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Appendix C

The Basics of Logic Design

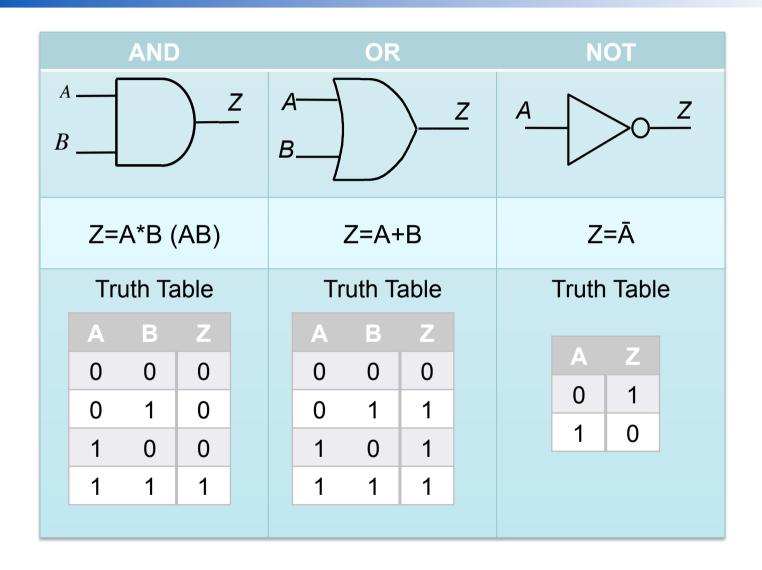
Outline

- Fundamental Boolean operations
- Deriving logic expressions from truth tables
- Boolean Identities
- Simplifying logic expressions using Boolean identities
- Combinational and sequential circuits
- Verilog basics

Boolean Algebra

- Boolean algebra is the basic math used in digital circuits and computers.
- A Boolean variable takes on only 2 values: {0,1}, {T,F}, {Yes, No}, etc.
- There are 3 fundamental Boolean operations:
 - AND, OR, NOT

Fundamental Boolean Operations



Boolean Algebra

- A truth table specifies output signal logic values for every possible combination of input signal logic values
- In evaluating Boolean expressions, the Operation Hierarchy is: 1) NOT 2) AND 3)
 OR. Order can be superseded using (...)
- **Example:** A = T, B = F, C = T, D = T
- What is the value of $Z = (\overline{A} + B) \cdot (C + \overline{B} \cdot D)$?

$$Z = (\overline{T} + F) \cdot (C + \overline{B} \cdot D) = (F + F) \cdot (C + \overline{B} \cdot D)$$
$$= F \cdot (C + \overline{B} \cdot D) = F$$

Deriving Logic Expressions From Truth Tables

Light must be ON when both switches A and B are OFF, or when both of them are ON.



Truth Table:

A	В	Z
0	0	1
0	1	0
1	0	0
1	1	1

What is the Boolean expression for Z?

Minterms and Maxterms

Minterms

- AND term of all input variables.
- For variables with value 0, apply complements
- Maxterms
 - OR factor with all input variables
 - For variables with value 1, apply complements

A	В	Z	Minterms	Maxterms
0	0	1	$ar{A}.ar{B}$	A + B
0	1	0	$ar{A}$. B	$A + \bar{B}$
1	0	0	$A.ar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

Minterms and Maxterms

- A function with n variables has 2ⁿ minterms (and Maxterms) – exactly equal to the number of rows in truth table
- Each minterm is true for exactly one combination of inputs
- Each Maxterm is false for exactly one combination of inputs

Α	В	Z	Minterms	Maxterms
0	0	1	$ar{A}$. $ar{B}$	A + B
0	1	0	$ar{A}$. B	$A + \bar{B}$
1	0	0	$A.ar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

Equivalent Logic Expressions

- Two <u>equivalent</u> logic expressions can be derived from Truth Tables:
- 1. Sum-of-Products (SOP) expressions:
 - Several AND terms OR'd together, e.g.

