Hashing I

cse2011

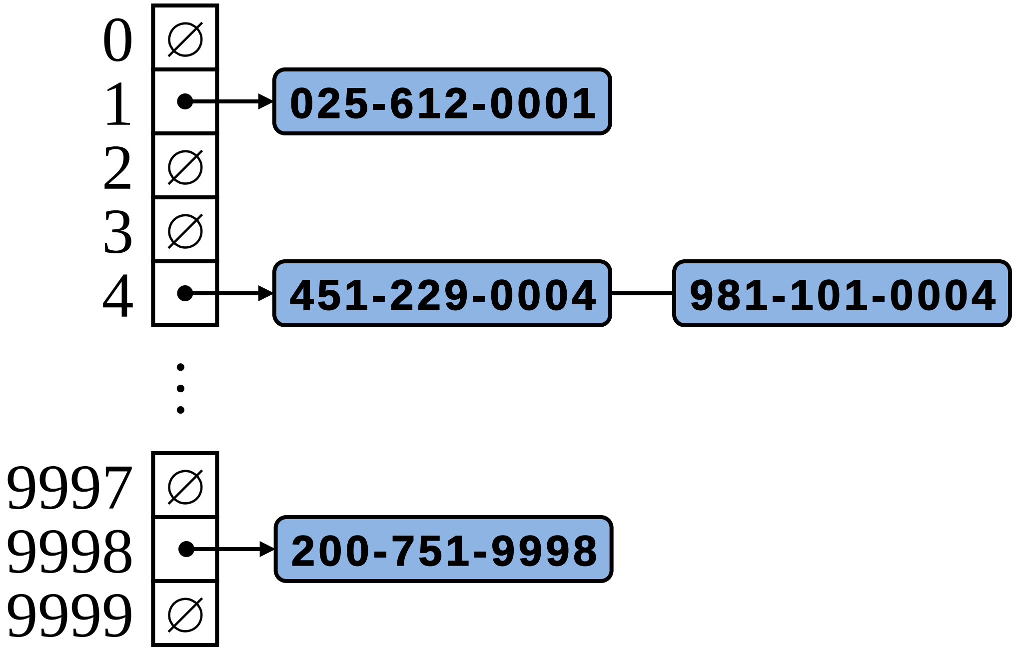
section 9.2 of textbook

Hashing

- BST, AVL trees: $O(\log N)$ for insertion, deletions and searches.
- Hashing is a technique used for performing insertion, deletions and searches in ***constant average*** time (i.e., O(1)).
- Finding min, finding max, printing the whole collection in sorted order in linear time are ***not*** supported.
- A hash table data structure consists of:
  - Hash function ***h***
  - Array of size ***N*** (**bucket array**)

Example

- We design a hash table for a dictionary storing items (SIN, Name), where SIN (social insurance number) is a ten-digit positive integer
- Our hash table uses an array of size $N = 10{,}000$ and the hash function $h(x) = x \bmod N$
- We use **chaining** to **handle collisions**
- Assuming *integer keys*, how do we map keys to hash table entries?



Hash Functions and Hash Tables

- A hash function ***h*** maps keys of a given type to integers in a fixed interval [0, ***N*** - 1]
- Example:

  $$h(x) = x \bmod N$$

  is a hash function for integer keys
- The integer ***h***(***x***) is called the hash value of key ***x***
- The goal of a hash function is to **uniformly** disperse keys in the range [0, ***N*** - 1]

- A hash table for a given key type consists of
  - Hash function ***h***
  - Array of size ***N***

- A **collision** occurs when two keys in the dictionary have the same hash value.
- Collision handing schemes:
  - Chaining: colliding items are stored in a sequence
  - Open addressing: the colliding item is placed in a different cell of the table

