

ENG2200

Electric Circuits

Chapter 7

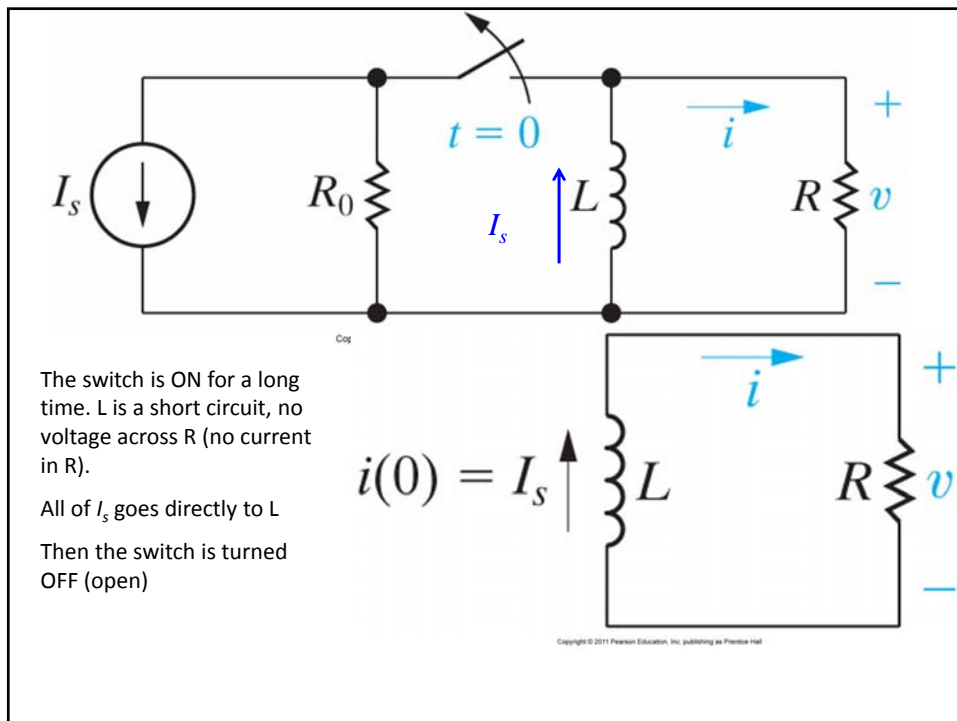
Response of First Order RL and RC Circuits

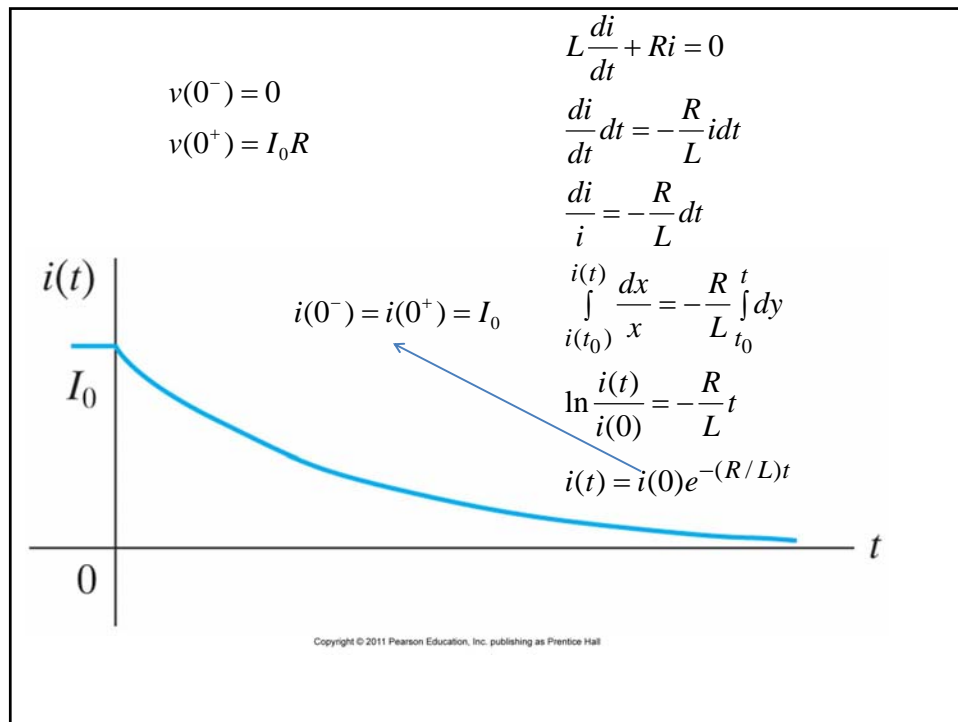
Objectives

- Be able to determine the natural response of RL and RC circuits.
- Be able to determine the step response of RL and RC circuits.
- Know how to analyze circuits with sequential switching.

