CSE 2021 Computer Organization

Appendix C

The Basics of Logic Design

Outline

- Fundamental Boolean operations
- Deriving logic expressions from truth tables
- Boolean Identities
- Simplifying logic expressions using Boolean identities
- Combinational and sequential circuits
- Verilog basics

Boolean Algebra

- Boolean algebra is the basic math used in digital circuits and computers.
- A Boolean variable takes on only 2 values: {0,1}, {T,F}, {Yes, No}, etc.
- There are 3 fundamental Boolean operations:
 - AND, OR, NOT

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Fundamental Boolean Operations Ζ Z=A*B (AB) Z=A+B Z=Ā **Truth Table** Truth Table Truth Table 0 0 1 0 1 1 0 1 0 1 1 1 1 1 4

Boolean Algebra

- A truth table specifies output signal logic values for every possible combination of input signal logic values
- In evaluating Boolean expressions, the Operation Hierarchy is: 1) NOT 2) AND 3)
 OR. Order can be superseded using (...)
- **Example:** A = T, B = F, C = T, D = T
- What is the value of $Z = (\overline{A} + B) \cdot (C + \overline{B} \cdot D)$?

$$Z = (\overline{T} + F) \cdot (C + \overline{B} \cdot D) = (F + F) \cdot (C + \overline{B} \cdot D)$$
$$= F \cdot (C + \overline{B} \cdot D) = F$$

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Deriving Logic Expressions From Truth Tables

Light must be ON when both switches A and B are OFF, or when both of them are ON.



Truth Table:

Α	В	
0	0	1
0	1	0
1	0	0
1	1	1

What is the Boolean expression for Z?

Minterms and Maxterms

- Minterms
 - AND term of all input variables.
 - For variables with value 0, apply complements
- Maxterms
 - OR factor with all input variables
 - For variables with value 1, apply complements

	В		Minterms	Maxterms
0	0	1	$\bar{A}.\bar{B}$	A + B
0	1	0	$\bar{A}.B$	$A + \bar{B}$
1	0	0	$A. \bar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

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Minterms and Maxterms

- A function with n variables has 2ⁿ minterms (and Maxterms) – exactly equal to the number of rows in truth table
- Each minterm is true for exactly one combination of inputs
- Each Maxterm is false for exactly one combination of inputs

Α	В		Minterms	Maxterms
0	0	1	$\bar{A}.\bar{B}$	A + B
0	1	0	$\bar{A}.B$	$A + \bar{B}$
1	0	0	$A. \bar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

Equivalent Logic Expressions

- Two <u>equivalent</u> logic expressions can be derived from Truth Tables:
- Sum-of-Products (SOP) expressions:
 - Several AND terms OR'd together, e.g.

$$\overrightarrow{ABC} + \overrightarrow{ABC} + \overrightarrow{ABC}$$

- 2. Product-of-Sum (POS) expressions:
 - Several OR terms AND'd together, e.g.

$$(\overline{A} + \overline{B} + C)(A + B + \overline{C})$$

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Rules for Deriving SOP Expressions

- Find each row in TT for which output is 1 (rows 1 & 4)
- 2. For those rows write a minterm of all input variables.
- OR together all minterms found in (2):
 Such an expression is called a *Canonical* SOP

Α	В		Minterms	Maxterms
0	0	1	$\bar{A}.\bar{B}$	A + B
0	1	0	$\bar{A}.B$	$A + \bar{B}$
1	0	0	$A. \bar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

 $Z = \overline{A} \, \overline{B} + AB$

Rules for Deriving POS Expressions

- Find each row in TT for which output is 0 (rows 2 & 3)
- 2. For those rows write a maxterm
- 3. AND together all maxterm found in (2): Such an expression is called a Canonical POS.

Α	В	Z	Minterms	Maxterms
0	0	1	$\bar{A}.\bar{B}$	A + B
0	1	0	\bar{A} . B	$A + \bar{B}$
1	0	0	$A.ar{B}$	$\bar{A} + B$
1	1	1	AB	$\bar{A} + \bar{B}$

$$Z = (A + \overline{B})(\overline{A} + B)$$

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CSOP and CPOS

- Canonical SOP: $Z = \overline{A} \overline{B} + AB$
- Canonical POS: $Z = (A + \overline{B})(\overline{A} + B)$
- Since they represent the same truth table, they should be identical

Verify that
$$Z = \overline{A} \overline{B} + AB = (A + \overline{B})(\overline{A} + B)$$

 CPOS and CSOP expressions for the same TT are logically equivalent. Both represent the same information.