Design Issues

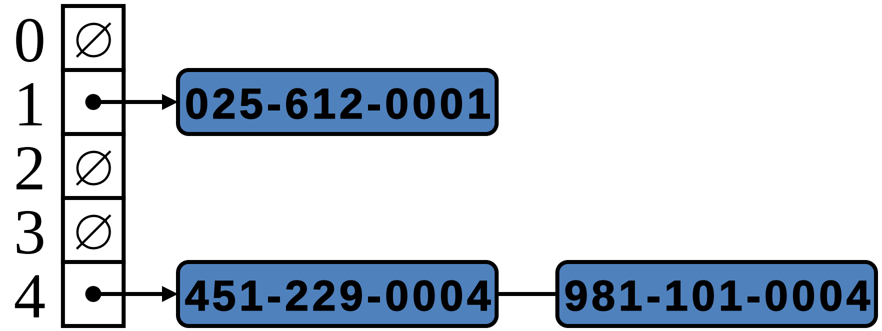
- Hash function
  - For integer keys (compression functions)
  - For strings
- Collision handling
  - Separate chaining
  - Probing (open addressing)
    - Linear probing
    - Quadratic probing
    - Double hashing
- Table size (should be a prime number)

Hash Functions

- Division:
  - $h_2(y) = y \bmod N$
  - The size N of the hash table is usually chosen to be a prime number to minimize the number of collisions
  - The reason has to do with number theory and is beyond the scope of this course
- Multiply, Add and Divide (MAD):
  - $h_2(y) = (ay + b) \bmod N$
  - a and b are nonnegative integers such that
    $a \bmod N \neq 0$
  - Otherwise, every integer would map to the same value b

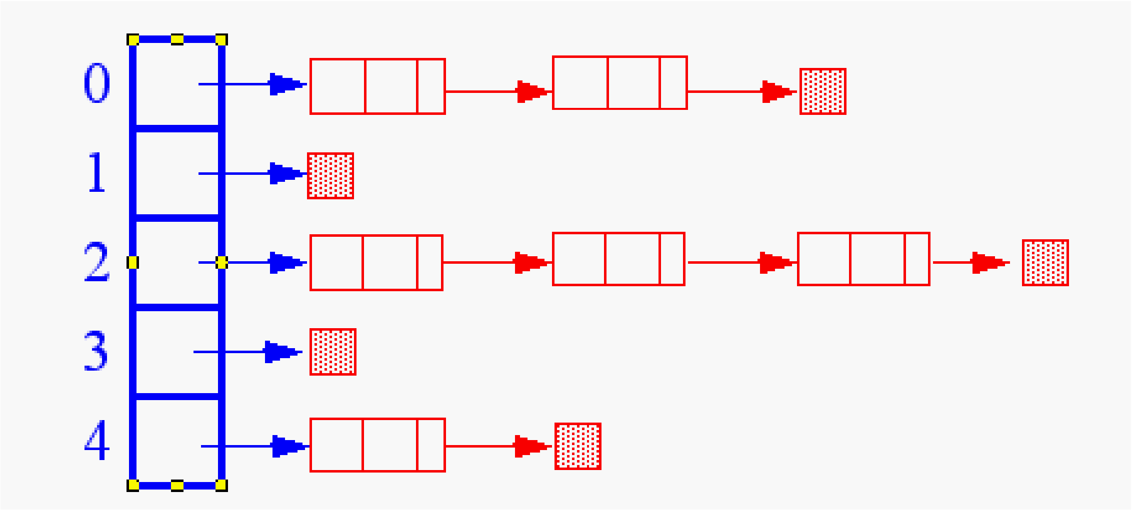
Collision Handling

- Collisions occur when different elements are mapped to the same cell
- **Separate Chaining**: let each cell in the table point to a **linked list** of entries that map there



- Separate chaining is simple, but **requires additional memory** outside the table

Separate Chaining



- Use ***chaining*** to set up ***lists*** of items with same index
- The ***expected*** search/insertion/removal time is O(n/N), provided that the indices are **uniformly** distributed
  - N = hash table size
  - n = number of elements in the table
- If n = O(N), the expected running time is O(1)

Load Factor – Separate Chaining

- Define the load factor $\lambda = n/N$
  - n = number of elements in the hash table
  - N = hash table size (prime number)

- To obtain best performance with separate chaining, ensure $\lambda \leq 1$.

