ENG2200
Electric Circuits

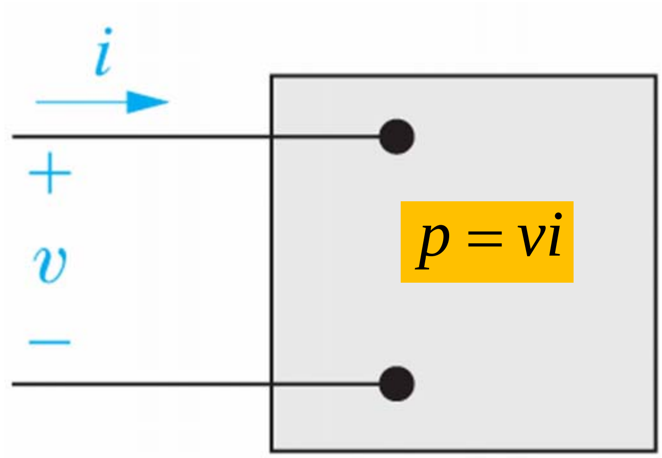
Chapter 10

Sinusoidal Steady Power Calculation

Objectives

- Understanding the difference between instantaneous power, average power reactive power, complex power and how to calculate them.
- Understanding power factor and how to calculate it.
- Understand the condition for a maximum real power delivered to the load.

Figure 10.1 The black box representation of a circuit used for calculating power.



Instantaneous Power

- $v = V_m \cos(\omega t + \theta_v) \quad i = I_m \cos(\omega t + \theta_i)$

$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$i = I_m \cos(\omega t)$$

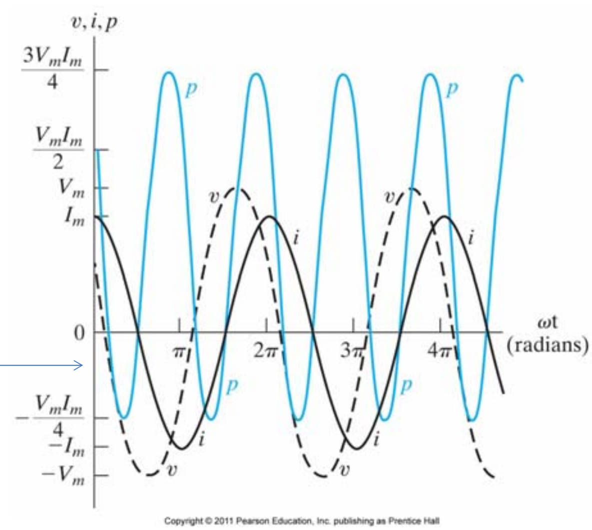
$$p = I_m V_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$$

$$p = \frac{1}{2} I_m V_m \{\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v - \theta_i)\}$$

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$



Negative Power?

Average and Reactive Power

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

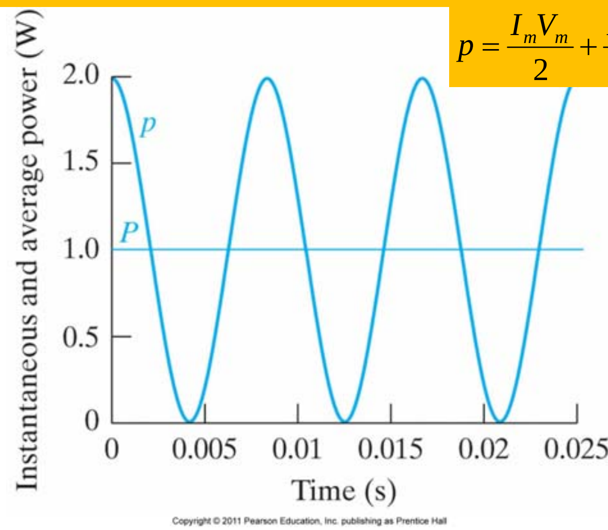
- P is the average power (real power) the power transferred from electric to non-electric (the consumer made use of it)
- Q is the reactive power