Introduction

- **Natural Response:** The current and voltages that arise when the stored energy in the L or C is released.
- **Step Response:** The current and voltage that arise when energy is being acquired by the L or C when a sudden application of voltage or current is applied to the circuit.





Power and Energy

$$p = vi = i^2 R$$

$$p = I_0^2 R e^{-2(R/L)t}$$

$$w = \int_0^t p(x) dx$$

$$w = \int_0^t I_0^2 R e^{-2(R/L)x} dx$$

$$w = \frac{-1}{2R/L} I_0^2 R e^{-2(R/L)x} \Big|_0^t$$

$$w = \frac{1}{2} L I_0^2 (1 - e^{-2(R/L)t})$$

TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors

Inductors

$$v = L \frac{di}{dt} \quad (\text{V})$$

$$i = \frac{1}{L} \int_{t_0}^t v \, d\tau + i(t_0) \quad (\text{A})$$

$$p = vi = Li \frac{di}{dt} \quad (\text{W})$$

$$w = \frac{1}{2} Li^2 \quad (\text{J})$$

Capacitors

$$v = \frac{1}{C} \int_{t_0}^t i \, d\tau + v(t_0) \quad (\text{V})$$

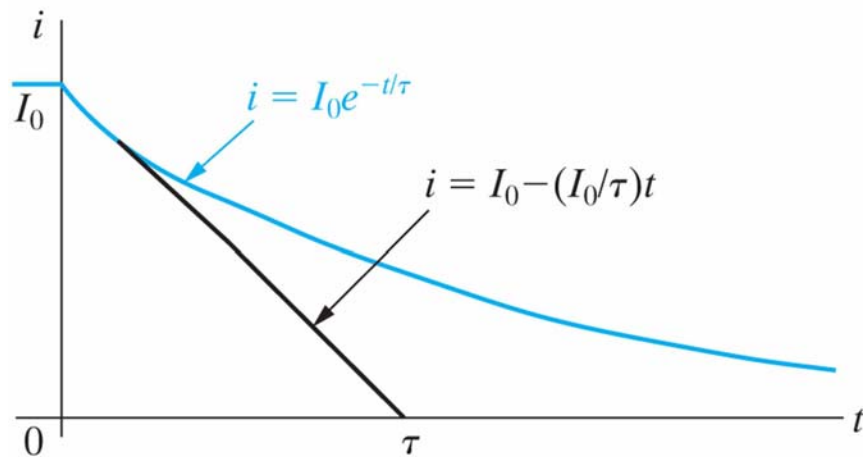
$$i = C \frac{dv}{dt} \quad (\text{A})$$

$$p = vi = Cv \frac{dv}{dt} \quad (\text{W})$$

$$w = \frac{1}{2} Cv^2 \quad (\text{J})$$

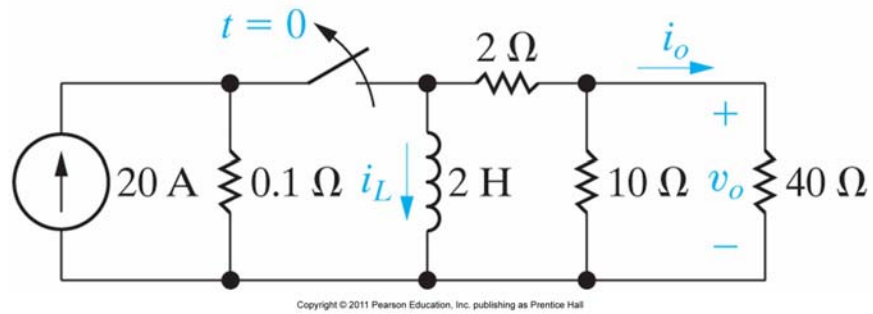
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$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = \frac{L}{R} \text{ time constant}$$