- As we add more elements to the hash table, λ goes up $\Rightarrow$ **rehashing** (allocate a bigger table, define a new hash function, and copy the elements to the new array).

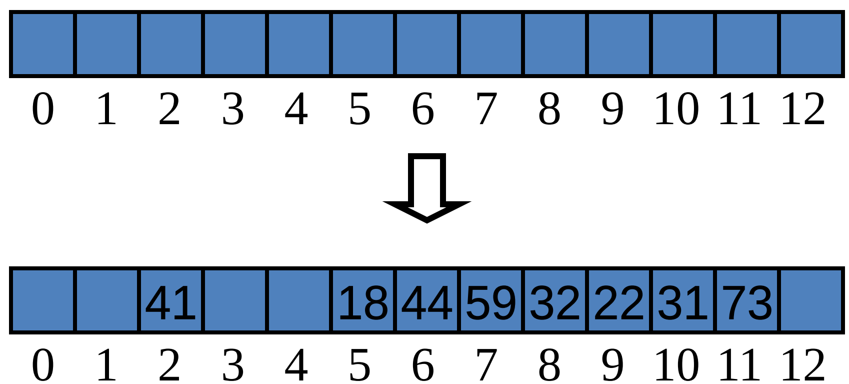
Collision Handling

- Separate chaining
- Probing (open addressing)
  - Linear probing
  - Quadratic probing
  - Double hashing

Linear Probing

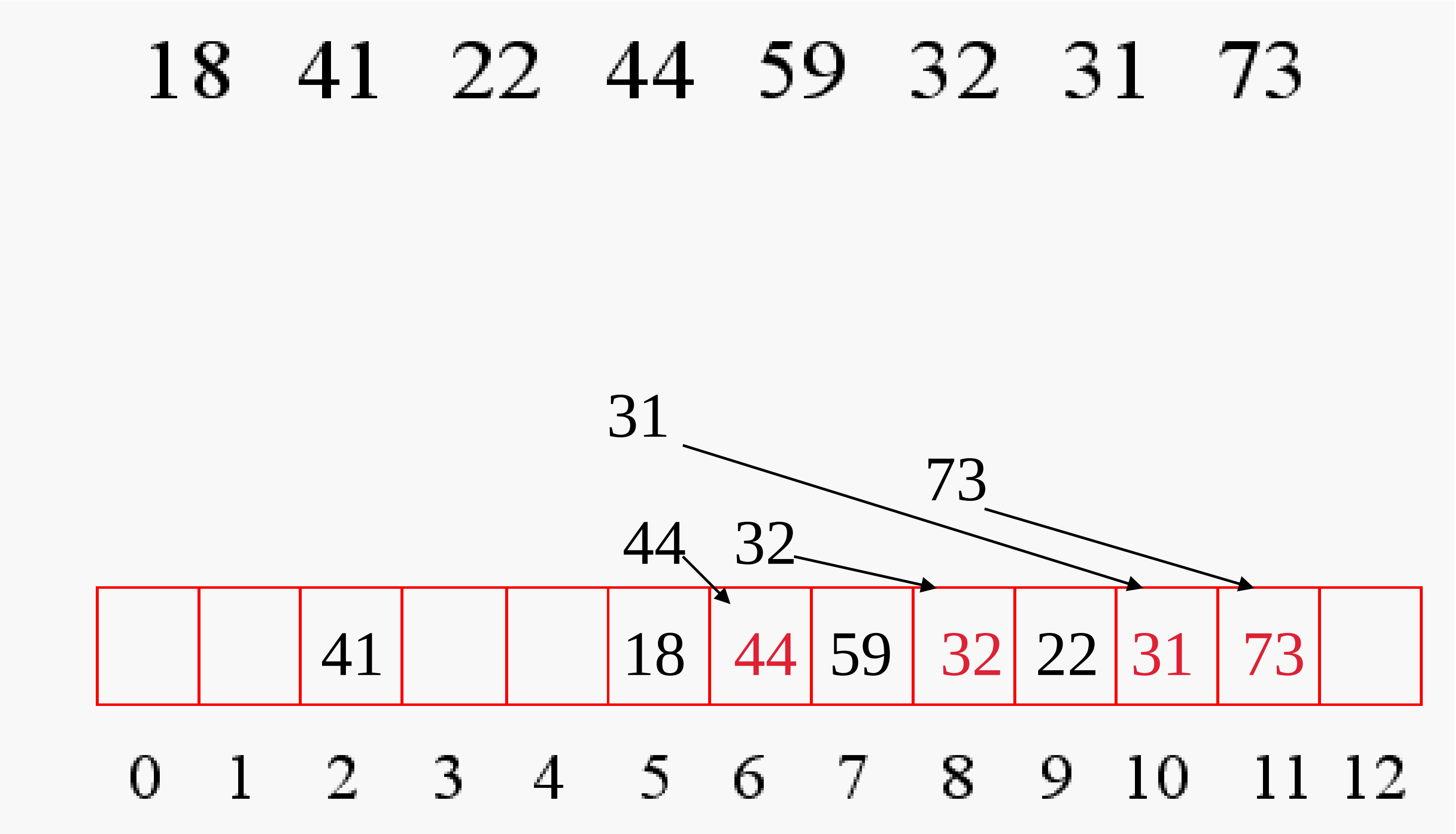
- Linear probing handles collisions by placing the colliding item in the next (circularly) available table cell
- Each table cell inspected is referred to as a “probe”
- Colliding items lump together; future collisions will cause a longer sequence of probes