Activity 1

Derive SOP and POS expressions for the following TT.

A	В	Carry
0	0	0
0	1	0
1	0	0
1	1	1

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Boolean Identities

Boolean Identities Useful for simplifying logic equations. $\overline{\overline{A}} = A$ $\overline{\overline{A}} = A$ A + false = A (A + 0 = A) $A \cdot true = A (A \cdot 1 = A)$ $A + true = true \quad (A + 1 = 1)$ $A \cdot \text{false} = \text{false} \quad (A \cdot 0 = 0)$ A + A = A $A + \overline{A} = \text{true} \quad (A + \overline{A} = 1)$ $\begin{array}{ccc} A \cdot \underline{A} &= A \\ A \cdot \overline{A} &= \text{ false } (A \cdot \overline{A} = 0) \end{array}$ $A + A = \text{true} \quad (A + A = 1)$ A + B = B + A A + B + C = (A + B) + C = A + (B + C) $A \cdot (B + C) = A \cdot B + A \cdot C$ $A \cdot B = A \cdot B + A \cdot C$ $A \cdot A = \text{taise} \quad (A \cdot A = 0)$ $A \cdot B = B \cdot A$ $A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$ $A + \frac{B \cdot C}{A \cdot B} = \frac{(A + B)(A + C)}{A + B}$ 6 8 $A \cdot B + A \cdot \overline{B} = A$ 10 $(A + B)(A + \overline{B}) = A$ 11 $A + A \cdot B = A$ A(A + B) = A $A(\overline{A} + B) = A \cdot B$ 12 $A + \overline{A} \cdot B = A + B$ 13 $A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$ $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$ 15

Boolean Identities Single variable, foundations of Boolean 1-5 manipulation 6 Commutative 7 Associative 8 Distributive 9 De Morgan's 10 Combining Absorption 11 Consensus 13 16

Boolean Identities

- The right side is the dual of the left side
 - Duals formed by replacing

$$AND \rightarrow OR$$
 $OR \rightarrow AND$
 $0 \rightarrow 1$
 $1 \rightarrow 0$

The dual of any true statement in Boolean algebra is also a true statement.

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Boolean Identities

• DeMorgan's laws very useful: 9a and 9b

$$\overline{A+B} = \overline{A}.\overline{B}$$

$$B \longrightarrow \overline{A+B} = A \longrightarrow \overline{A}.\overline{B}$$
NOR gate Alt gate rep.

Activity 2

Proofs of some Identities:

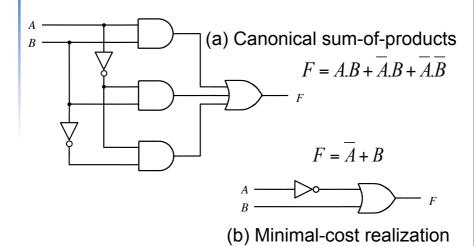
12b:
$$A + \overline{AB} = A + B$$

13a:
$$AB + \overline{AC} + BC = AB + \overline{AC}$$

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Simplifying Logic Expressions Using Boolean Identities

Simplifying Logic Equations – Why?



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Simplifying Logic Equations

- Simplifying logic expressions can lead to using smaller number of gates (parts) to implement the logic expression
- Can be done using
 - Boolean Identities (algebraic)
 - Karnaugh Maps (graphical)
- A minimum SOP (MSOP) expression is one that has no more AND terms or variables than any other equivalent SOP expression.
- A minimum POS (MPOS) expression is one that has no more OR factors or variables than any other equivalent POS expression.
- There may be several MSOPs of an expression