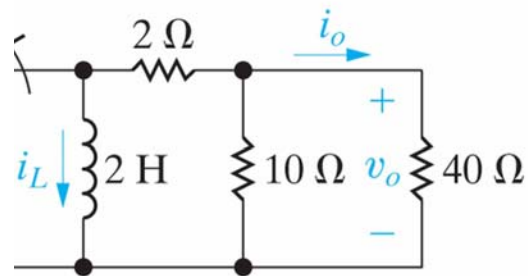


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Example



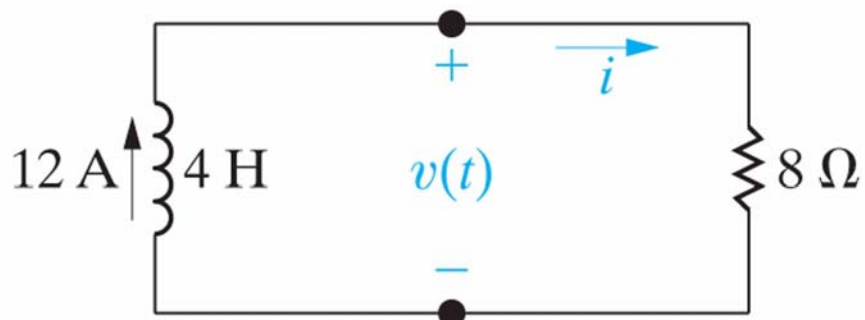
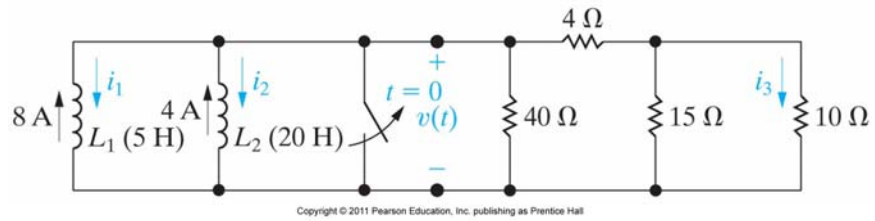
Example



$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = \frac{L}{R} \text{ time constant}$$

$$i(t) = 20e^{-4000t}$$

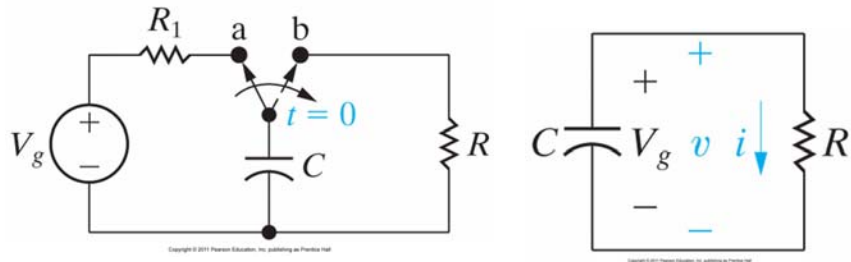
Example



$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = 0.5 \text{ sec}$$

$$i(t) = 12e^{-2t}$$

An RC circuit.



Switch in position A for a long time
The capacitor charged to V_g

An RC circuit.

