# ENG2200
# Electric Circuits

Chapter 7

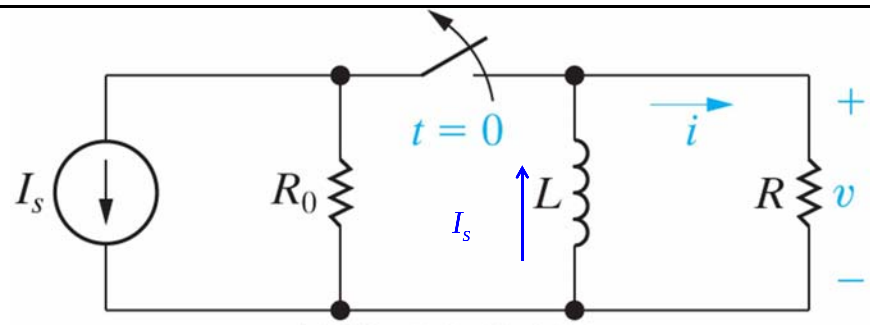
Response of First Order RL and RC Circuits

## Objectives

- Be able to determine the natural response of RL and RC circuits.
- Be able to determine the step response of RL and RC circuits.
- Know how to analyze circuits with sequential switching.

## Introduction

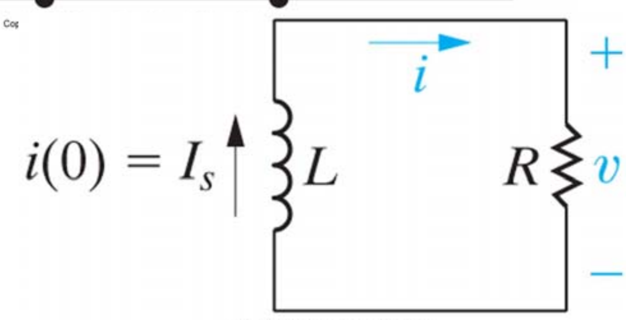
- **Natural Response**: The current and voltages that arise when the stored energy in the L of C is released.
- **Step Response**: The current and voltage that arise when energy is being acquired by the L or C when a sudden application f voltage or current is applied to the circuit.




The switch is ON for a long time. L is a short circuit, no voltage across R (no current in R).

All of $I_s$ goes directly to L

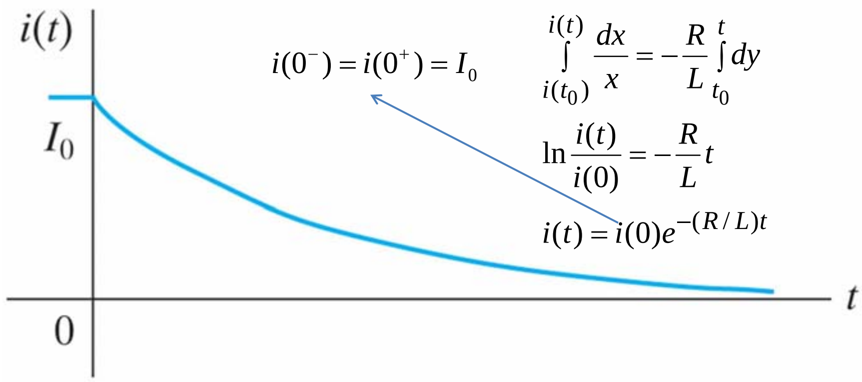
Then the switch is turned OFF (open)



$$v(0^-) = 0$$
$$v(0^+) = I_0 R$$

$$L\frac{di}{dt} + Ri = 0$$
$$\frac{di}{dt}dt = -\frac{R}{L}idt$$
$$\frac{di}{i} = -\frac{R}{L}dt$$
$$\int_{i(t_0)}^{i(t)} \frac{dx}{x} = -\frac{R}{L}\int_{t_0}^{t} dy$$
$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t$$
$$i(t) = i(0)e^{-(R/L)t}$$