Example of Using Boolean Identities

Find an MSOP for

$$F = \overline{X}W + Y + \overline{Z}(Y + \overline{X}W)$$

$$= \overline{X}W + Y + \overline{Z}Y + \overline{Z}\overline{X}W$$

$$= \overline{X}W(1 + \overline{Z}) + Y(1 + \overline{Z})$$

$$= \overline{X}W + Y$$

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Activity 3

Find an MSOP for

$$F = V\overline{W}XY + VWYZ + V\overline{X}YZ$$

CSE 2021 Computer Organization Combinational and Sequential Circuits

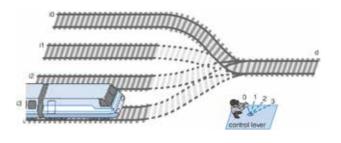
Digital Circuit Classification

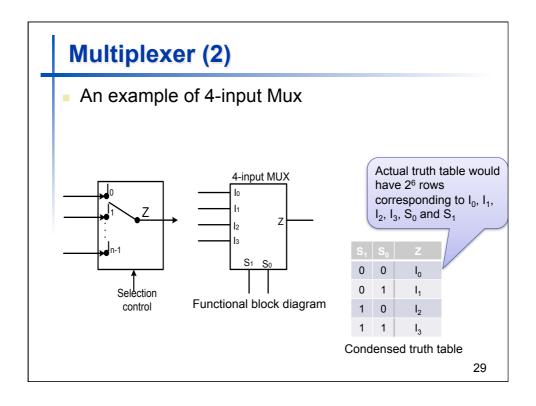
- Combinational circuits
 - Output depends only solely on the current combination of circuit inputs
 - Same set of input will always produce the same outputs
 - Consists of AND, OR, NOR, NAND, and NOT gates
- Sequential circuits
 - Output depends on the current inputs and state of the circuit (or past sequence of inputs)
 - Memory elements such as flip-flops and registers are required to store the "state"
 - Same set of input can produce completely different outputs

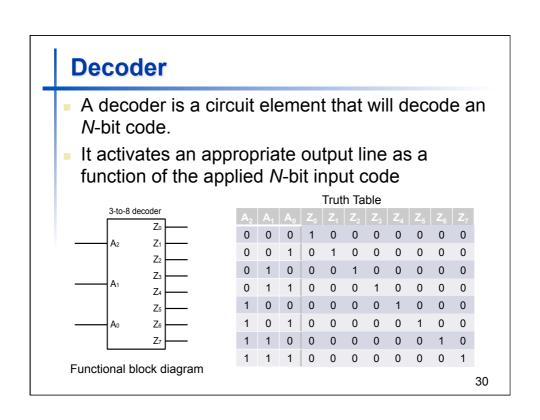
CSE 2021 Computer Organization Combinational Circuits

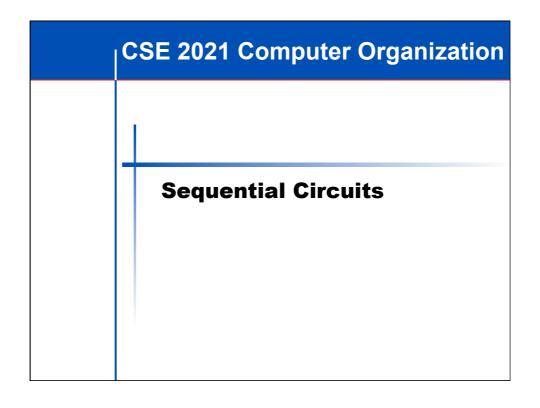
Multiplexer

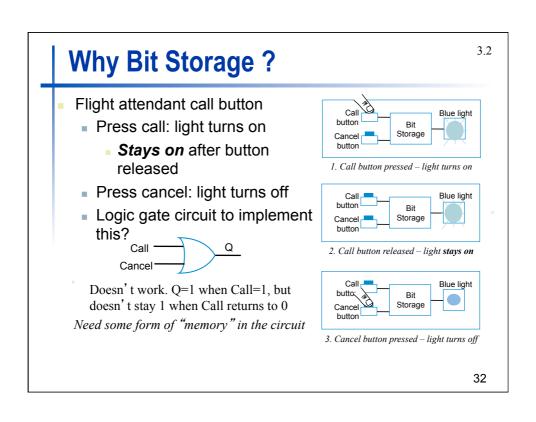
- A multiplexer (MUX) selects data from one of N inputs and directs it to a single output, just like a railyard switch
 - 4-input Mux needs 2 select lines to indicate which input to route through
 - N-input Mux needs log₂(N) selection lines





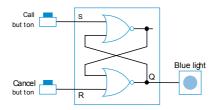






Bit Storage Using SR Latch

- Simplest memory elements are Latch and Flip-Flops
- SR (set-reset) latch is an un-clocked latch
 - Output Q=1 when S=1, R=0 (set condition)
 - Output Q=0 when S=0, R=1 (reset condition)
 - Problem Q is undefined if S=1 and R=1



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Period

0.01 ns

0.1 ns

1 ns

10 ns

100 ns

Freq

100 GHz

10 GHz

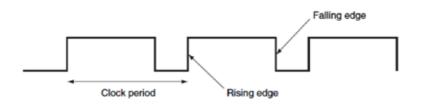
1 GHz

100 MHz

10 MHz

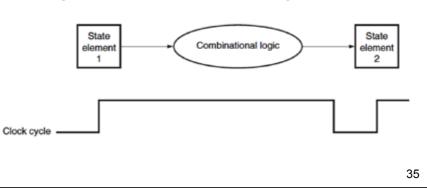
Clocks

- Clock period: time interval between pulses
 - example: period = 20 ns
- Clock frequency: 1/period
 - example: frequency = 1 / 20 ns = 50 MHz
- Edge-triggered clocking: all state changes occur on a clock edge.



