

# ENG2200

## Electric Circuits

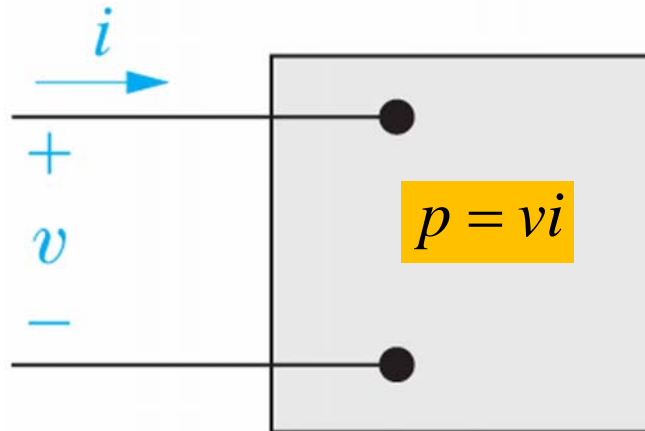
### Chapter 10

#### Sinusoidal Steady Power Calculation

### Objectives

- Understanding the difference between instantaneous power, average power reactive power, complex power and how to calculate them.
- Understanding power factor and how to calculate it.
- Understand the condition for a maximum real power delivered to the load.

**Figure 10.1** The black box representation of a circuit used for calculating power.



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## Instantaneous Power

- $v = V_m \cos(\omega t + \theta_v)$   $i = I_m \cos(\omega t + \theta_i)$

$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$i = I_m \cos(\omega t)$$

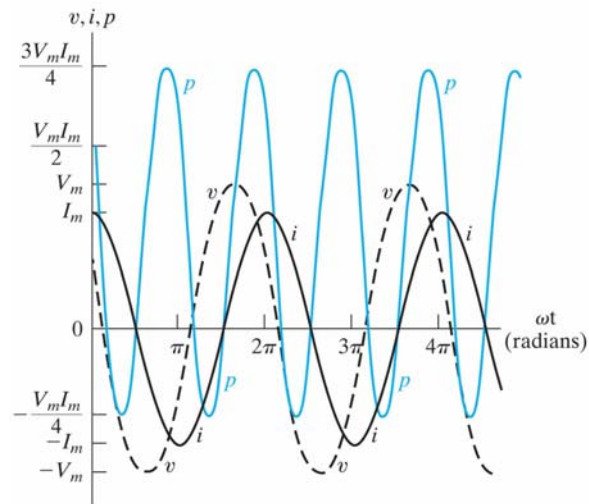
$$p = I_m V_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

$$p = \frac{1}{2} I_m V_m \{ \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v - \theta_i) \}$$

$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(2\omega t + \theta_v - \theta_i)$$

$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$

$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$



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## Average and Reactive Power

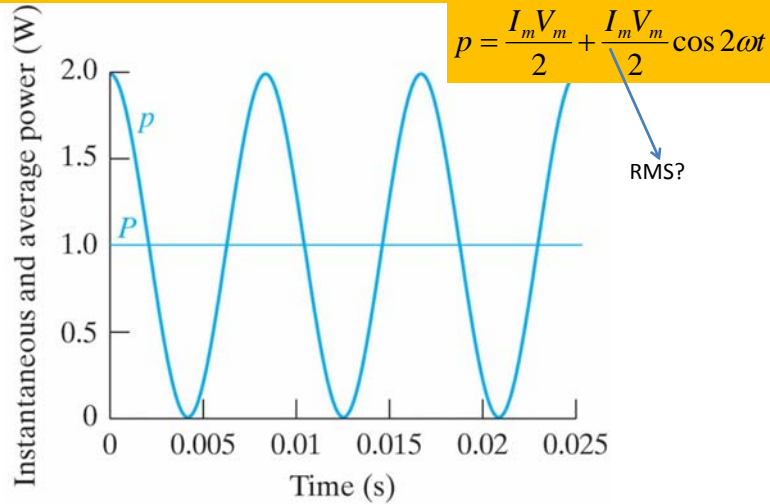
$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$

$$p = P + P \cos 2\omega t - Q \sin 2\omega t$$

- P is the average power (real power) the power transferred from electric to non-electric (the consumer made use of it)
- Q is the reactive power