- Example:
  – $h(x) = x \bmod 13$
  – Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order
  – Remove 44, 32, 73, 31



Linear Probing Example

1. $h(x) = x \bmod 13$
2. $h(x) = (h(x) + 1) \bmod 13$
3. $h(x) = (h(x) + 2) \bmod 13$



Search with Linear Probing

- Consider a hash table A that uses linear probing
- $get(k)$
  - We start at cell $h(k)$
  - We probe consecutive locations until one of the following occurs
    - An item with key k is found, or
    - An empty cell (null) is found, or
    - N cells have been unsuccessfully probed

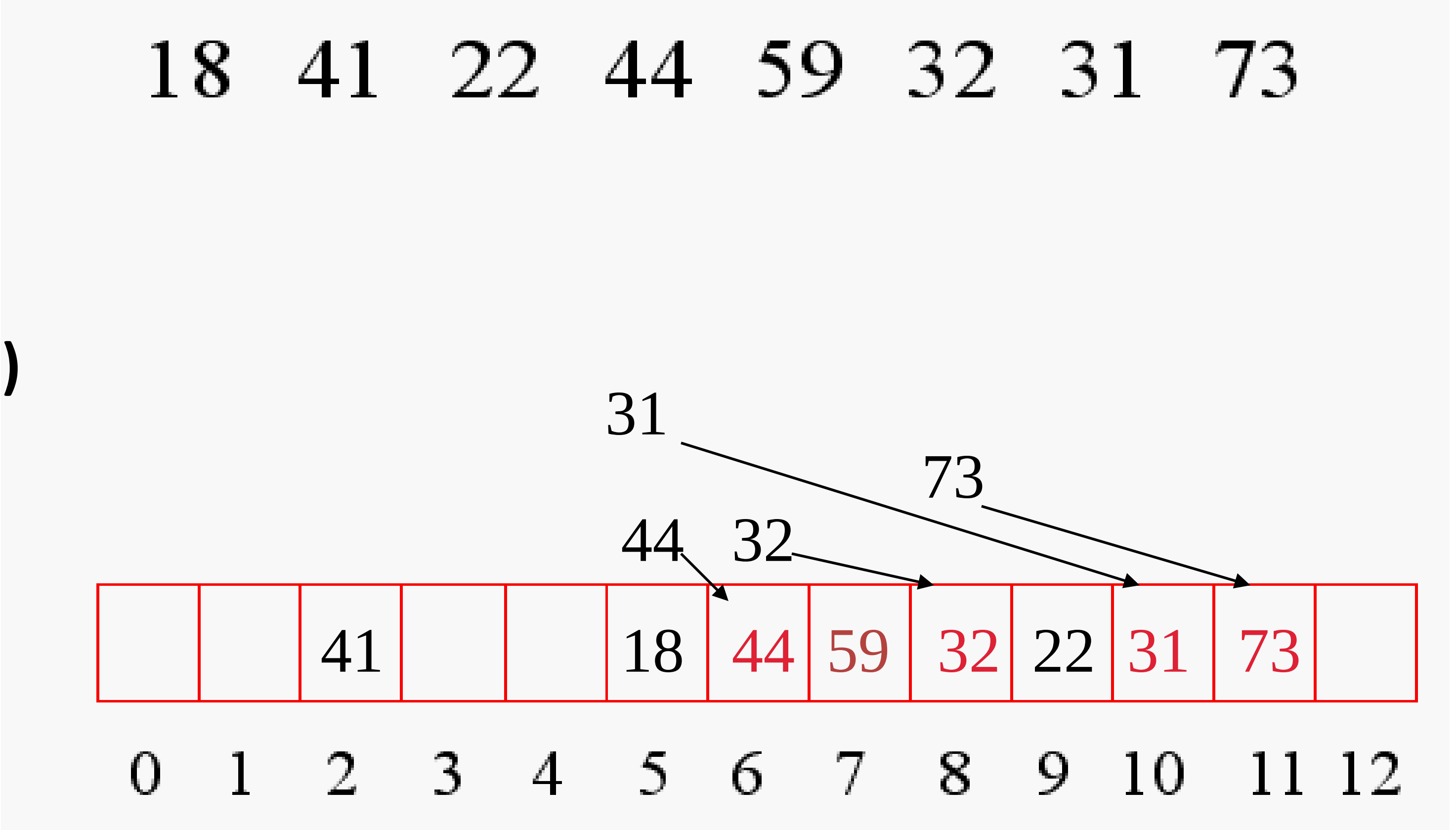
```
Algorithm get(k)
    i ← h(k)
    p ← 0
    repeat
        c ← A[i]
        if c == ∅
            return NULL
        else if c.key () = k
            return c.element()
        else
            i ← (i + 1) mod N
            p ← p + 1
    until p = N
    return NULL
```

Removal and Insertion with Probing

- *remove(k)*
  - Call *get*(*k*) to get the element.
  - Should we set the now empty cell to NULL?
    - **No.** It would mess up the search procedure. See example on the next slide.
  - Return the element.
- A cell has **three** states:
  - **null**: brand new, never used. *get*(*x*) stops when a null cell is reached.
  - **in use**: currently used.
  - **available**: previously used, now available but unused. *get*(*x*) <u>continues</u> the search when encountering an available cell.
