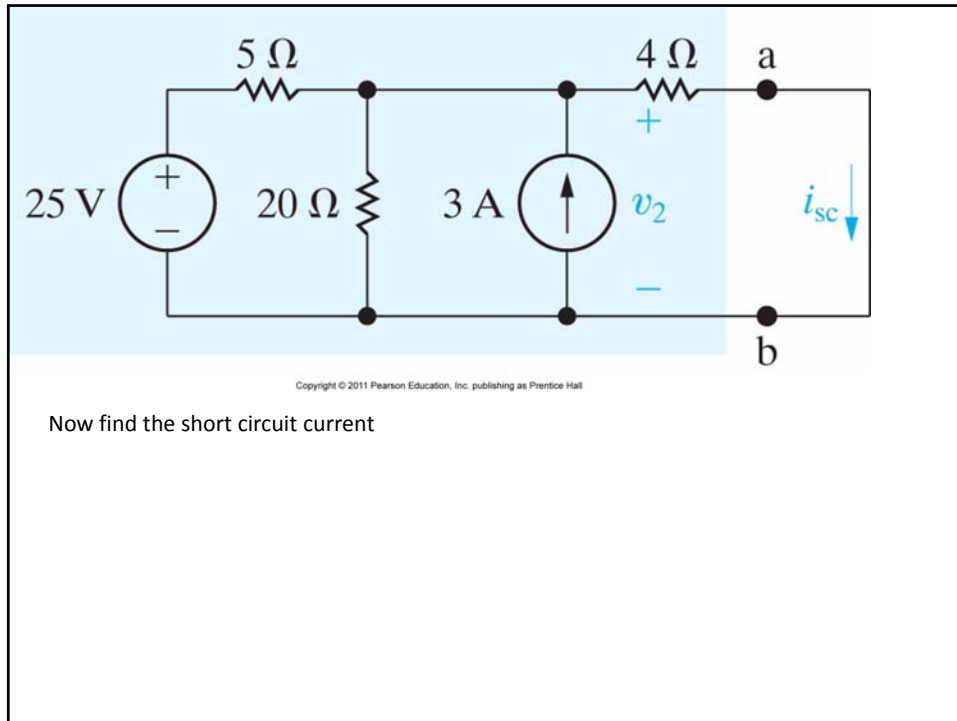
$R_{TH} = V_{TH}/I_{sc}$  where  $I_{sc}$  is the short circuit current from a to b