$$\overline{ABC} + \overline{ABC} + \overline{ABC}$$

- 2. Product-of-Sum (POS) expressions:
 - Several OR terms AND'd together, e.g.

$$(\overline{A} + \overline{B} + C)(A + B + \overline{C})$$

Rules for Deriving SOP Expressions

- Find each row in TT for which output is 1 (rows 1 & 4)
- 2. For those rows write a minterm of all input variables.
- OR together all minterms found in (2): Such an expression is called a Canonical SOP

Α	В	Z	Minterms	Maxterms
0	0	1	$ar{A}.ar{B}$	A + B
0	1	0	$ar{A}$. B	$A + \bar{B}$
1	0	0	$A.ar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

$$Z = \overline{A} \overline{B} + AB$$

Rules for Deriving POS Expressions

- Find each row in TT for which output is 0 (rows 2 & 3)
- 2. For those rows write a maxterm
- AND together all maxterm found in (2): Such an expression is called a Canonical POS.

A	В	Z	Minterms	Maxterms
0	0	1	$ar{A}$. $ar{B}$	A + B
0	1	0	$ar{A}$. B	$A + \bar{B}$
1	0	0	$A.ar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

$$Z = (A + \overline{B})(\overline{A} + B)$$

CSOP and CPOS

- Canonical SOP: $Z = \overline{A} \overline{B} + AB$
- Canonical POS: Z = (A + B)(A + B)
- Since they represent the same truth table, they should be identical

Verify that
$$Z = \overline{A} \overline{B} + AB \equiv (A + \overline{B})(\overline{A} + B)$$

 CPOS and CSOP expressions for the same TT are logically equivalent. Both represent the same information.

Activity 1

Derive SOP and POS expressions for the following TT.

A	В	Carry
0	0	0
0	1	0
1	0	0
1	1	1

Useful for simplifying logic equations.

	(a)	(b)
1	= A = A	$\overline{\overline{A}} = A$
2	A + false = A (A + 0 = A)	$A \cdot true = A (A \cdot 1 = A)$
3	A + true = true (A + 1 = 1)	$A \cdot false = false (A \cdot 0 = 0)$
4	A + A = A	$A \cdot A = A$
5	$A + \overline{A} = \text{true} (A + \overline{A} = 1)$	$A \cdot \overline{A} = \text{false } (A \cdot \overline{A} = 0)$
6	A + B = B + A	$A \cdot B = B \cdot A$
7	A + B + C = (A + B) + C = A + (B + C)	$A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$
8	$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + B \cdot C = (A + B)(A + C)$
9	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{\mathbf{A} \cdot \mathbf{B}} = \overline{\mathbf{A}} + \overline{\mathbf{B}}$
10	$A \cdot B + A \cdot \overline{B} = A$	$(A + B)(A + \overline{B}) = A$
11	$A + A \cdot B = A$	A(A + B) = A
12	$A(\overline{A} + B) = A \cdot B$	$A + \overline{A} \cdot B = A + B$
13	$A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$	$(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Identities	Property
1-5	Single variable, foundations of Boolean manipulation
6	Commutative
7	Associative
8	Distributive
9	De Morgan's
10	Combining
11	Absorption
13	Consensus

- The right side is the dual of the left side
 - Duals formed by replacing

AND
$$\rightarrow$$
 OR OR \rightarrow AND 0 \rightarrow 1 1 \rightarrow 0

The dual of any true statement in Boolean algebra is also a true statement.

DeMorgan's laws very useful: 9a and 9b

$$\overline{A+B}=\overline{A.B}$$

$$A \longrightarrow \overline{A+B}$$

$$B \longrightarrow \overline{AB}$$
NOR gate
$$A \longrightarrow \overline{AB}$$
Alt gate rep.

$$\overline{AB} = \overline{A} + \overline{B}$$

$$A \longrightarrow \overline{AB}$$

$$B \longrightarrow \overline{AB}$$

$$B \longrightarrow \overline{AB}$$

$$B \longrightarrow \overline{AB}$$

$$A \longrightarrow \overline{A+B}$$

$$B \longrightarrow \overline{A+B}$$
Alt gate rep.