Purely Resistive Circuits $\theta_i=\theta_v$

$$p=\frac{I_m V_m}{2}\cos(\theta_v-\theta_i)+\frac{I_m V_m}{2}\cos(\theta_v-\theta_i)\cos 2\omega t-\frac{I_m V_m}{2}\sin(\theta_v-\theta_i)\sin 2\omega t$$

$$p=\frac{I_m V_m}{2}+\frac{I_m V_m}{2}\cos 2\omega t$$

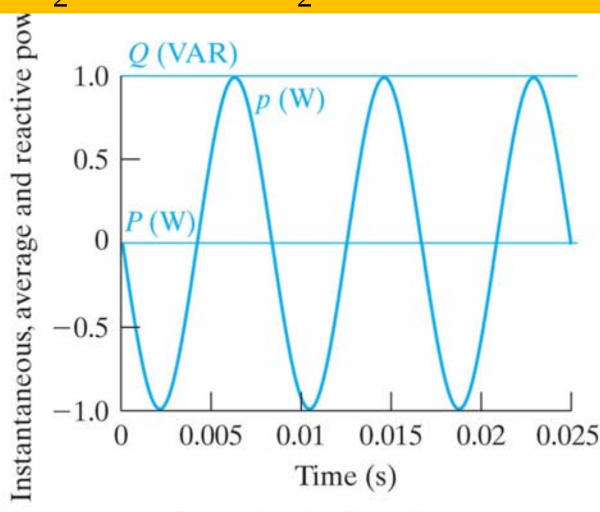
RMS?



Purely Inductive $\theta_i=\theta_v-90^{\circ}$

$$p=\frac{I_m V_m}{2}\cos(\theta_v-\theta_i)+\frac{I_m V_m}{2}\cos(\theta_v-\theta_i)\cos 2\omega t-\frac{I_m V_m}{2}\sin(\theta_v-\theta_i)\sin 2\omega t$$

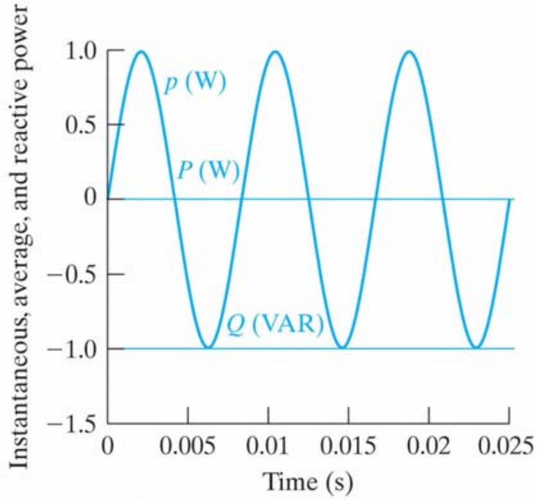
$$p=-Q\sin 2\omega t$$



Purely Capacitive Circuits $\theta_i=\theta_v+90^\circ$

$$p = \frac{I_m V_m}{2}\cos(\theta_v-\theta_i) + \frac{I_m V_m}{2}\cos(\theta_v-\theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v-\theta_i)\sin 2\omega t$$

$$p = Q\sin 2\omega t$$



Power Factor

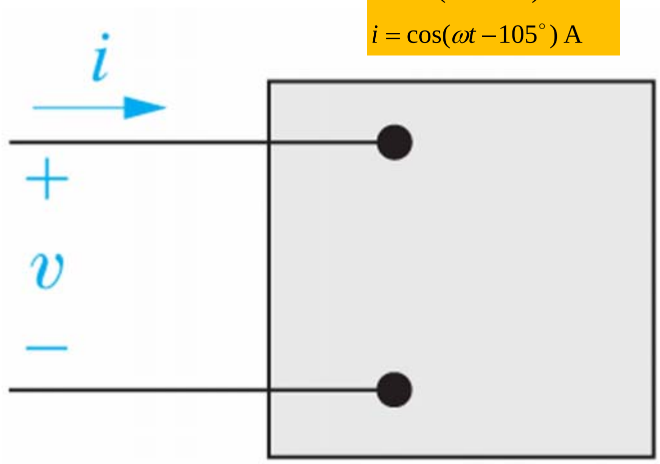
- The units for p is Watt (W)
- The units for Q is VAR (Volt Ampere Reactive)
- θ_i-θ_v power factor angle
- PF = $cos(\theta_i-\theta_v)$
- Note that $cos(\theta_i-\theta_v)=cos(\theta_v-\theta_i)$
- PF is defines as lagging (current lags voltage – inductive)or leading (currents leads voltage – capacitive)