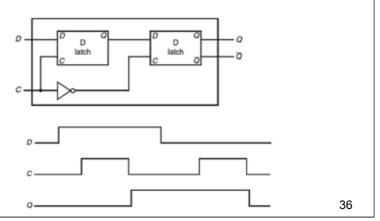
Clock and Change of State

- Clock controls when the state of a memory element changes
- Edge-triggered clocking: all state changes occur on a clock edge.



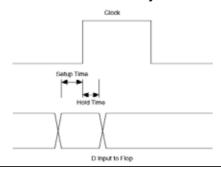
Clock Edge Triggered Bit Storage

- Flip-flop Bit storage that stores on clock edge, not level
- D Flip-flop
 - Two latches, master and slave latches.
 - Output of the first goes to input of second, slave latch has inverted clock signal (falling-edge trigger)



Setup and Hold Time

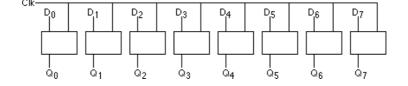
- Setup time
 - The minimum amount of time the data signal should be held steady before the clock edge arrives.
- Hold time
 - The minimum amount of time the data signal should be held steady after the clock edge.



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N-Bit Register

- Cascade N number of D flip-flops to form a N-bit register
- An example of 8-bit register formed by 8 edge-triggered D flip-flops



Verilog Basics

What is an HDL?

- A Hardware Description Language (HDL) is a software programming language used to model the intended operation of a piece of hardware.
- The difference between an HDL and "C"
 - Concurrency
 - Timing
- A powerful feature of the Verilog HDL is that we can use the same language for describing, testing and debugging the system.

An Example

```
module pound_one;
reg [7:0] a,a$b,b,c; // register declarations
reg clk;

initial
begin
    clk=0; // initialize the clock
    c = 1;
    forever #25 clk = !clk;
end
/* This section of code implements
    a pipeline */
always @ (posedge clk)
begin
    a = b;
    b = c;
end
endmodule
```

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Identifiers

- Identifiers are names assigned by the user to Verilog objects such as modules, variables, tasks etc.
- An identifier may contain any sequence of letters, digits, a dollar sign '\$', and the underscore '_' symbol.
- The first character of an identifier must be a letter or underscore; it cannot be a dollar sign '\$', for example. We cannot use characters such as '-' (hyphen), brackets, or '#' in Verilog names (escaped identifiers are an exception).

Escaped Identifiers

- The use of escaped identifiers allow any character to be used in an identifier.
 - Escaped identifiers start with a backslash (\) and end with white space (White space characters are space, tabs, carriage returns).
 - Gate level netlists generated by EDA tools (like DC) often have escaped identifiers
- Examples:
 - Vclock = 0;
 - a*b = 0;
 - \5-6
 - \bus_a[0]
 - \bus_a[1]

```
module identifiers; /* Multiline comments in Verilog look like C comments
and // is OK in here. */
// Single-line comment in Verilog
 reg legal_identifier, two__underscores;
 reg _OK,OK_,OK_$,OK_123,CASE_SENSITIVE, case_sensitive;
 reg \text{Vclock ,\a*b ; // Add white_space after escaped identifier.}
 //reg $_BAD,123_BAD; // Bad names even if we declare them!
 initial begin
     legal_identifier = 0; // Embedded underscores are OK,
     two_underscores = 0; // even two underscores in a row.
      OK = 0; // Identifiers can start with underscore
      OK = 0; // and end with underscore.
     OK$ = 0; // $ sign is OK.
     OK_123 =0; // Embedded digits are OK.
     CASE_SENSITIVE = 0; // Verilog is case-sensitive (unlike VHDL).
     case sensitive = 1;
     Vclock = 0; // An escaped identifier with \ breaks rules
      \a*b = 0; // but be careful to watch the spaces!
      $display("Variable CASE_SENSITIVE= %d",CASE_SENSITIVE);
      $display("Variable case_sensitive= %d",case_sensitive);
      $display("Variable \clock = \%d",\clock );
      An Example
endmodule
```

Simulation Result of the Example

Variable CASE_SENSITIVE= 0

Variable case sensitive= 1

Variable /clock = 0

Variable $\a^*b = 0$

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Logic values

- Verilog has 4 logic Values:
 - '0' represents zero, low, false, not asserted.
 - '1' represents one, high, true, asserted.
 - 'z' or 'Z' represent a high-impedance value, which is usually treated as an 'x' value.
 - 'x' or 'X' represent an uninitialized or an unknown logic value--an unknown value is either '1', '0', 'z', or a value that is in a state of change.