### Purely Resistive Circuits $\theta_i = \theta_v$

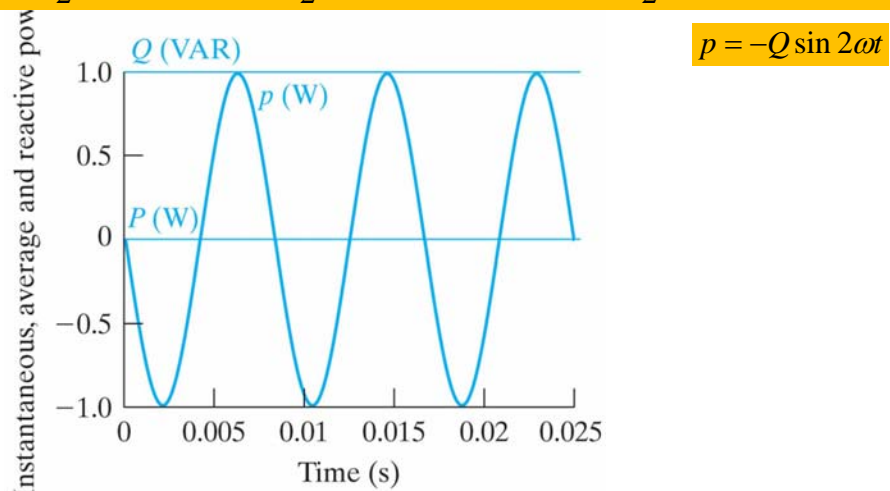
$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$



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### Purely Inductive $\theta_i = \theta_v - 90^\circ$

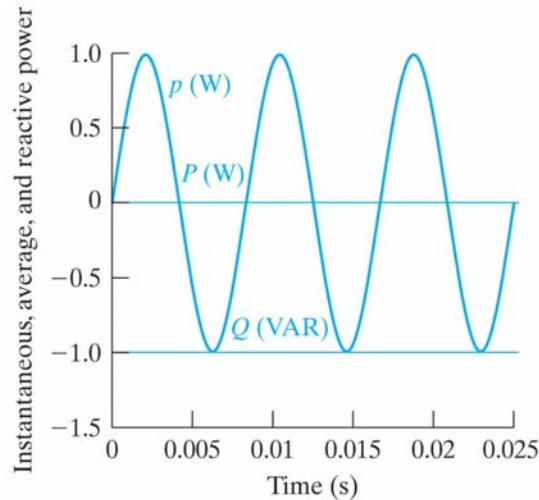
$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$



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## Purely Capacitive Circuits $\theta_i = \theta_v + 90^\circ$

$$p = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) + \frac{I_m V_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{I_m V_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t$$



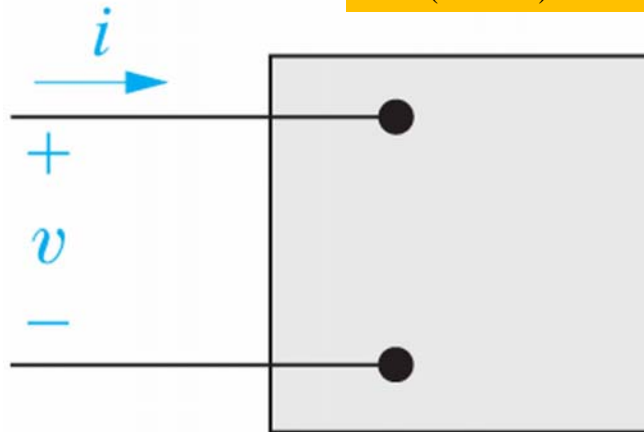
## Power Factor

- The units for  $p$  is Watt (W)
- The units for  $Q$  is VAR (Volt Ampere Reactive)
- $\theta_i - \theta_v$  power factor angle
- $PF = \cos(\theta_i - \theta_v)$
- Note that  $\cos(\theta_i - \theta_v) = \cos(\theta_v - \theta_i)$
- PF is defines as lagging (current lags voltage – inductive) or leading (currents leads voltage – capacitive)