Example

$v = 100\cos(\omega t + 15^\circ)$ V

$i = \sin(\omega t - 15^\circ)$ A

$i = \cos(\omega t - 105^\circ)$ A



Calculate the average power, reactive power

TABLE 10.1 Annual Energy Requirements of Electric Household Appliances

Appliance	Average Wattage	Est. kWh Consumed Annually[a]
Food preparation		
Coffeemaker	1,200	140
Dishwasher	1,201	165
Egg cooker	516	14
Frying pan	1,196	100
Mixer	127	2
Oven, microwave (only)	1,450	190
Range, with oven	12,200	596
Toaster	1,146	39
Laundry		
Clothes dryer	4,856	993
Washing machine, automatic	512	103
Water heater	2,475	4,219
Quick recovery type	4,474	4,811
Comfort conditioning		
Air conditioner (room)	860	860[b]
Dehumidifier	257	377
Fan (circulating)	88	43
Heater (portable)	1,322	176

Appliance	Average Wattage	Est. kWh Consumed Annually[a]
Health and beauty		
Hair dryer	600	25
Shaver	15	0.5
Sunlamp	279	16
Home entertainment		
Radio	71	86
Television, color, tube type	240	528
Solid-state type	145	320
Housewares		
Clock	2	17
Vacuum cleaner	630	46

a) Based on normal usage. When using these figures for projections, such factors as the size of the specific appliance, the geographical area of use, and individual usage should be taken into consideration. Note that the wattages are not additive, since all units are normally not in operation at the same time.

b) Based on 1000 hours of operation per year. This figure will vary widely depending on the area and the specific size of the unit. See EEI-Pub #76-2, "Air Conditioning Usage Study," for an estimate for your location.

Source: Edison Electric Institute.

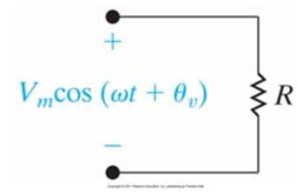
Figure 10.7 A sinusoidal voltage applied to the terminals of a resistor.

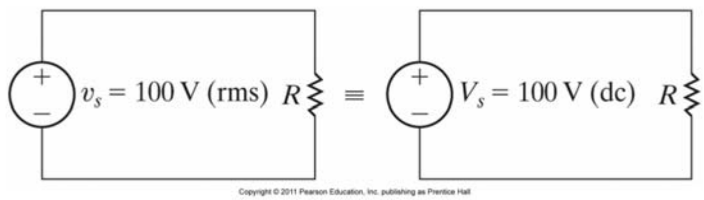
$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \theta_v)}{R} dt$$

$$P = \frac{1}{R}\left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \theta_v) dt\right]$$

$$P = \frac{V_{RMS}^2}{R}$$

$$P = I_{RMS}^2 R$$



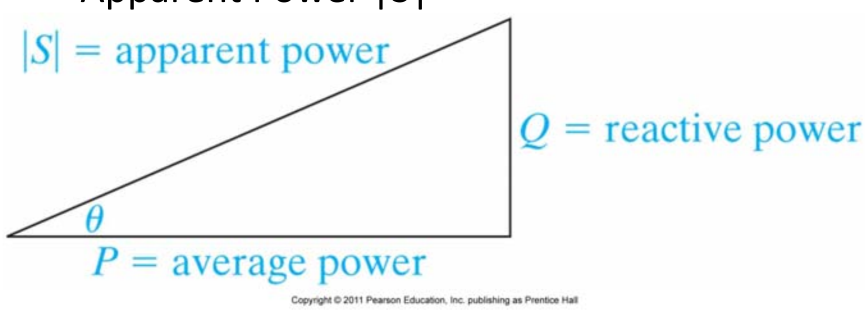


Complex Power

- Complex power $S = P + JQ$

$$\frac{Q}{P} = \frac{(V_m I_m / 2)\sin(\theta_v - \theta_i)}{(V_m I_m / 2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i)$$

- Apparent Power |S|



Example

- An electrical motor operates at 240 V rms. The average power is 8 kW at a lagging power factor of 0.8

Power calculation

$$S = (V_m I_m / 2)\cos(\theta_v - \theta_i) + j(V_m I_m / 2)\sin(\theta_v - \theta_i)$$

$$S = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$S = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{V_m I_m}{2} \angle(\theta_v - \theta_i)$$

$$S = V_{rms} \angle \theta_v \times I_{rme} \angle -\theta_i$$

$$S = V_{rms} I_{rms}^* = \frac{1}{2} V I^*$$

Power calculation

$$S = V_{rms} I^*_{rms}$$
$$V_{rms} = I_{rms} Z$$
$$S = I_{rms} I_{rms} Z$$
$$S = |I_{rms}|^2 Z$$
$$S = |I_{rms}|^2 (R + jX)$$
$$S = |I_{rms}|^2 R + j|I_{rms}|^2 X$$

Figure 10.11 The phasor voltage and current associated with a pair of terminals.