**EXCEPT for an ideal current source**

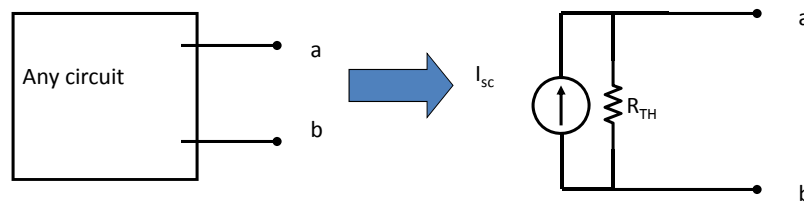


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Find Thevenin equivalent



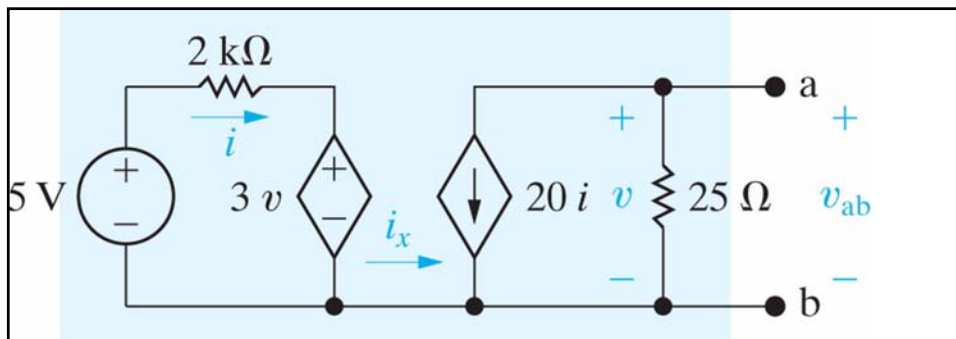