### Example

$$v = 100 \cos(\omega t + 15^\circ) \text{ V}$$

$$i = \sin(\omega t - 15^\circ) \text{ V}$$



Calculate the average power, reactive power

TABLE 10.1 Annual Energy Requirements of Electric Household Appliances

Appliance	Average Wattage	Est. kWh Consumed Annually <sup>a</sup>	Appliance	Average Wattage	Est. kWh Consumed Annually <sup>a</sup>
<b>Food preparation</b>			<b>Health and beauty</b>		
Coffeemaker	1,200	140	Hair dryer	600	25
Dishwasher	1,201	165	Shaver	15	0.5
Egg cooker	516	14	Sunlamp	279	16
Frying pan	1,196	100	<b>Home entertainment</b>		
Mixer	127	2	Radio	71	86
Oven, microwave (only)	1,450	190	Television, color, tube type	240	528
Range, with oven	12,200	596	Solid-state type	145	320
Toaster	1,146	39	<b>Housewares</b>		
<b>Laundry</b>			Clock	2	17
Clothes dryer	4,856	993	Vacuum cleaner	630	46
Washing machine, automatic	512	103	a) Based on normal usage. When using these figures for projections, such factors as the size of the specific appliance, the geographical area of use, and individual usage should be taken into consideration. Note that the wattages are not additive, since all units are normally not in operation at the same time.		
Water heater	2,475	4,219	b) Based on 1000 hours of operation per year. This figure will vary widely depending on the area and the specific size of the unit. See EEL-Pub #76-2, "Air Conditioning Usage Study," for an estimate for your location.		
Quick recovery type	4,474	4,811	Source: Edison Electric Institute.		
<b>Comfort conditioning</b>			Copyright © 2011 Pearson Education, Inc. publishing as Prentice Hall		
Air conditioner (room)	860	860 <sup>b</sup>			
Dehumidifier	257	377			
Fan (circulating)	88	43			
Heater (portable)	1,322	176			

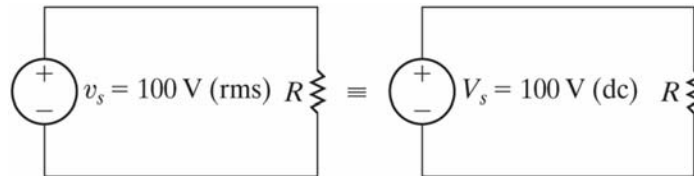
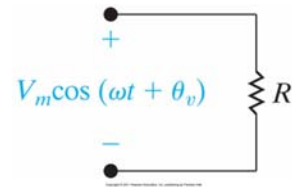
**Figure 10.7** A sinusoidal voltage applied to the terminals of a resistor.

$$P = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \theta_v)}{R} dt$$

$$P = \frac{1}{R} \left[ \frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \theta_v) dt \right]$$

$$P = \frac{V_{RMS}^2}{R}$$

$$P = I_{RMS}^2 R$$



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## Complex Power

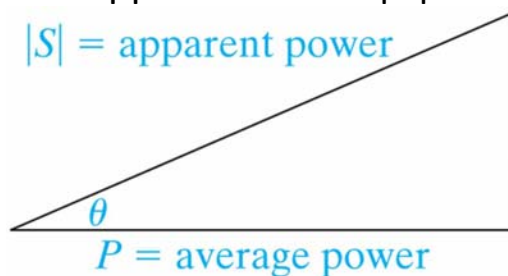
- Complex power  $S = P + jQ$

$$\frac{Q}{P} = \frac{(V_m I_m / 2) \sin(\theta_v - \theta_i)}{(V_m I_m / 2) \cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i)$$