Activity 2

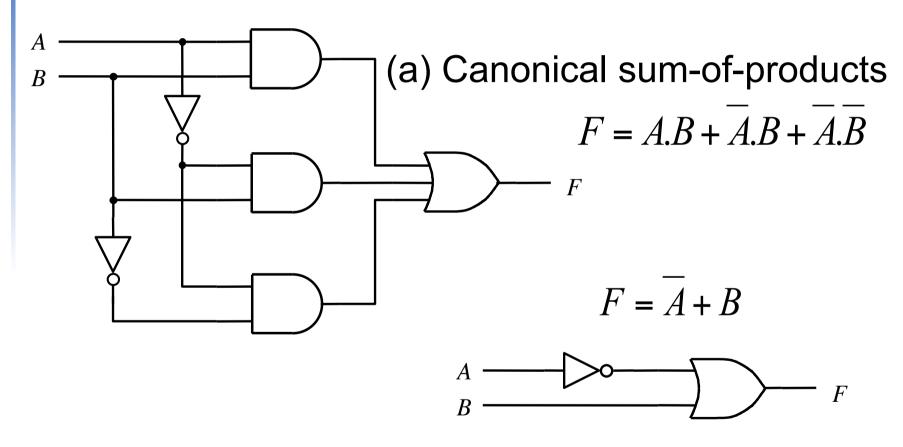
Proofs of some Identities:

12b:
$$A + AB = A + B$$

13a:
$$AB + \overline{AC} + BC = AB + \overline{AC}$$

Simplifying Logic Expressions Using Boolean Identities

Simplifying Logic Equations – Why?



(b) Minimal-cost realization

Simplifying Logic Equations

- Simplifying logic expressions can lead to using smaller number of gates (parts) to implement the logic expression
- Can be done using
 - Boolean Identities (algebraic)
 - Karnaugh Maps (graphical)
- A minimum SOP (MSOP) expression is one that has no more AND terms or variables than any other equivalent SOP expression.
- A minimum POS (MPOS) expression is one that has no more OR factors or variables than any other equivalent POS expression.
- There may be several MSOPs of an expression

Example of Using Boolean Identities

Find an MSOP for

$$F = \overline{X}W + Y + \overline{Z}(Y + \overline{X}W)$$

$$= \overline{X}W + Y + \overline{Z}Y + \overline{Z}\overline{X}W$$

$$= \overline{X}W(1 + \overline{Z}) + Y(1 + \overline{Z})$$

$$= \overline{X}W + Y$$

Activity 3

Find an MSOP for

$$F = V\overline{W}XY + VWYZ + V\overline{X}YZ$$

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Combinational and Sequential Circuits

Digital Circuit Classification

Combinational circuits

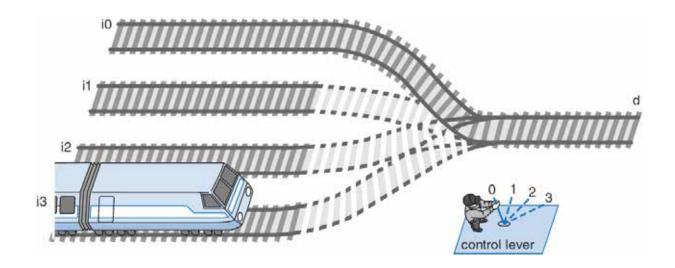
- Output depends only solely on the current combination of circuit inputs
- Same set of input will always produce the same outputs
- Consists of AND, OR, NOR, NAND, and NOT gates
- Sequential circuits
 - Output depends on the current inputs and state of the circuit (or past sequence of inputs)
 - Memory elements such as flip-flops and registers are required to store the "state"
 - Same set of input can produce completely different outputs