$$v + iR = 0$$

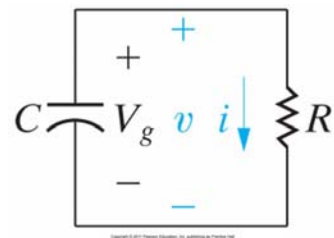
$$v = -RC \frac{dv}{dt}$$

$$dt = -RC dv \frac{dv}{v}$$

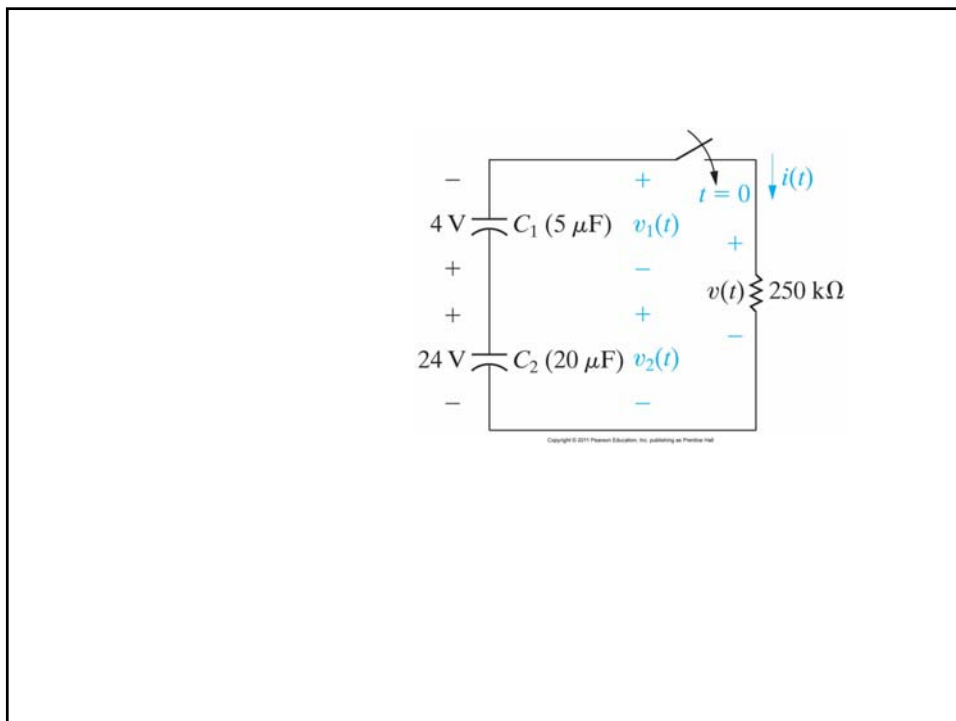
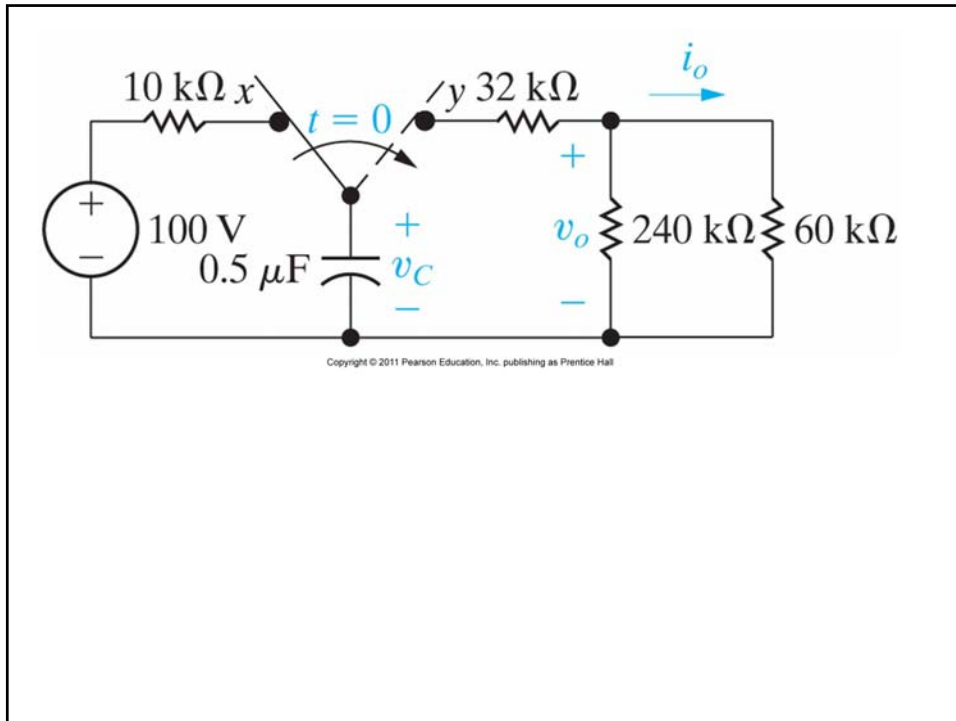
$$\int_{i(t_0)}^{i(t)} -RC \frac{dv}{v} = \int_{t_0}^t dy$$