- Apparent Power  $|S|$

$|S|$  = apparent power

$Q$  = reactive power



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## Example

- An electrical motor operates at 240 V rms. The average power is 8 kW at a lagging power factor of 0.8

## Power calculation

$$S = (V_m I_m / 2) \cos(\theta_v - \theta_i) + j(V_m I_m / 2) \sin(\theta_v - \theta_i)$$

$$S = \frac{V_m I_m}{2} [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$$

$$S = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{V_m I_m}{2} \angle(\theta_v - \theta_i)$$

$$S = V_{rms} \angle \theta_v \times I_{rms} \angle -\theta_i$$

$$S = V_{rms} I_{rms}^* = \frac{1}{2} VI^*$$



## Power calculation

$$S = V_{rms} I_{rms}^*$$

$$V_{rms} = I_{rms} Z$$

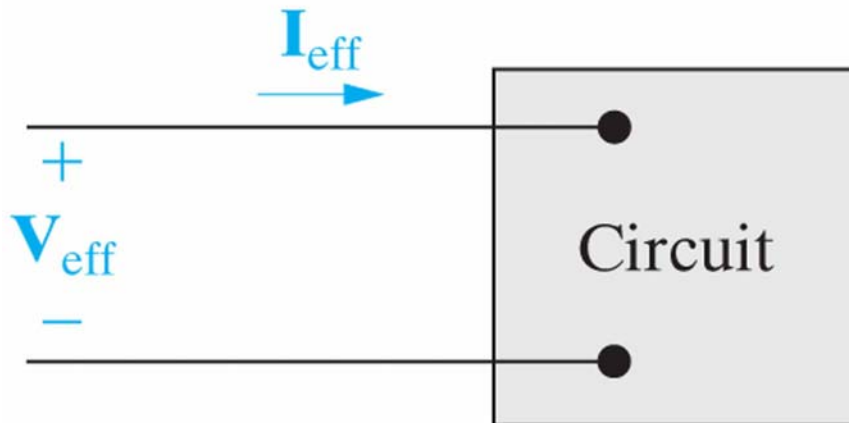
$$S = I_{rms} I_{rms}^* Z$$

$$S = |I_{rms}|^2 Z$$

$$S = |I_{rms}|^2 (R + jX)$$

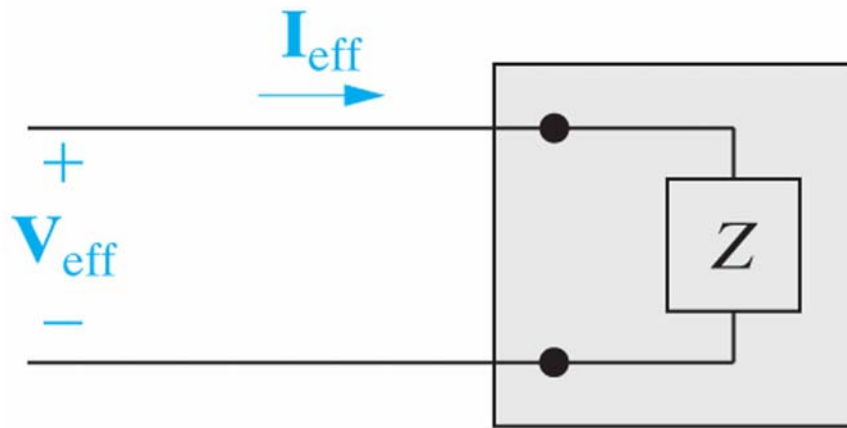
$$S = |I_{rms}|^2 R + j |I_{rms}|^2 X$$

**Figure 10.11** The phasor voltage and current associated with a pair of terminals.



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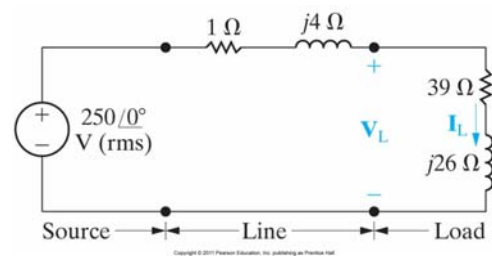
**Figure 10.12** The general circuit of Fig. 10.11 replaced with an equivalent impedance.