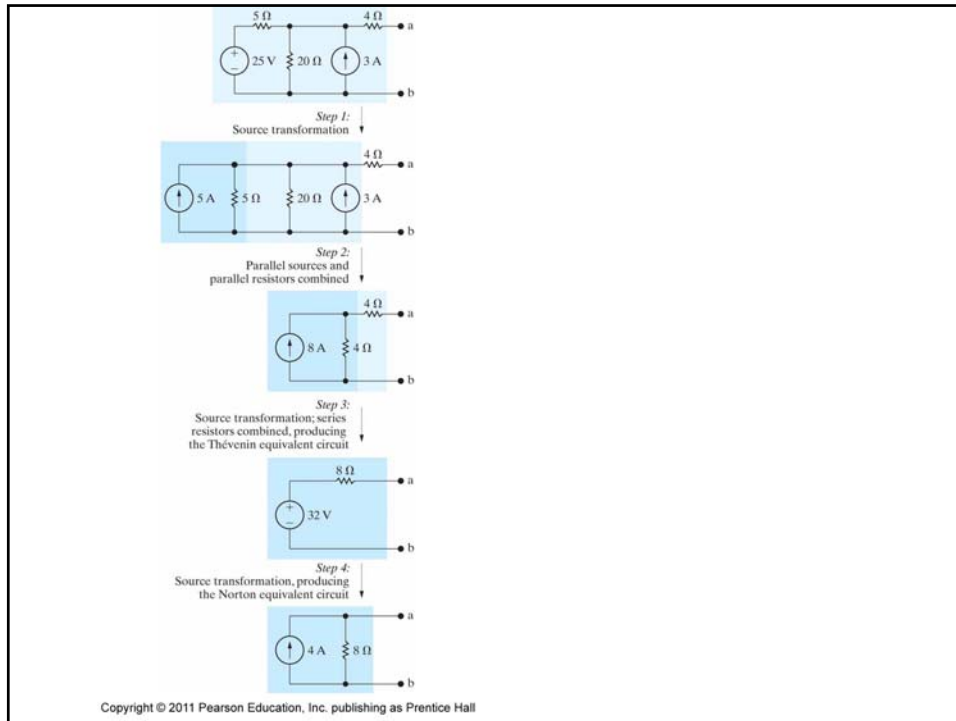
## Norton Equivalent Circuit



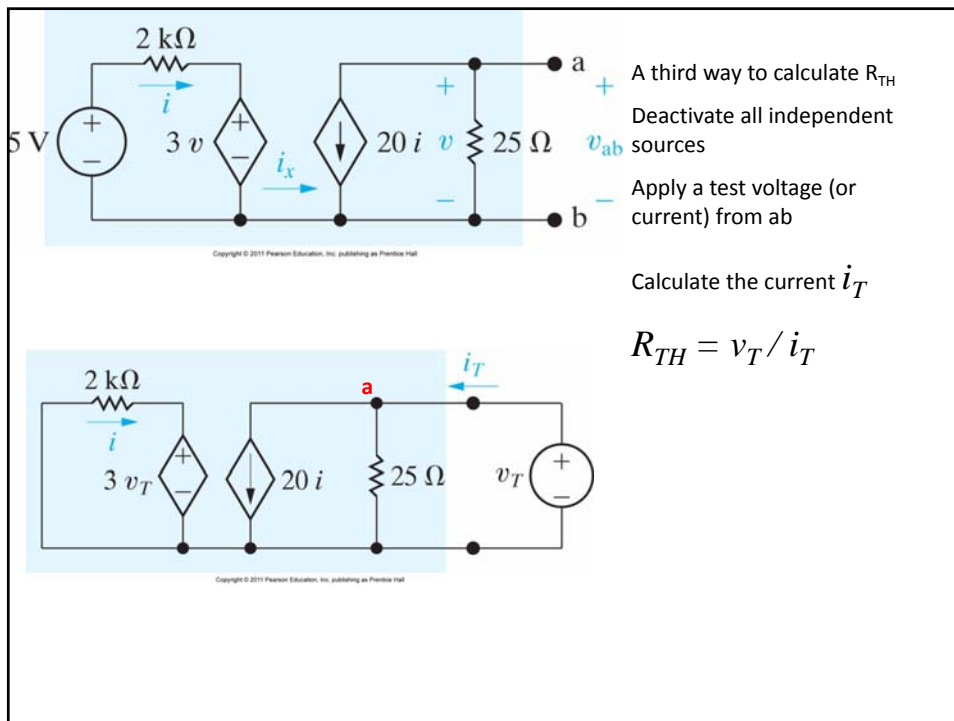
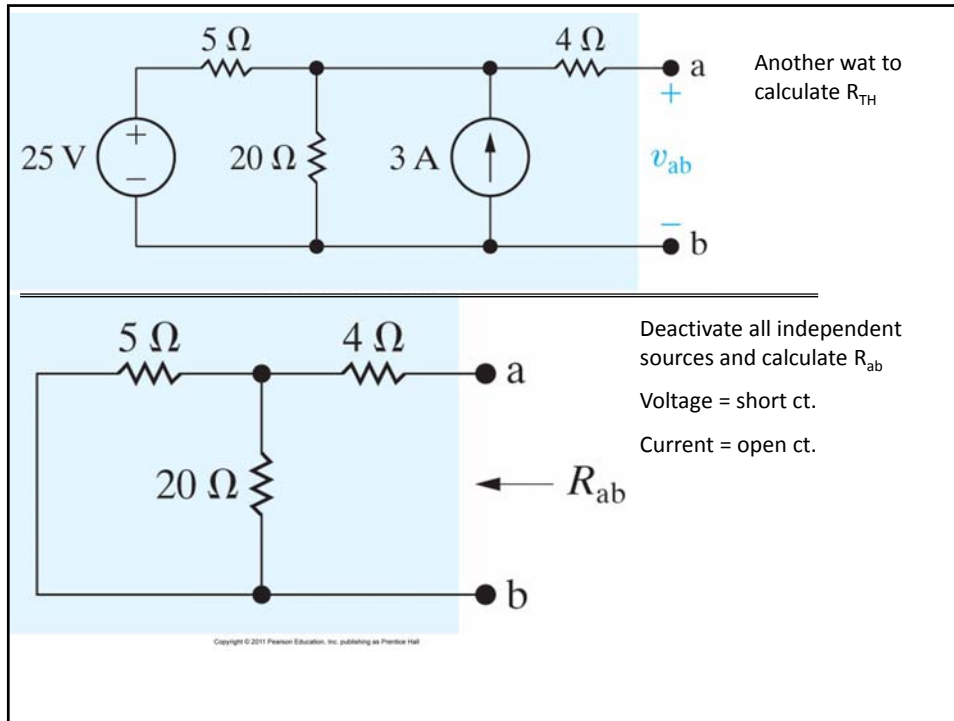
$I_{sc}$  = the short circuit current between a and b