$$\ln \frac{v(t)}{v(0)} = -\frac{1}{RC} t$$

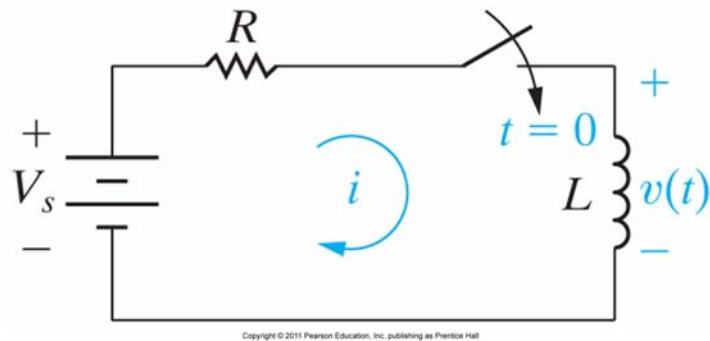
$$v(t) = v(0)e^{-t/\tau}, \tau = RC$$



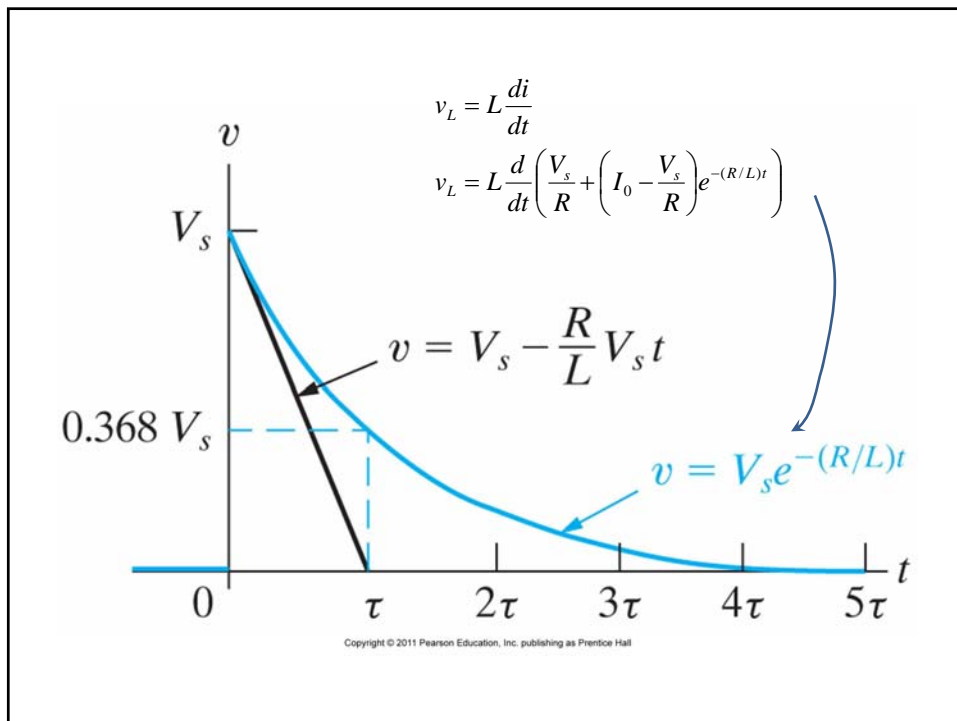
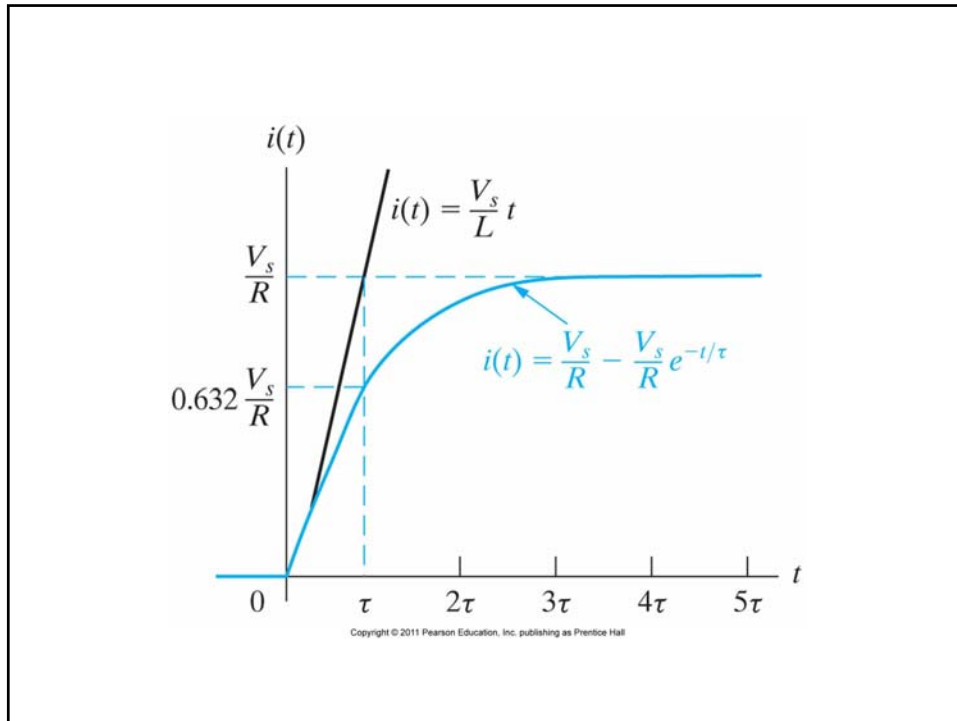
Why $v+iR=0$ not $v-iR=0$