$$i(0^-) = i(0^+) = I_0$$





# Power and Energy

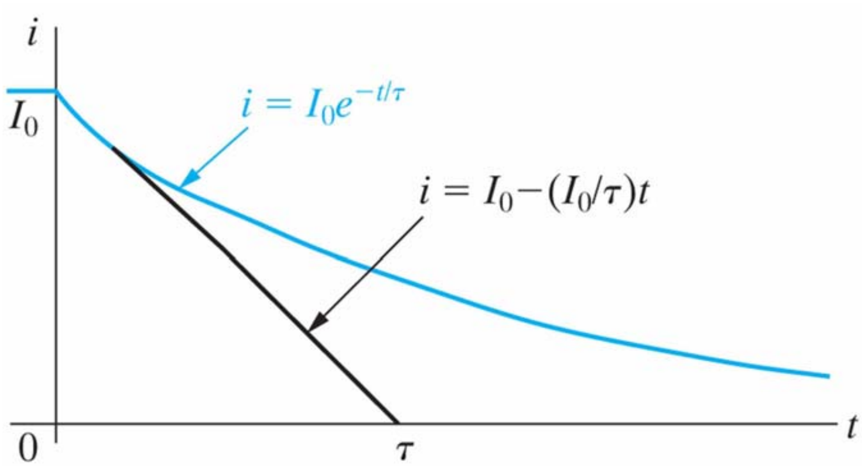
$$p = vi = i^2 R$$
$$p = I_0^2 R e^{-2(R/L)t}$$
$$w = \int_0^t p(x)dx$$
$$w = \int_0^t I_0^2 R e^{-2(R/L)x} dx$$
$$w = \frac{-1}{2R/L} I_0^2 R \quad e^{-2(R/L)x}\Big|_0^t$$
$$w = \frac{1}{2}LI_0^2(1 - e^{-2(R/L)t})$$

**TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors**

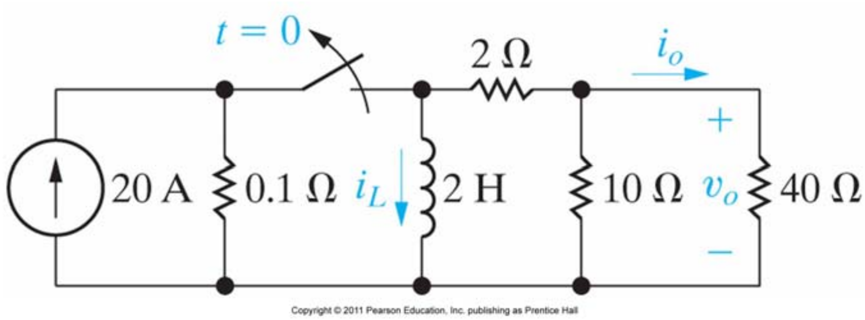
| | |
|---|---|
| **Inductors** | |
| $v = L\frac{di}{dt}$ | (V) |
| $i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$ | (A) |
| $p = vi = Li\frac{di}{dt}$ | (W) |
| $w = \frac{1}{2}Li^2$ | (J) |
| **Capacitors** | |
| $v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$ | (V) |
| $i = C\frac{dv}{dt}$ | (A) |
| $p = vi = Cv\frac{dv}{dt}$ | (W) |
| $w = \frac{1}{2}Cv^2$ | (J) |



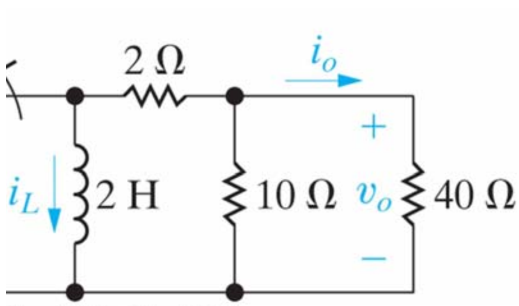
$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = \frac{L}{R} \text{ time constant}$$