    - Example of available cells: key has value -1.

Example with *remove*(*k*)

***remove*(59)**
***get*(31)**



Linear Probing: Removal and Insertion

- To handle insertions and deletions, we marked the deleted cells as “available” instead of null.
- *remove*$(\boldsymbol{k})$
  - We search for a cell with key $\boldsymbol{k}$
  - If such an item is found, we mark the cell as “available” and we return the element.
  - Else, we return $\boldsymbol{NULL}$
- *put*$(\boldsymbol{k, e})$

  If table is not full, we start at cell $\boldsymbol{h}(\boldsymbol{k})$. If this cell is occupied:
  - We probe consecutive cells until a cell $\boldsymbol{i}$ is found that is either null or marked as “available”.
  - We store item $(\boldsymbol{k, e})$ in cell $\boldsymbol{I}$

Load Factor – Linear Probing

- Define the load factor $\lambda = n/N$
  - n = number of elements in the hash table
  - N = hash table size (prime number)

- To obtain best performance with linear probing, ensure that $\lambda \leq 0.5$.

- As we add more elements to the hash table, λ goes up $\Rightarrow$ rehashing (allocate a bigger table, define a new hash function, and copy the elements to the new array).

Next lecture …

- Probing (open addressing)
  - Linear probing
  - Quadratic probing
  - Double hashing
- Rehashing
- Hash functions for strings