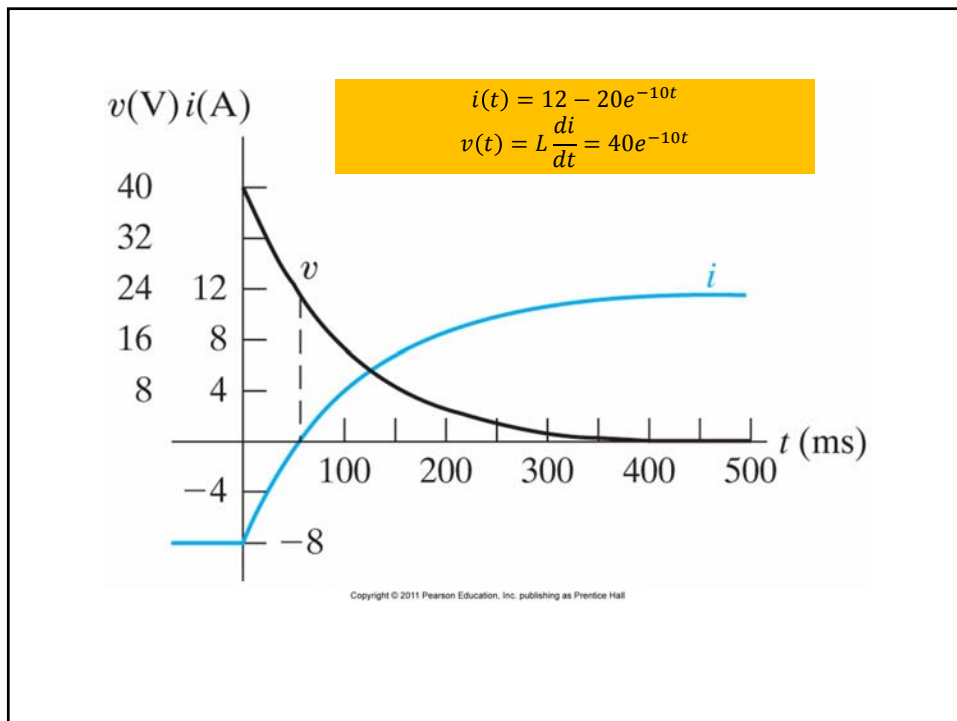
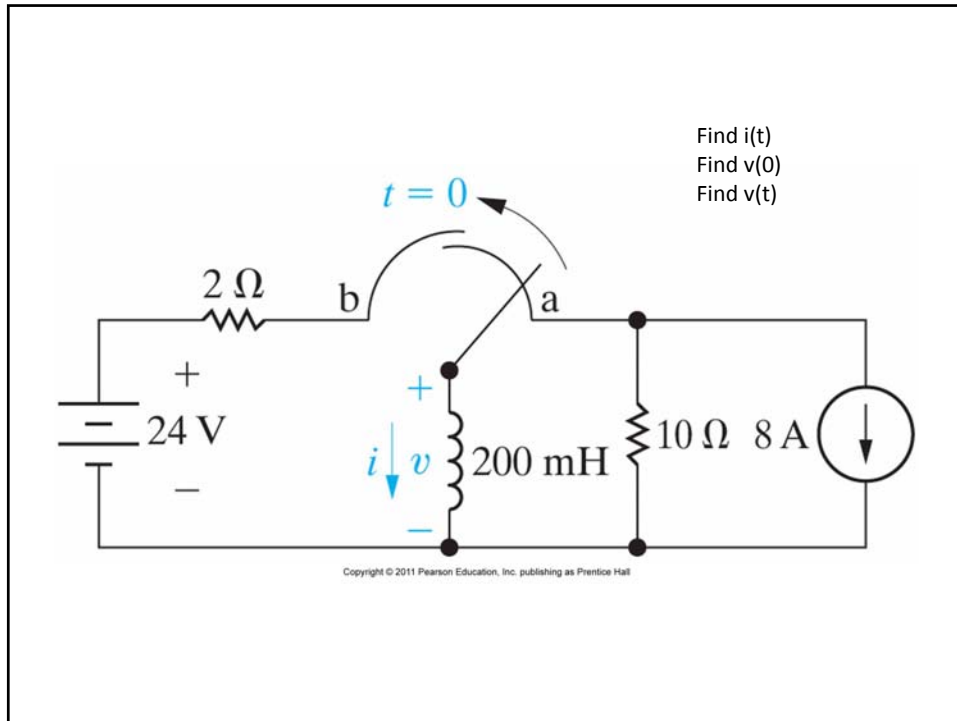