## Example



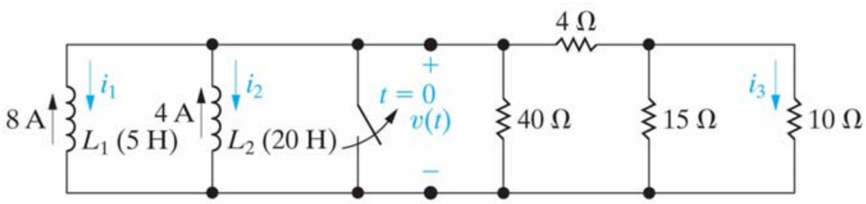

## Example



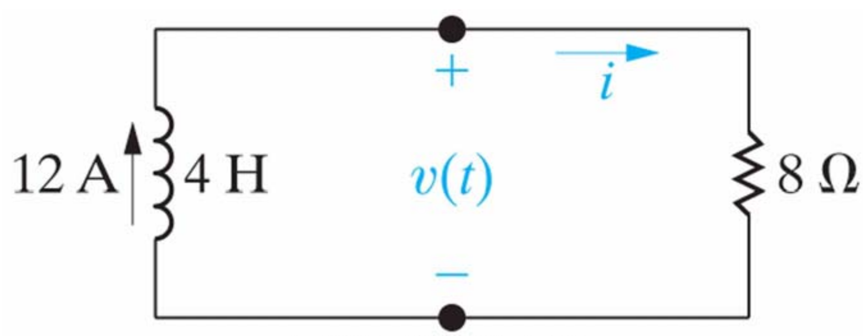

$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = \frac{L}{R} \text{ time constant}$$

$$\mathrm{i(t)} = 20e^{-4000t}$$

# Example





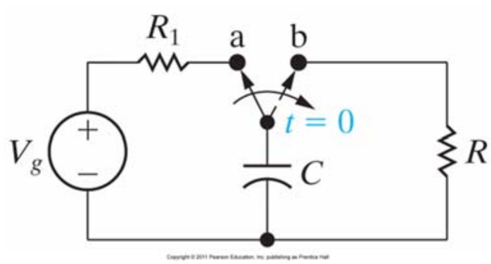



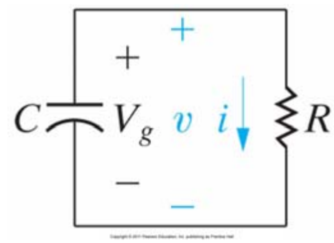



$$i(t) = I_0 e^{-(R/L)t}, \quad \tau = 0.5\,\text{sec}$$

$$\text{i(t)} = 12e^{-2t}$$

# An *RC circuit.*




Switch in position A for a long time
The capacitor charged to $V_g$

# An *RC circuit.*

$$v + iR = 0$$

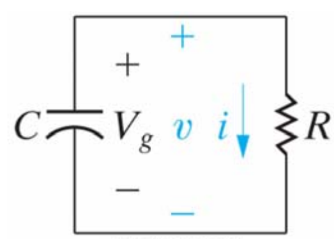
$$v = -RC\frac{dv}{dt}$$

$$dt = -RCdv\frac{dv}{v}$$

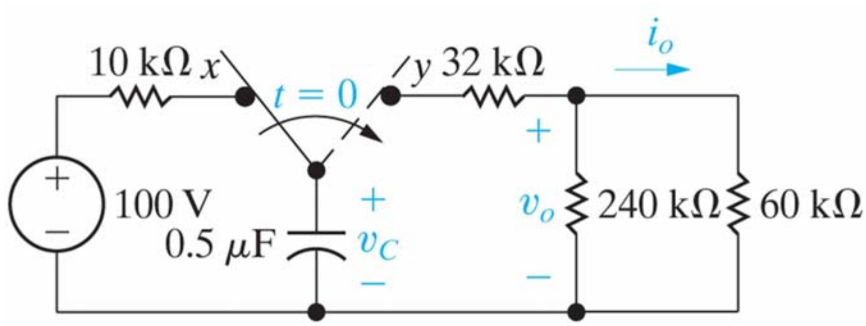
$$\int_{i(t_0)}^{i(t)} -RC\frac{dv}{v} = \int_{t_0}^{t} dy$$