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Combinational Circuits

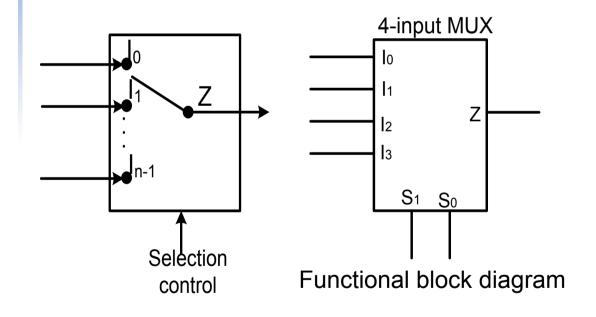
Multiplexer

- A multiplexer (MUX) selects data from one of N inputs and directs it to a single output, just like a railyard switch
 - 4-input Mux needs 2 select lines to indicate which input to route through
 - N-input Mux needs $log_2(N)$ selection lines



Multiplexer (2)

An example of 4-input Mux



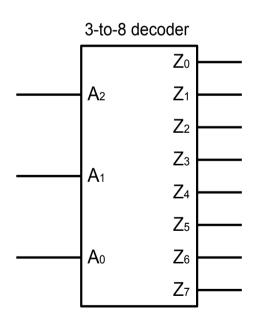
Actual truth table would have 2^6 rows corresponding to I_0 , I_1 , I_2 , I_3 , S_0 and S_1

S ₁	S ₀	Z
0	0	I _o
0	1	I ₁
1	0	l ₂
1	1	l ₃

Condensed truth table

Decoder

- A decoder is a circuit element that will decode an N-bit code.
- It activates an appropriate output line as a function of the applied N-bit input code



Functional block diagram

				Hut	пта	DIC				
A_2	A ₁	A_0	Z_0	Z ₁	Z_2	Z_3	Z_4	Z_5	Z_6	Z ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

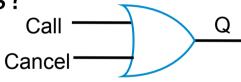
Truth Table

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Sequential Circuits

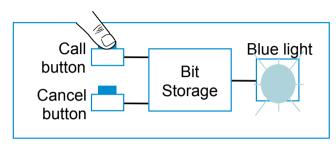
Why Bit Storage?

- Flight attendant call button
 - Press call: light turns on
 - Stays on after button released
 - Press cancel: light turns off
 - Logic gate circuit to implement this?

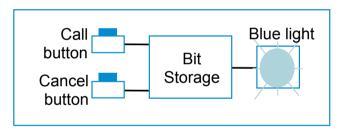


Doesn't work. Q=1 when Call=1, but doesn't stay 1 when Call returns to 0

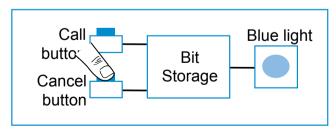
Need some form of "memory" in the circuit



1. Call button pressed – light turns on



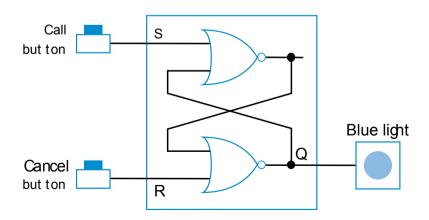
2. Call button released – light stays on



3. Cancel button pressed – light turns off

Bit Storage Using SR Latch

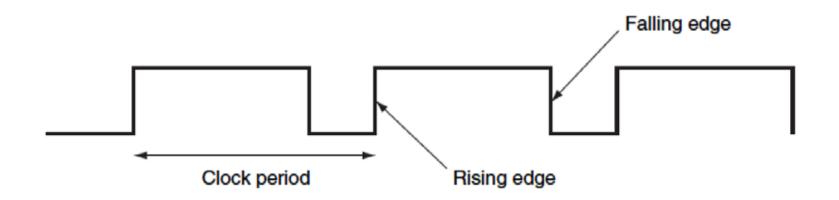
- Simplest memory elements are Latch and Flip-Flops
- SR (set-reset) latch is an un-clocked latch
 - Output Q=1 when S=1, R=0 (set condition)
 - Output Q=0 when S=0, R=1 (reset condition)
 - Problem Q is undefined if S=1 and R=1