Step Response or RL Circuits

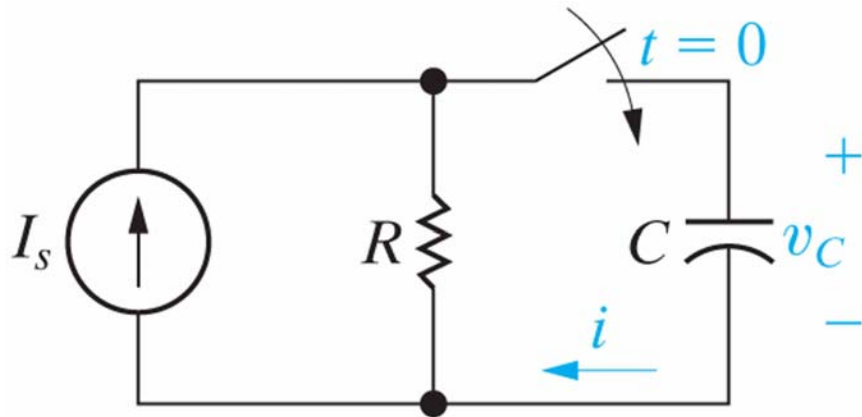


$$\begin{aligned}
 V_s &= Ri + L \frac{di}{dt} \\
 \frac{di}{dt} &= \frac{-Ri + V_s}{L} = \frac{-R}{L} \left(i - \frac{V_s}{R} \right) \\
 di &= \frac{-R}{L} \left(i - \frac{V_s}{R} \right) dt \\
 \frac{di}{i - (V_s/R)} &= \frac{-R}{L} dt \\
 \int_{I_0}^{i(t)} \frac{di}{i - (V_s/R)} &= \int_0^t \frac{-R}{L} dt \\
 \ln \frac{i(t) - (V_s/R)}{I_0 - (V_s/R)} &= \frac{-R}{L} t \\
 \frac{i(t) - (V_s/R)}{I_0 - (V_s/R)} &= e^{-(R/L)t} \\
 i(t) &= \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-(R/L)t}
 \end{aligned}$$





Step Response of RC Circuits



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Step Response of RC Circuit

$$C \frac{dv}{dt} + \frac{v}{R} = I_s$$

$$\frac{dv}{dt} = \frac{-v/R + I_s}{C} = \frac{-v + I_s R}{RC}$$

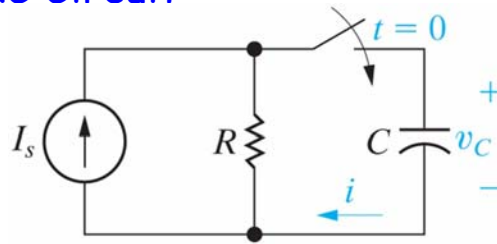
$$\frac{dv}{v - I_s R} = \frac{-1}{RC} dt$$