$$\ln\frac{v(t)}{v(0)} = -\frac{1}{RC}t$$

$$v(t) = v(0)e^{-t/\tau}, \tau = RC$$

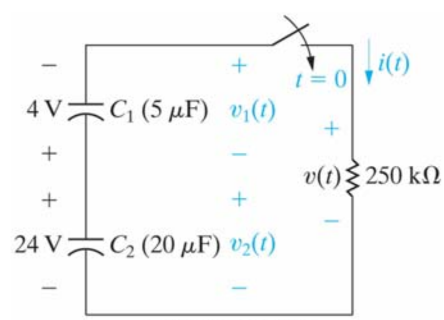



Why v+iR=0 not v-iR=0



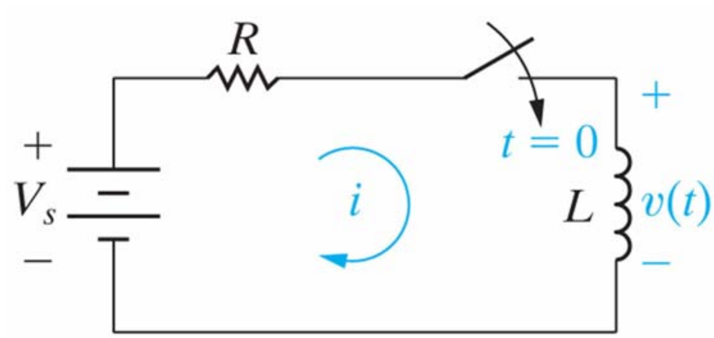
10 kΩ x
t = 0
y 32 kΩ
$i_o$
+
100 V
0.5 μF
+
$v_C$
−
+
$v_o$
240 kΩ
60 kΩ
−



−
+
4 V
$C_1$ (5 μF)
$v_1(t)$
t = 0
$i(t)$
+
−
$v(t)$
250 kΩ
+
+
24 V
$C_2$ (20 μF)
$v_2(t)$
−
−

## Step Response or RL Circuits






$$V_S = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{-Ri + V_S}{L} = \frac{-R}{L}\left(i - \frac{Vs}{R}\right)$$

$$di = \frac{-R}{L}\left(i - \frac{Vs}{R}\right)dt$$

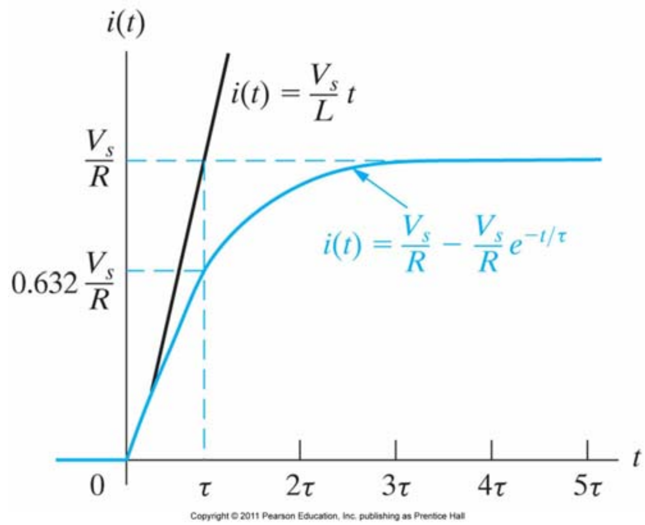
$$\frac{di}{i - (V_S / R)} = \frac{-R}{L}dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i - (V_S / R)} = \int_0^t \frac{-R}{L}dt$$