Clocks

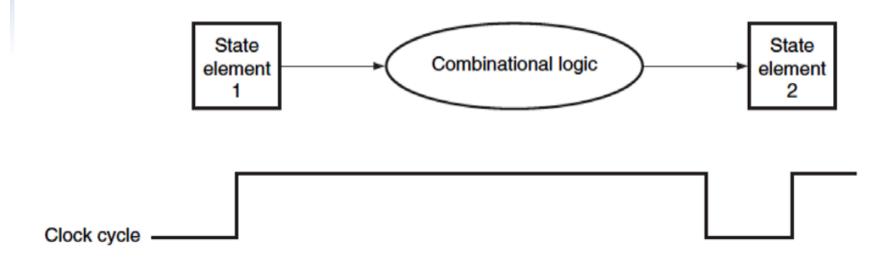
- Clock period: time interval between pulses
 - example: period = 20 ns
- Clock frequency: 1/period
 - example: frequency = 1 / 20 ns = 50MHz
- Edge-triggered clocking: all state changes occur on a clock edge.

Freq	Period
100 GHz	0.01 ns
10 GHz	0.1 ns
1 GHz	1 ns
100 MHz	10 ns
10 MHz	100 ns



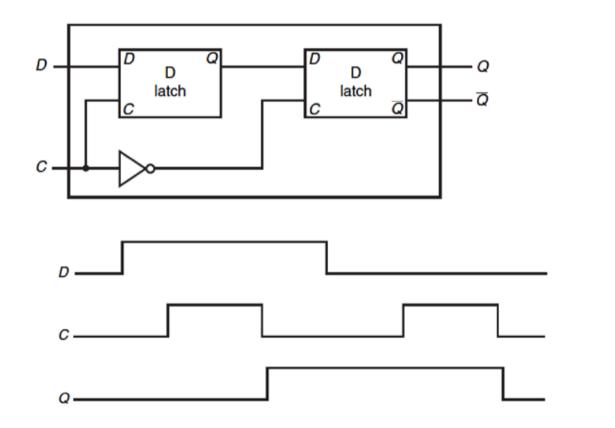
Clock and Change of State

- Clock controls when the state of a memory element changes
- Edge-triggered clocking: all state changes occur on a clock edge.



Clock Edge Triggered Bit Storage

- Flip-flop Bit storage that stores on clock edge, not level
- D Flip-flop
 - Two latches, master and slave latches.
 - Output of the first goes to input of second, slave latch has inverted clock signal (falling-edge trigger)



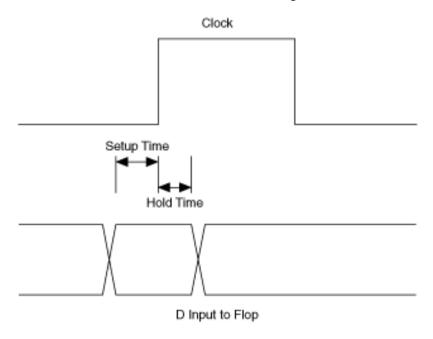
Setup and Hold Time

Setup time

The minimum amount of time the data signal should be held steady before the clock edge arrives.

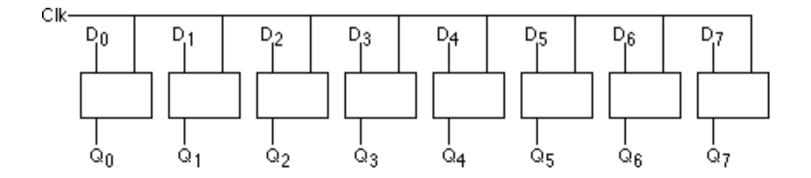
Hold time

The minimum amount of time the data signal should be held steady after the clock edge.



N-Bit Register

- Cascade N number of D flip-flops to form a N-bit register
- An example of 8-bit register formed by 8 edge-triggered D flip-flops



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Verilog Basics

What is an HDL?