$R_{TH} = V_{TH}/I_{sc}$  where  $I_{sc}$  is the short circuit current from a to b

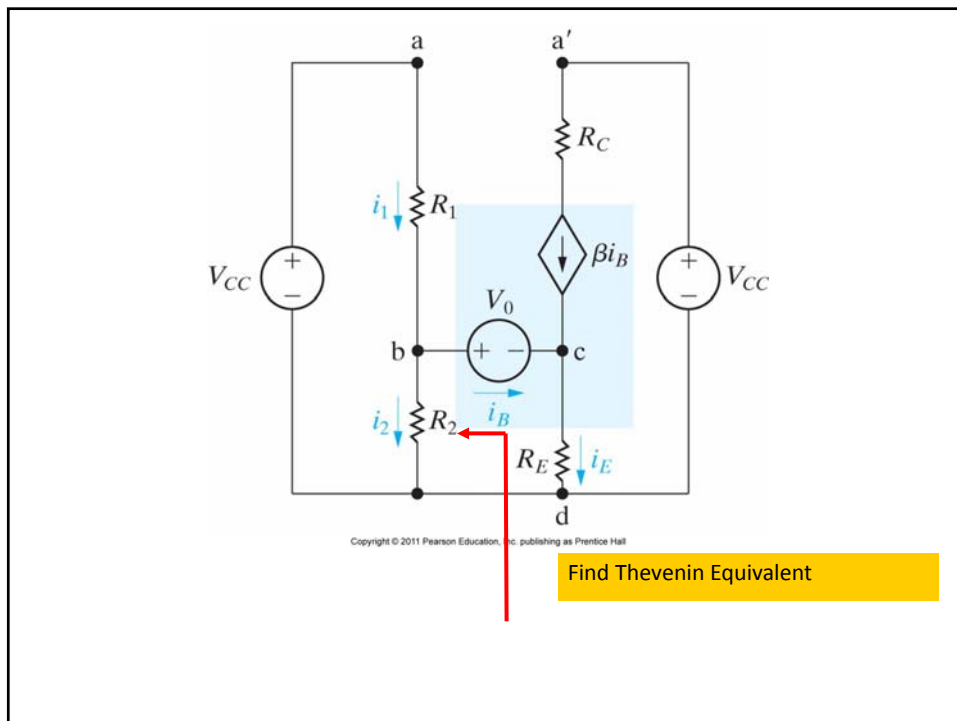
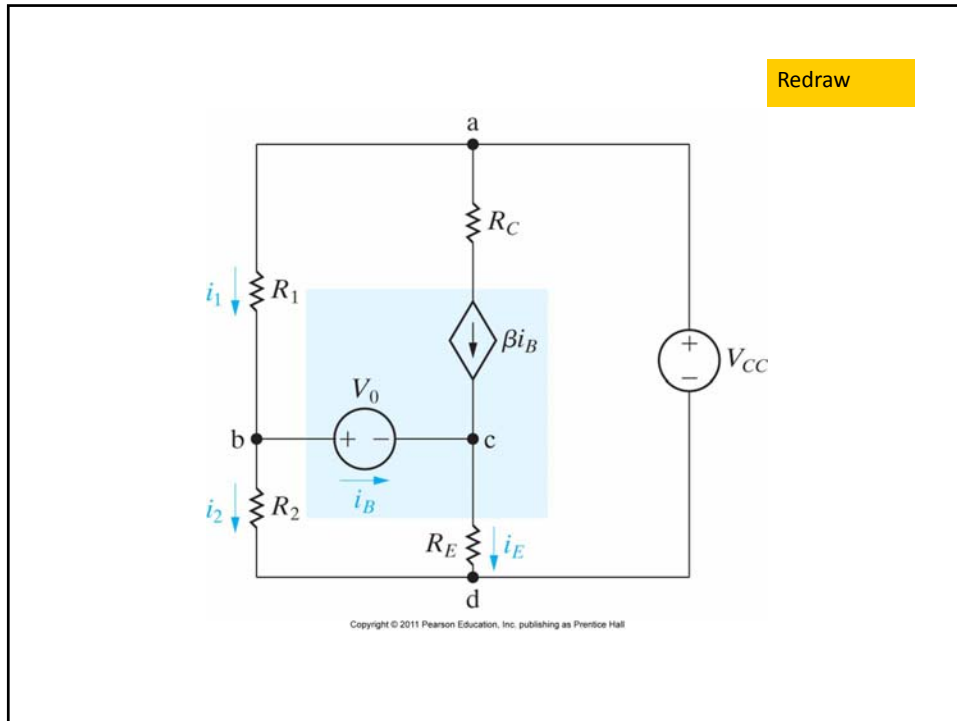
**EXCEPT for an ideal voltage source**

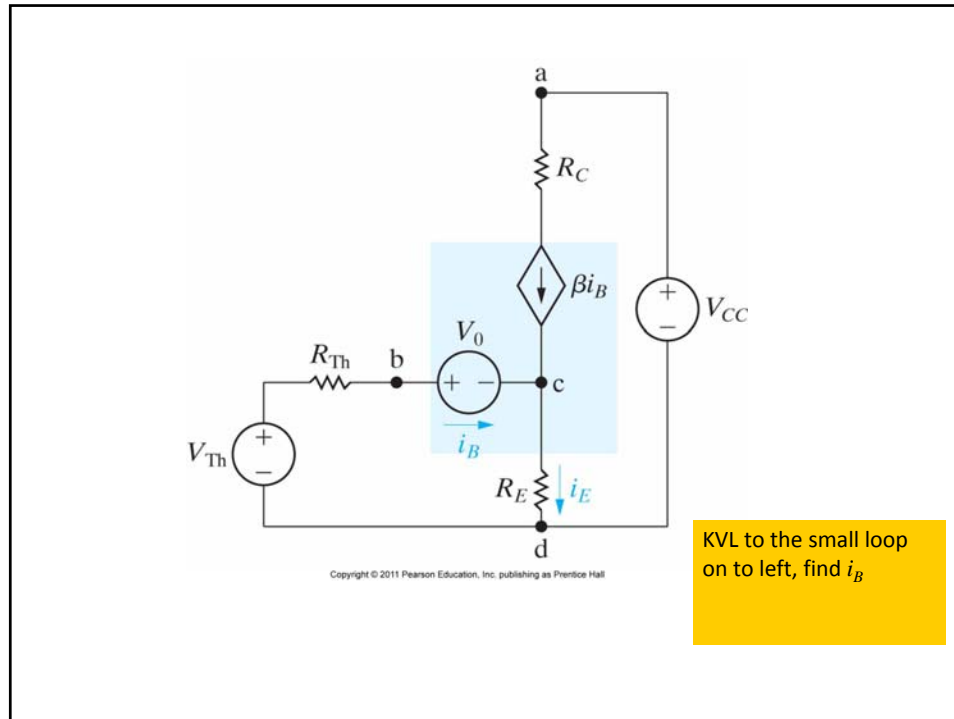


1. Note that  $i_x = 0$  (why?)
2. KCL at node a
3. KVL at the loop to the left get  $v_{ab} = V_{th}$
4. Short circuit a and b
5.  $I_{sc} = -20 i$ , find  $i$  from KVL o the left