$$\ln\frac{i(t) - (V_S / R)}{I_0 - (V_S / R)} = \frac{-R}{L}t$$

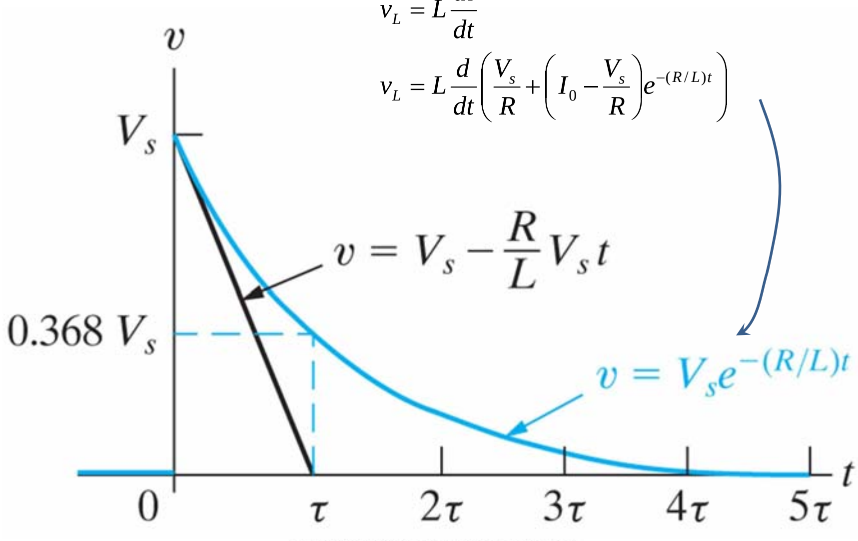
$$\frac{i(t) - (V_S / R)}{I_0 - (V_S / R)} = e^{-(R/L)t}$$

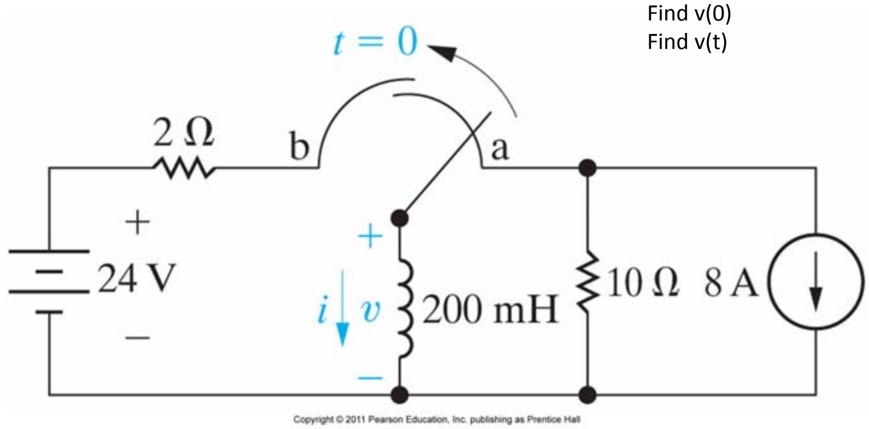
$$i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right)e^{-(R/L)t}$$




$$v_L = L\frac{di}{dt}$$

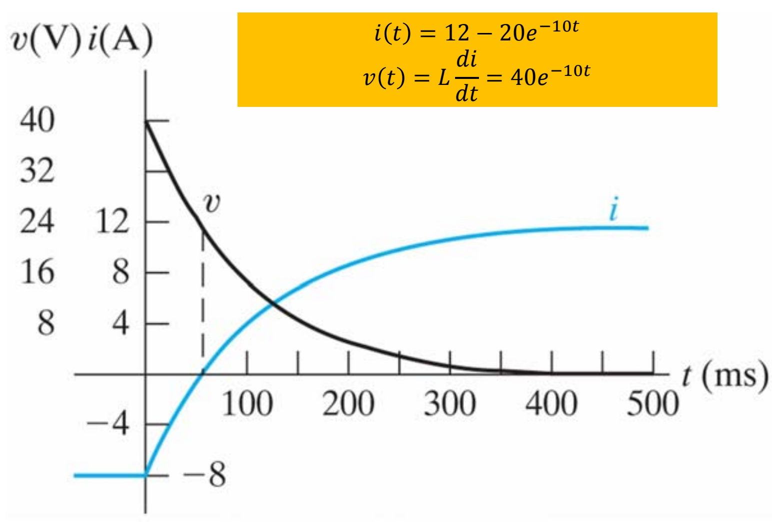
$$v_L = L\frac{d}{dt}\left(\frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-(R/L)t}\right)$$