- A Hardware Description Language (HDL) is a software programming language used to model the intended operation of a piece of hardware.
- The difference between an HDL and "C"
 - Concurrency
 - Timing
- A powerful feature of the Verilog HDL is that we can use the same language for describing, testing and debugging the system.

An Example

```
module pound_one;
reg [7:0] a,a$b,b,c; // register declarations
reg clk;
initial
begin
    clk=0; // initialize the clock
    c = 1:
    forever #25 clk = !clk;
end
/* This section of code implements
   a pipeline */
always @ (posedge clk)
begin
    a = b:
    b = c;
end
endmodule
```

Identifiers

- Identifiers are names assigned by the user to Verilog objects such as modules, variables, tasks etc.
- An identifier may contain any sequence of letters, digits, a dollar sign '\$', and the underscore '_' symbol.
- The first character of an identifier must be a letter or underscore; it cannot be a dollar sign '\$', for example. We cannot use characters such as '-' (hyphen), brackets, or '#' in Verilog names (escaped identifiers are an exception).

Escaped Identifiers

- The use of escaped identifiers allow any character to be used in an identifier.
 - Escaped identifiers start with a backslash (\) and end with white space (White space characters are space, tabs, carriage returns).
 - Gate level netlists generated by EDA tools (like DC) often have escaped identifiers

Examples:

- Vclock = 0;
- a*b = 0;
- \5-6
- \bus_a[0]
- \bus_a[1]

```
module identifiers; /* Multiline comments in Verilog look like C comments
and // is OK in here. */
// Single-line comment in Verilog
 reg legal_identifier, two__underscores;
 reg _OK,OK_,OK_$,OK_123,CASE_SENSITIVE, case_sensitive;
  reg Vclock ,\a*b ; // Add white space after escaped identifier.
  //reg $ BAD,123 BAD; // Bad names even if we declare them!
 initial begin
      legal identifier = 0; // Embedded underscores are OK,
      two underscores = 0; // even two underscores in a row.
      _OK = 0; // Identifiers can start with underscore
      OK = 0; // and end with underscore.
      OK$ = 0; // $ sign is OK.
      OK 123 =0; // Embedded digits are OK.
      CASE SENSITIVE = 0; // Verilog is case-sensitive (unlike VHDL).
      case sensitive = 1;
      Vclock = 0; // An escaped identifier with \ breaks rules
      \a*b = 0; // but be careful to watch the spaces!
      $display("Variable CASE SENSITIVE= %d", CASE SENSITIVE);
      $display("Variable case_sensitive= %d",case_sensitive);
      $display("Variable Vclock = %d", Vclock );
      \frac{\a^*b = \%d',\a^*b)}{}
                                                          An Example
  end
endmodule
```

Simulation Result of the Example

Variable CASE_SENSITIVE= 0

Variable case_sensitive= 1

Variable /clock = 0

Variable a*b = 0

Logic values

- Verilog has 4 logic Values:
 - '0' represents zero, low, false, not asserted.
 - '1' represents one, high, true, asserted.
 - 'z' or 'Z' represent a high-impedance value, which is usually treated as an 'x' value.
 - 'x' or 'X' represent an uninitialized or an unknown logic value--an unknown value is either '1', '0', 'z', or a value that is in a state of change.

Data Types

- Three data type classes:
 - Nets
 - Physical connections between devices
 - Example: wire a, b;
 - Registers
 - Storage devices, variables.
 - Example: reg a; reg [7:0] bus;
 - Parameters
 - Constants
 - Example: parameter width=32; parameter A string ="hello";

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Code Structure

Design Entities Verilog Module Basics

Design Entities

- The module is the basic unit of code in the Verilog language.
- Example

```
module holiday_1(sat, sun,weekend);
input sat, sun;
output weekend;
assign weekend = sat | sun;
endmodule
```