$$\int_{V_0}^{v(t)} \frac{dv}{v - I_s R} = \int_0^t \frac{-1}{RC} dt$$

$$\ln \frac{v(t) - (I_s R)}{V_0 - (I_s R)} = \frac{-1}{RC} t$$

$$\frac{v(t) - (I_s R)}{V_0 - (I_s R)} = e^{-t/RC}$$

$$v(t) = I_s R + (V_0 - I_s R) e^{-t/RC}$$



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$$i = C \frac{dv}{dt}$$

$$i = C(V_0 - I_s R) \left(\frac{-1}{RC} \right) e^{-t/RC}$$

$$i = \left(I_s - \frac{V_0}{R} \right) e^{-t/RC}$$

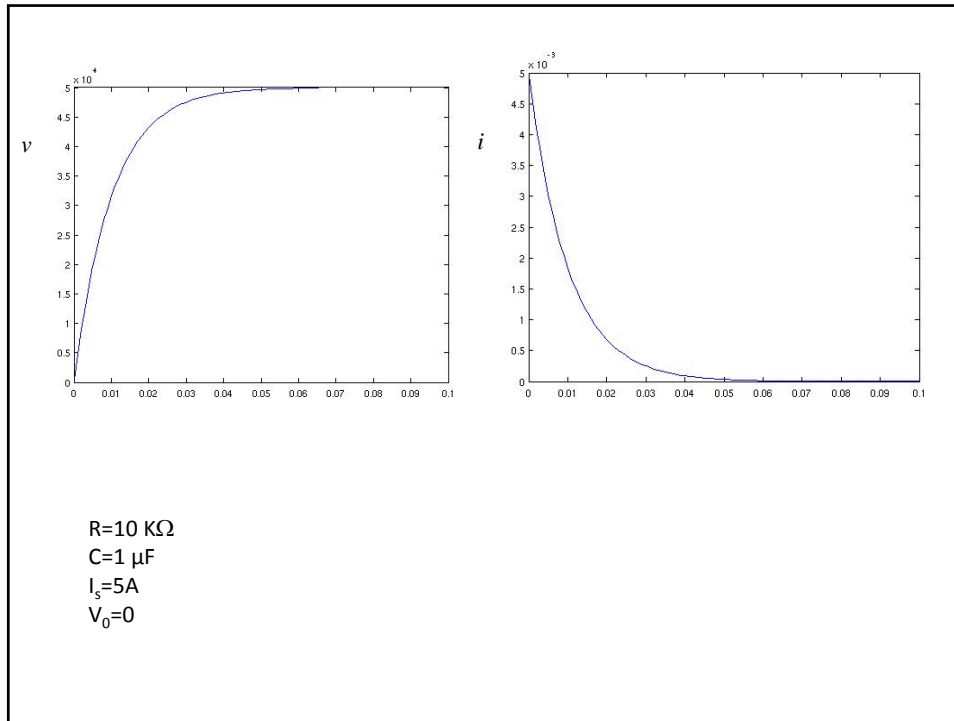
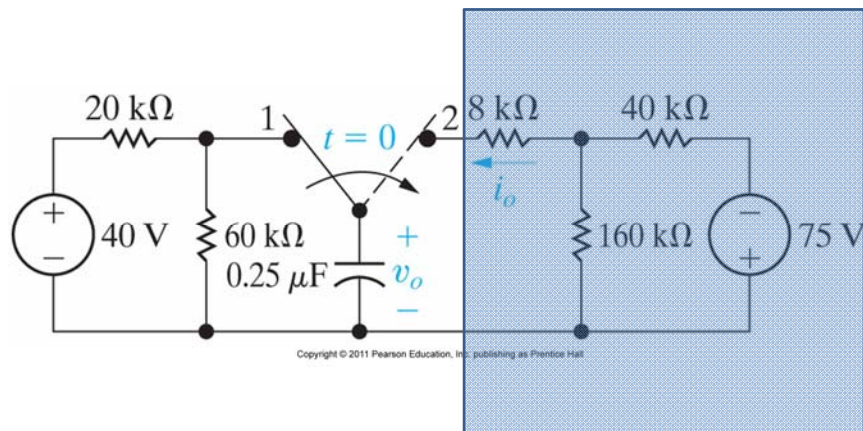
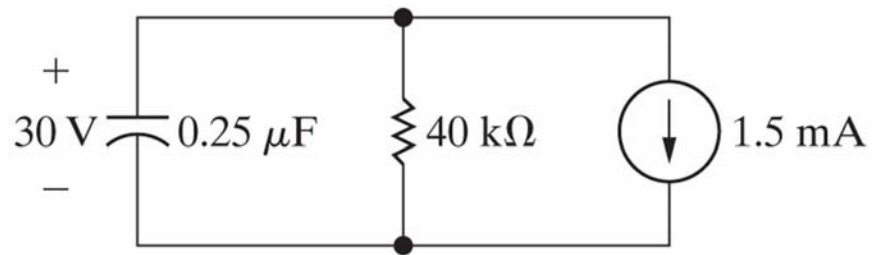
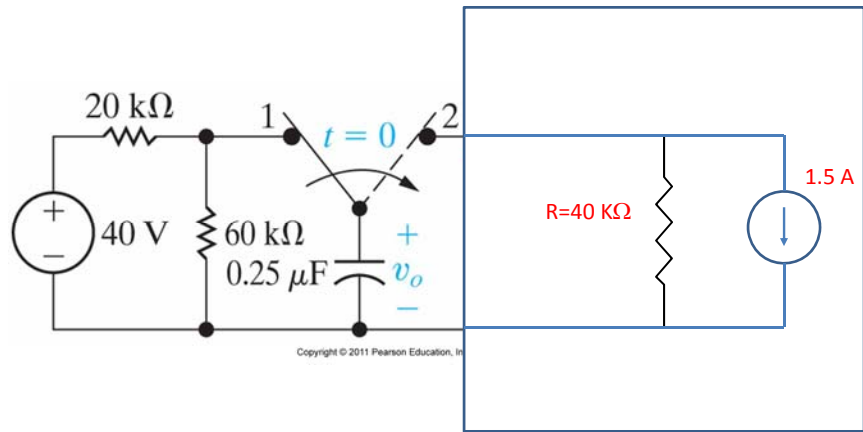


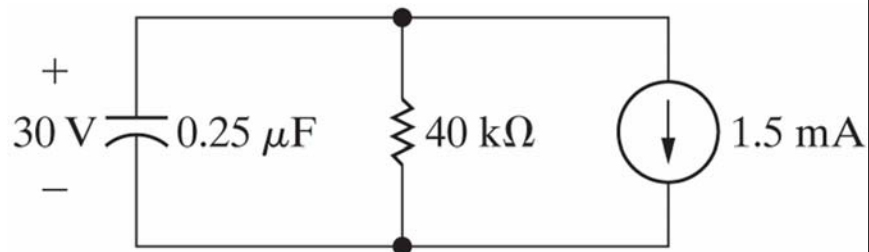
Figure 7.22 The circuit for Example 7.6.



Get Norton's Equivalent

Figure 7.22 The circuit for Example 7.6.





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$$v(t) = I_s R + (V_0 - I_s R) e^{-t/RC}$$

$$v(t) = (-1.5) \times 40K + (30 - (-60)) e^{-100t}$$

$$v(t) = -60 + 90 e^{-100t}$$