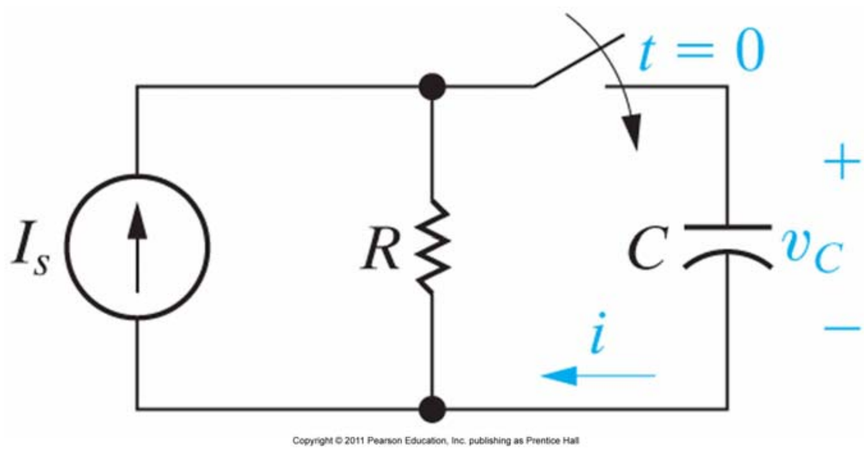



Find i(t)
Find v(0)
Find v(t)

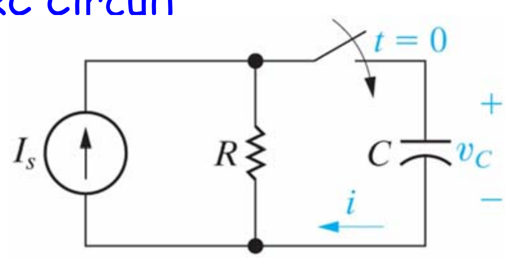
$$i(t) = 12 - 20e^{-10t}$$
$$v(t) = L\frac{di}{dt} = 40e^{-10t}$$



## Step Response of RC Circuits




## Step Response of RC Circuit




$$C\frac{dv}{dt}+\frac{v}{R}=I_s$$

$$\frac{dv}{dt}=\frac{-v/R+I_s}{C}=\frac{-v+I_sR}{RC}$$

$$\frac{dv}{v-I_sR}=\frac{-1}{RC}dt$$

$$\int_{V_0}^{v(t)}\frac{dv}{v-I_sR}=\int_0^t\frac{-1}{RC}dt$$

$$\ln\frac{v(t)-(I_sR)}{V_0-(I_sR)}=\frac{-1}{RC}t$$

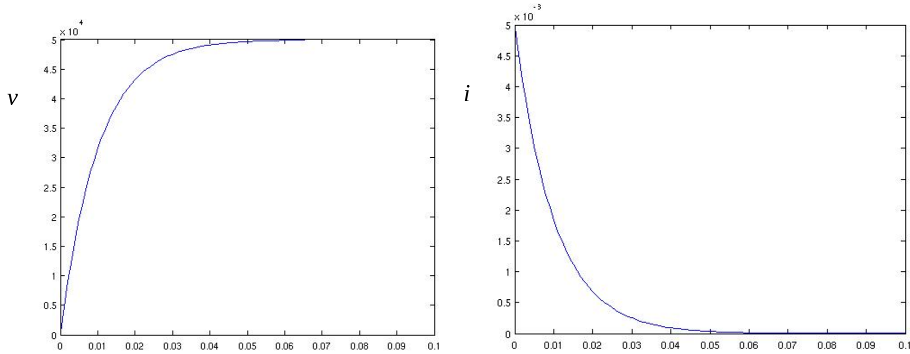
$$\frac{v(t)-(I_sR)}{V_0-(I_sR)}=e^{-t/RC}$$