Verilog Module

- Modules contain
 - declarations
 - functionality
 - timing

```
module name (port_names);

module port declarations

data type declarations

procedural blocks

continuous assignments

user defined tasks & functions

primitive instances

module instances

specify blocks
```

endmodule

```
syntax:
module module_name (signal, signal,... signal );
. ; //content of module
.
..
..
endmodule
```

Module Port Declarations

- Scalar (1bit) port declarations:
 - port_direction port_name, port_name ...;
- Vector (Multiple bit) port declarations:
 - port_direction [port_size] port_name, port_name ...;
- port_direction : input, inout (bi-directional) or output
- port_name : legal identifier
- port_size : is a range from [msb:lsb]

```
input a, into_here, george;// scalar ports
input [7:0] in_bus, data; //vectored ports
output [31:0] out_bus; //vectored port
inout [maxsize-1:0] a_bus;//parameterized port
```

Module Instances

- A module may be instantiated within another module.
- There may be multiple instances of the same module.

```
syntax for instantiation:

module_name instance_name (signal, signal,...);
```

```
module example (a,b,c,d);
input a,b;
output c,d;
. . . .
endmodule

example ex_inst_1(in_1, in_2, w, z);
example ex_inst_2(in_1, in_2, , z); // skip a port
```

Gate-level Primitives

- Verilog has pre-defined primitives that implement basic logic functions.
- Structural modeling with the primitives is similar to schematic level design.

```
and nand or nor xor xnor buf not bufif0 bufif1 notif0 notif1
```

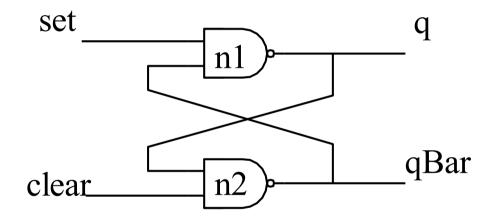
```
module
gate_level_ex(in_1,in_2,c);
output c;
input in_1,in_2;

nand (a, in_1, in_2);
not (b, a);
or or_1(c, in_2, b);
endmodule
```

$$in_{1}$$
 a b or_{1} c

Activity 4

Given the circuit below, develop a Verilog module for the circuit



User-Defined Primitives

- We can define primitive gates (a user-defined primitive or UDP) using a truth-table specification. The first port of a UDP must be an output port, and this must be the only output port (we may not use vector or inout ports).
- An example primitive Adder(Sum, InA, InB); output Sum; input InA, InB; table // inputs : output 00 : 0; 01 : 1; 10 : 1; 11 : 0; endtable endprimitive

Operators

Verilog operators (in increasing order of precedence)

```
?: (conditional)
 || (logical or)
&& (logical and)
| (bitwise or)
~| (bitwise nor)
^ (bitwise xor)
^~ ~^ (bitwise xnor, equivalence)
& (bitwise and)
~& (bitwise nand)
= == (logical) != (logical) === (case) !== (case)
(It)
<= (It or equal)</p>
> (gt)
>= (gt or equal)
<< (shift left)</p>
>> (shift right)
+ (addition)
 - (subtraction)
* (multiply)
 / (divide)
% (modulus)
```

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Procedural Assignment Continuous Assignment Control Statement

Procedures

- A Verilog procedure is an always or initial statement, a task, or a function.
- The statements within a sequential block (statements that appear between a begin and an end) that is part of a procedure execute sequentially in the order in which they appear, but the procedure executes concurrently with other procedures.