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## Example

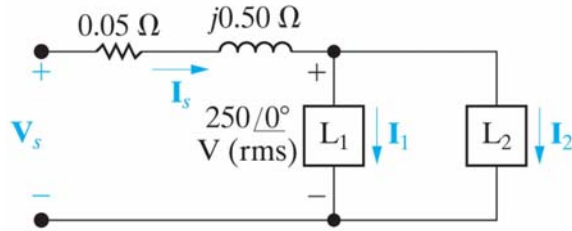
- Find  $I_L$  and  $V_L$
- Calculate  $S$  supplied by the source
- Calculate  $S$  delivered to the load
- Calculate  $S$  delivered to the line



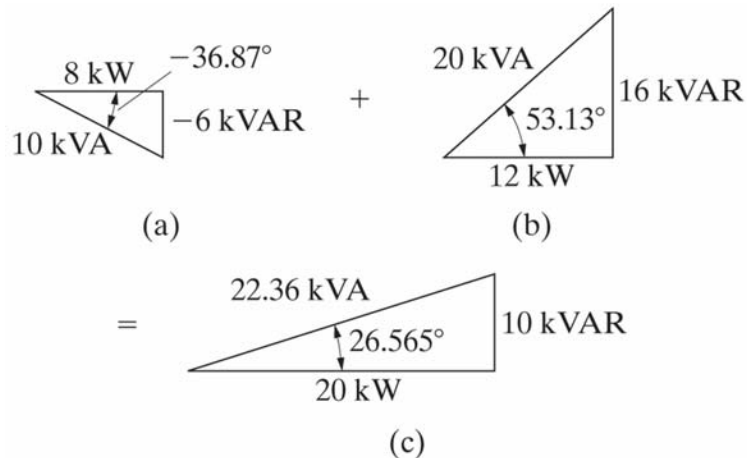
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## Example

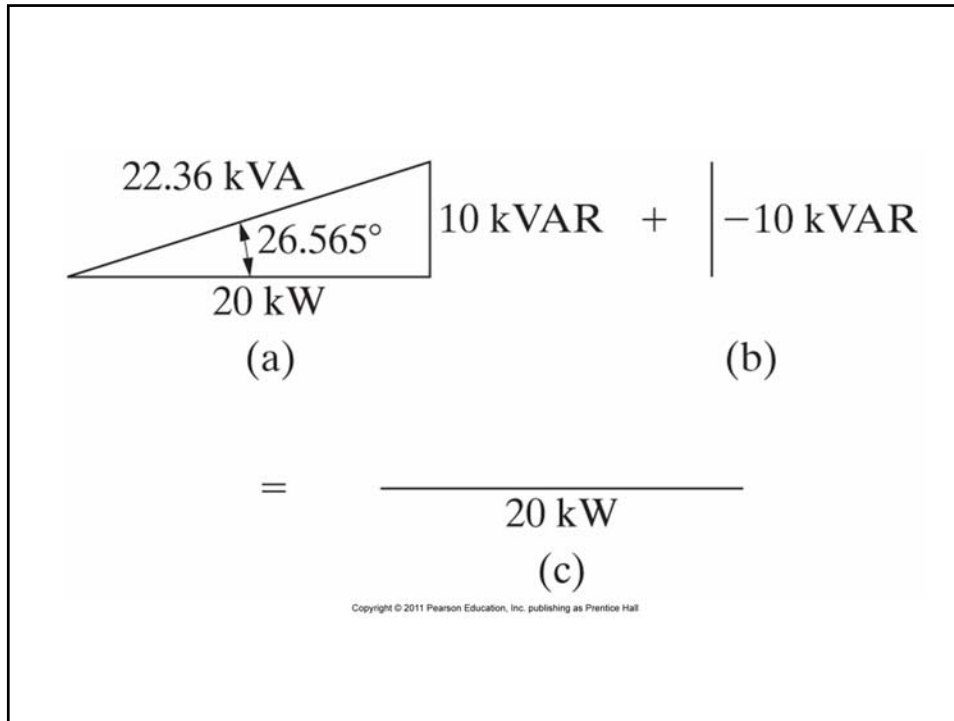
Load 1  
8 kW leading pf 0.8  
Load 2  
20 kVA at lagging pf 0.6