$$v(t)=I_sR+(V_0-I_sR)e^{-t/RC}$$

$$i=C\frac{dv}{dt}$$

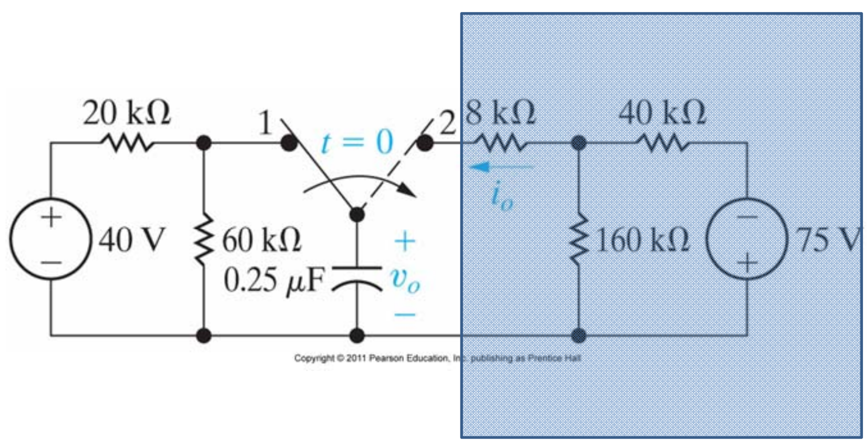
$$i=C(V_0-I_sR)\left(\frac{-1}{RC}\right)e^{-t/RC}$$

$$i=\left(I_s-\frac{V_0}{R}\right)e^{-t/RC}$$



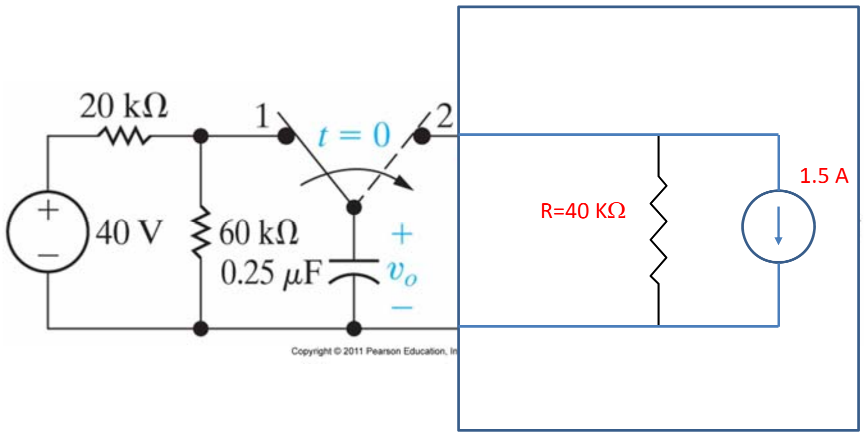

R=10 KΩ
C=1 μF
$I_s$=5A
$V_0$=0

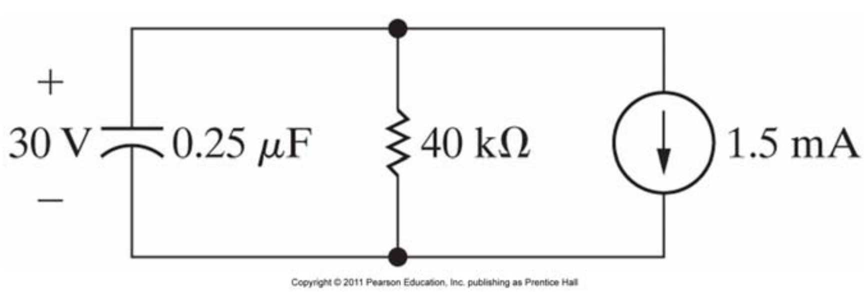
**Figure 7.22** The circuit for Example 7.6.

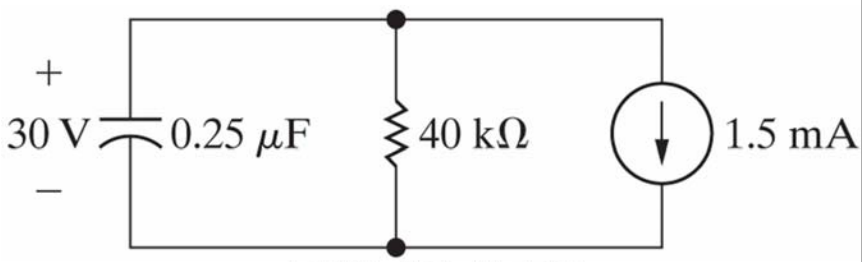



Get Norton's Equivalent

**Figure 7.22** The circuit for Example 7.6.








$$v(t) = I_s R + (V_0 - I_s R)e^{-t/RC}$$
$$v(t) = (-1.5) \times 40K + (30 - (-60))e^{-100t}$$
$$v(t) = -60 + 90e^{-100t}$$