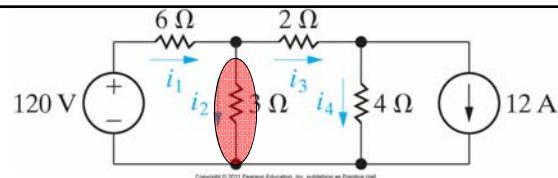


## Maximum Power Transfer

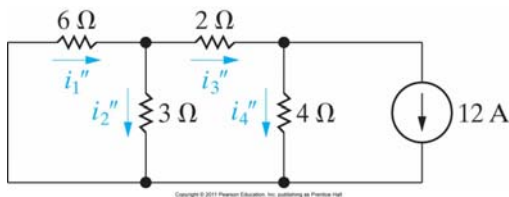
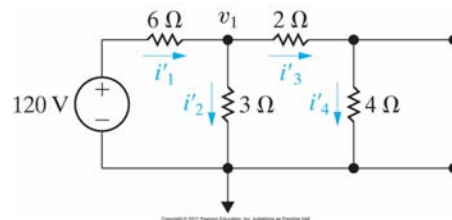
- Assume a voltage source,  $V$  with internal resistance  $R_0$  is driving a load  $R_L$
- What is the maximum power  $R_L$  can get?
- Calculate  $p(R_L, R_0)$  and differentiate  $=0$
- What if it is not a simple source with an internal resistance?
- Thevenin equivalent

## Superposition

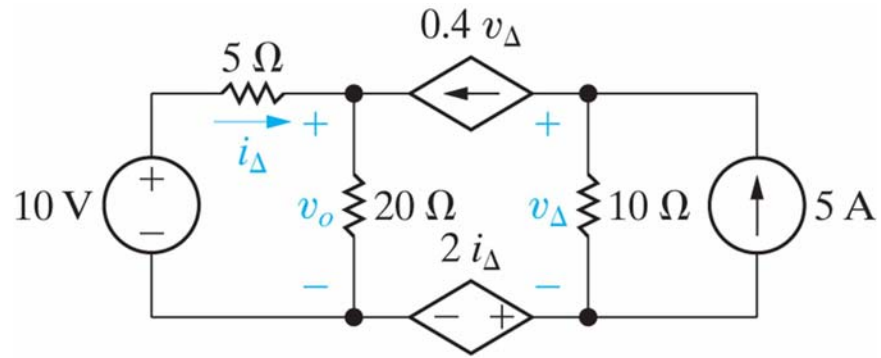
- When a linear system is excited by more than one independent source, the total response is the sum of the individual responses where each response is the result of one of the independent sources acting alone.



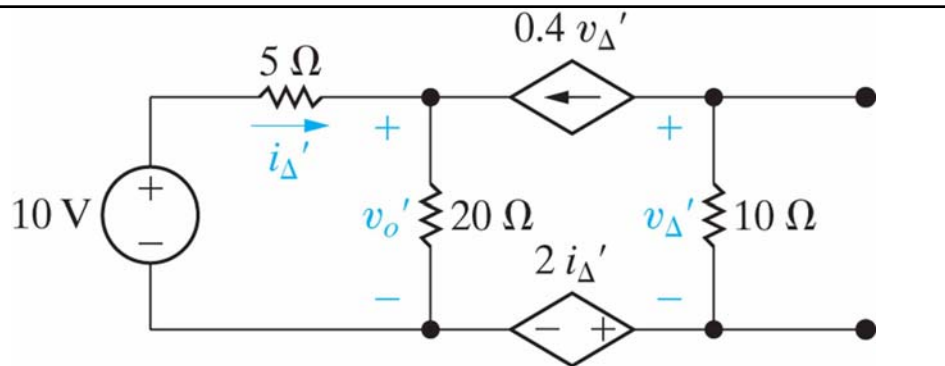
Find the current here



Find  $v_o$  using superposition



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