Procedural Blocks

- There are two types of procedural blocks:
 - initial blocks executes only once
 - always blocks executes in a loop
- Multiple Procedural blocks may be used, if so the multiple blocks are <u>concurrent</u>.
- Procedural blocks may have:
 - Timing controls which delays when a statement may be executed
 - Procedural assignments
 - Programming statements

Procedural Statement Groups

- When there is more than one statement within a procedural block the statements <u>must</u> be grouped.
- Sequential grouping: statements are enclosed within the keywords begin and end.
 - An example

 always

 begin

 a = 5; // executed 1st

 c = 4; // executed 2nd

 wake_up = 1; // executed 3rd

 end

Timing Controls (procedural delays)

- #delay simple delay
 - Delays execution for a specific number of time steps.

```
#5 reg_a = reg_b;
```

- @ (edge signal) edge-triggered timing control
 - Delays execution until a transition on signal occurs.
 - edge is optional and can be specified as either posedge or negedge.
 - Several *signal* arguments can be specified using the keyword *or*.
 - •An example : always @ (posedge clk) reg_a = reg_b;
- wait (expression) level-sensitive timing control
 - Delays execution until expression evaluates true.
 - wait (cond_is_true) reg_a = reg_b;

Procedural assignments

- Assignments made within procedural blocks are called procedural assignments.
 - Value of the RHS of the equal sign is transferred to the LHS
 - LHS must be a register data type (reg, integer, real). NO NETS!
 - RHS may be any valid expression or signal

```
always @ (posedge clk)
begin
a = 5; // procedural assignment
c = 4*32/6; // procedural assignment
wake_up =$time; // procedural assignment
end
```

Continuous Assignment

- Continuous assignment assigns a value to a wire in a similar way that a real logic gate drives a real wire.
- The main use for continuous assignments is to model combinatorial logic.

```
syntax: Explicit continuous assignment:
    assign net_name = expression;
    where net_name is a net that has been previously declared
```

```
module continuous (Ain, Aout);
input Ain;
output Aout;
assign Aout = ~Ain //continuous assignment.
endmodule

Ain

Aout
63
```

Illustration of Assignment Statements

```
module assignments
 //... Continuous assignments go here.
 always // beginning of a procedure
     begin // beginning of sequential block
    //... Procedural assignments go here.
     end
endmodule
```

Control Statements

- Two types of programming statements:
 - Conditional
 - Looping
- Programming statements only used in procedural blocks

if and if-else

syntax:

if(expression) statement

If the expression evaluates to true then execute the statement

if(expression) statement1 else statement2

If the expression evaluates to true then execute statement1, if false, then execute statement2.

```
module if_ex(clk);
input clk;
reg red,blue,pink,yellow,orange,color,green;
always @ (posedge clk)
if (red || (blue && pink))
begin
$display ("color is mixed up");
color <= 0; // reset the color
end
else if (blue && yellow)
$display ("color is greenish");
else if (yellow && (green || orange))
$display ("not sure what color is");
else $display ("color is black");
endmodule
```

for

syntax:

for (assignment_init; expression; assignment) statement or statement_group

- The assignment_init is executed once at the start of the loop.
- Loop executes as long as expression is true.
- The *assignment* is executed at the completion of each loop.

```
module for_ex1 (clk);
input clk;
reg [31:0] mem [0:9]; // 10x32 memory
integer i;
always @ (posedge clk)
for (i = 9; i >= 0; i = i-1)
mem[i] = 0; // init the memory to zeros
endmodule
```

Simulating the Verilog Code

Verilog code of NAND Latch Module simple latch (q, qBar, set, clear); input set, clear; output q, qBar; nand #2 n1(q,qBar,set); nand #2 n2(qBar,q,clear); endmodule set

clear

qBar

Testbench

- A testbench generates a sequence of input values (we call these input vectors) that test or exercise the verilog code.
- It provides stimulus to the statement that will monitor the changes in their outputs.
- Testbenchs do not have a port declaration but must have an instantiation of the circuit to be tested.

A testbench for NAND Latch

```
Module test simple latch;
    wire q, qBar;
    reg set, clear;
    simple_latch SL1(q,qBar,set,clear);
    initial
          begin
            #10 \text{ set} = 0; clear = 1;
            #10 \text{ set} = 1:
            #10 clear = 0:
            #10 clear = 1:
            #10 $stop;
            #10 $finish;
          end
    initial
          begin
            $monitor ("%d set= %b clear= %b q=%b qBar=%b",$time,
                       set,clear,q,qBar);
          end
endmodule
```