Data Types

- Three data type classes:
 - Nets
 - Physical connections between devices
 - Example: wire a, b;
 - Registers
 - Storage devices, variables.
 - Example: reg a; reg [7:0] bus;
 - Parameters
 - Constants
 - Example: parameter width=32; parameter A_string ="hello";

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Code Structure

Design Entities Verilog Module Basics

Design Entities

- The module is the basic unit of code in the Verilog language.
- Example

```
module holiday_1(sat, sun,weekend);
input sat, sun;
output weekend;
assign weekend = sat | sun;
endmodule
```

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Verilog Module

- Modules contain
 - declarations
 - functionality
 - timing

module name (port_names);

module port declarations

data type declarations

procedural blocks

continuous assignments

user defined tasks & functions

primitive instances

module instances specify blocks

endmodule

```
syntax:
```

```
module_name (signal, signal,... signal );
```

; //content of module

.

endmodule

Module Port Declarations

- Scalar (1bit) port declarations:
 - port_direction port_name, port_name ...;
- Vector (Multiple bit) port declarations:
 - port_direction [port_size] port_name, port_name ...;
- port direction : input, inout (bi-directional) or output
- port_name : legal identifier
- port_size : is a range from [msb:lsb]

input a, into_here, george;// scalar ports
input [7:0] in_bus, data; //vectored ports
output [31:0] out_bus; //vectored port
inout [maxsize-1:0] a bus;//parameterized port

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Module Instances

- A module may be instantiated within another module.
- There may be multiple instances of the same module.

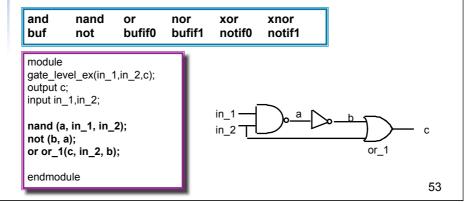
```
syntax for instantiation:
module_name instance_name (signal, signal,...);
```

```
module example (a,b,c,d);
input a,b;
output c,d;
. . . .
endmodule

example ex_inst_1(in_1, in_2, w, z);
example ex_inst_2(in_1, in_2, , z); // skip a port
```

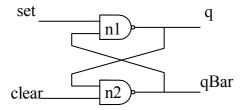
Gate-level Primitives

- Verilog has pre-defined primitives that implement basic logic functions.
- Structural modeling with the primitives is similar to schematic level design.



Activity 4

Given the circuit below, develop a Verilog module for the circuit



User-Defined Primitives

- We can define primitive gates (a user-defined primitive or UDP) using a truth-table specification. The first port of a UDP must be an output port, and this must be the only output port (we may not use vector or inout ports).
- An example
 primitive Adder(Sum, InA, InB);
 output Sum;
 input InA, InB;
 table // inputs : output
 00 : 0;
 01 : 1;
 10 : 1;
 11 : 0;
 endtable
 endprimitive

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Operators

```
Verilog operators (in increasing order of precedence)
```

```
?: (conditional)
  || (logical or)
  && (logical and)
 (bitwise or)
  ~| (bitwise nor)
  ^ (bitwise xor)
  ^~ ~^ (bitwise xnor, equivalence)
  & (bitwise and)
  ~& (bitwise nand)
  == (logical) != (logical) === (case) !== (case)
< (It)</p>
  <= (It or equal)
  > (gt)
 >= (gt or equal)
  << (shift left)
>> (shift right)
  + (addition)
  - (subtraction)
  * (multiply)
  / (divide)
  % (modulus)
```

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Procedural Assignment Continuous Assignment Control Statement

Procedures

- A Verilog procedure is an always or initial statement, a task, or a function.
- The statements within a sequential block (statements that appear between a begin and an end) that is part of a procedure execute sequentially in the order in which they appear, but the procedure executes concurrently with other procedures.