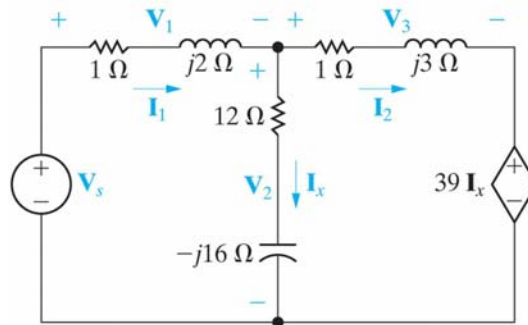
- Find the pf of the 2 loads in parallel
- Find  $I_s$  and the apparent power to supply the load
- Assuming 60 Hz, what is the capacitor required to correct the power factor



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**Figure 10.17** The circuit, with solution, for Example 10.7.

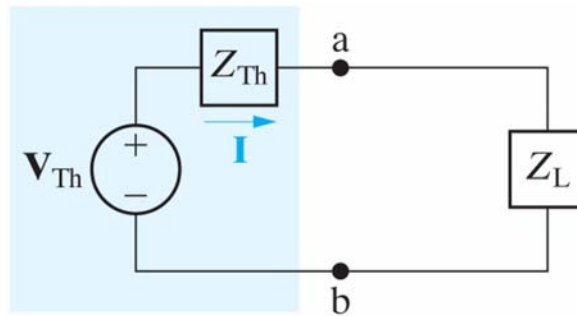


$$\begin{aligned} \mathbf{V}_s &= 150 \angle 0^\circ \text{ V} \\ \mathbf{V}_1 &= (78 - j104) \text{ V} & \mathbf{I}_1 &= (-26 - j52) \text{ A} \\ \mathbf{V}_2 &= (72 + j104) \text{ V} & \mathbf{I}_x &= (-2 + j6) \text{ A} \\ \mathbf{V}_3 &= (150 - j130) \text{ V} & \mathbf{I}_2 &= (-24 - j58) \text{ A} \end{aligned}$$

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## Maximum Power Transfer

- Assume the source is replaced by its Thevenin equivalent circuits.
- $V_{TH}$ ,  $Z_{TH}$  and a load or  $Z_L$  is connected



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## Maximum Power Transfer

$$I = \frac{V_{TH}}{(R_{TH} + R_L) + j(X_{TH} + X_L)}$$

$$P = |I|^2 R_L$$

$$P = \frac{|V_{TH}|^2 R_L}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{\partial P}{\partial X_L} = 0$$

$$X_L = X_{TH}, R_L = \sqrt{R_{TH}^2 + (X_L + X_{TH})^2}$$

$$Z_L = Z_{TH}^*$$

## Restriction

- Sometimes, we have restrictions on the load impedance.
- First, set  $X_L$  as close as possible to  $-X_{TH}$ , then calculate  $R_L$  as close as possible to  $\sqrt{R_{TH}^2 + (X_{TH} + X_L)^2}$
- If we can change the magnitude of the load impedance, but not the phase; set the magnitude of the load impedance to the magnitude of Thevenin impedance