Procedural Blocks

- There are two types of procedural blocks:
 - initial blocks executes only once
 - always blocks executes in a loop
- Multiple Procedural blocks may be used, if so the multiple blocks are <u>concurrent</u>.
- Procedural blocks may have:
 - Timing controls which delays when a statement may be executed
 - Procedural assignments
 - Programming statements

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Procedural Statement Groups

- When there is more than one statement within a procedural block the statements <u>must</u> be grouped.
- Sequential grouping: statements are enclosed within the keywords begin and end.
- An example

```
always

begin

a = 5;  // executed 1st

c = 4;  // executed 2nd

wake_up = 1; // executed 3rd

end
```

Timing Controls (procedural delays)

- #delay simple delay
 - Delays execution for a specific number of time steps.#5 reg a = reg b;
- @ (edge signal) edge-triggered timing control
 - Delays execution until a transition on signal occurs.
 - edge is optional and can be specified as either posedge or negedge.
 - Several **signal** arguments can be specified using the keyword **or**.
 - An example : always @ (posedge clk) reg_a = reg_b;
- wait (expression) level-sensitive timing control
 - Delays execution until expression evaluates true.
 - wait (cond_is_true) reg_a = reg_b;

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Procedural assignments

- Assignments made within procedural blocks are called procedural assignments.
 - Value of the RHS of the equal sign is transferred to the LHS
 - LHS must be a register data type (reg, integer, real). NO NETS!
 - RHS may be any valid expression or signal

```
always @ (posedge clk)
begin
a = 5; // procedural assignment
c = 4*32/6; // procedural assignment
wake_up =$time; // procedural assignment
end
```

Continuous Assignment

- Continuous assignment assigns a value to a wire in a similar way that a real logic gate drives a real wire.
- The main use for continuous assignments is to model combinatorial logic.

```
syntax: Explicit continuous assignment:
    assign net_name = expression;
    where net_name is a net that has been previously declared
```

```
module continuous (Ain, Aout);
input Ain;
output Aout;
assign Aout = ~Ain //continuous assignment.
endmodule

Ain
Aout
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```

Illustration of Assignment Statements

module assignments //... Continuous assignments go here. always // beginning of a procedure begin // beginning of sequential block //... Procedural assignments go here. end endmodule

Control Statements

- Two types of programming statements:
 - Conditional
 - Looping
- Programming statements only used in procedural blocks

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if and if-else

syntax:

if(expression) statement

If the expression evaluates to true then execute the statement

if(expression) statement1 else statement2

If the expression evaluates to true then execute statement1, if false, then execute statement2.

```
module if_ex(clk);
input clk;
reg red,blue,pink,yellow,orange,color,green;
always @ (posedge clk)
if (red || (blue && pink))
begin
$display ("color is mixed up");
color <= 0; // reset the color
end
else if (blue && yellow)
$display ("color is greenish");
else if (yellow && (green || orange))
$display ("not sure what color is");
else $display ("color is black");
endmodule
```

for

syntax:

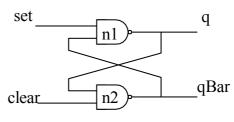
for (assignment_init; expression; assignment) statement or statement group

- The assignment_init is executed once at the start of the loop.
- Loop executes as long as expression is true.
- The assignment is executed at the completion of each loop.

```
module for_ex1 (clk);
input clk;
reg [31:0] mem [0:9]; // 10x32 memory
integer i;
always @ (posedge clk)
for (i = 9; i >= 0; i = i-1)
mem[i] = 0; // init the memory to zeros
endmodule
```

Simulating the Verilog Code

Verilog code of NAND Latch
Module simple_latch (q, qBar, set, clear);
input set, clear;
output q, qBar;
nand #2 n1(q,qBar,set);
nand #2 n2(qBar,q,clear);
endmodule
set



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Testbench

- A testbench generates a sequence of input values (we call these input vectors) that test or exercise the verilog code.
- It provides stimulus to the statement that will monitor the changes in their outputs.
- Testbenchs do not have a port declaration but must have an instantiation of the circuit to be tested.

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A testbench for NAND Latch

```
Module test simple latch;
   wire q, qBar;
   reg set, clear;
   simple_latch SL1(q,qBar,set,clear);
          begin
           #10 set = 0; clear = 1;
           #10 \text{ set} = 1;
           #10 clear = 0;
           #10 clear = 1;
           #10 $stop:
           #10 $finish;
          end
   initial
           $monitor ("%d set= %b clear= %b q=%b qBar=%b",$time,
                      set,clear,q,qBar);
endmodule
```