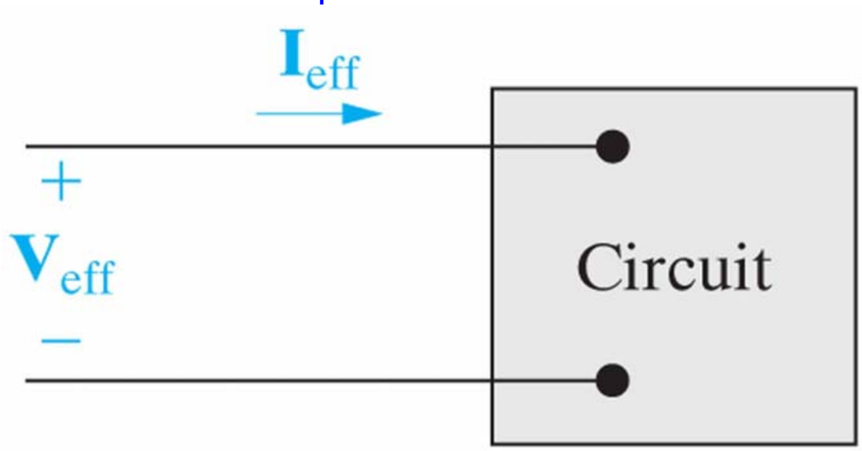
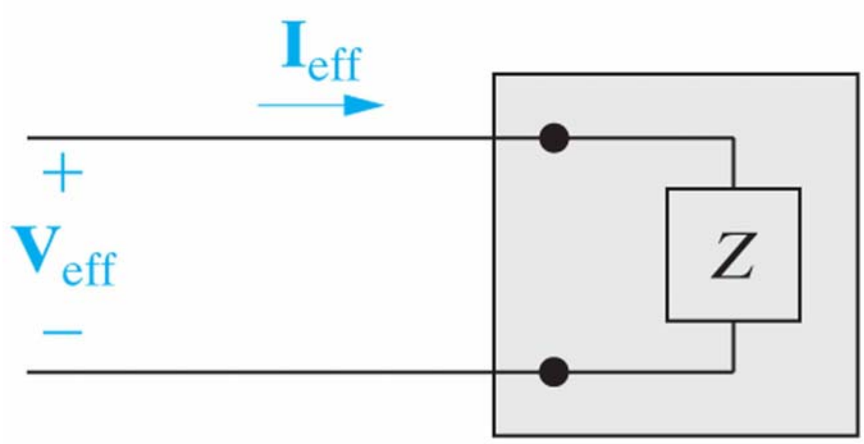
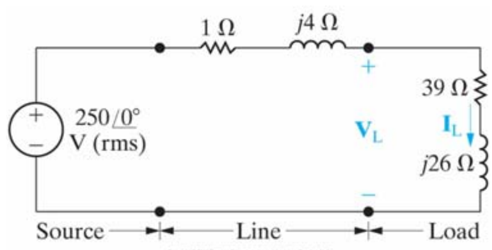


Figure 10.12 The general circuit of Fig. 10.11 replaced with an equivalent impedance.



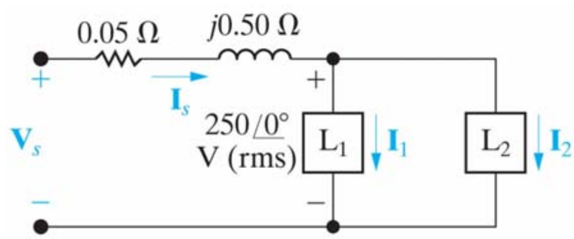
Example

- Find $\mathbf{I}_L$ and $\mathbf{V}_L$
- Calculate S supplied by the source
- Calculate S delivered to the load
- Calculate S delivered to the line

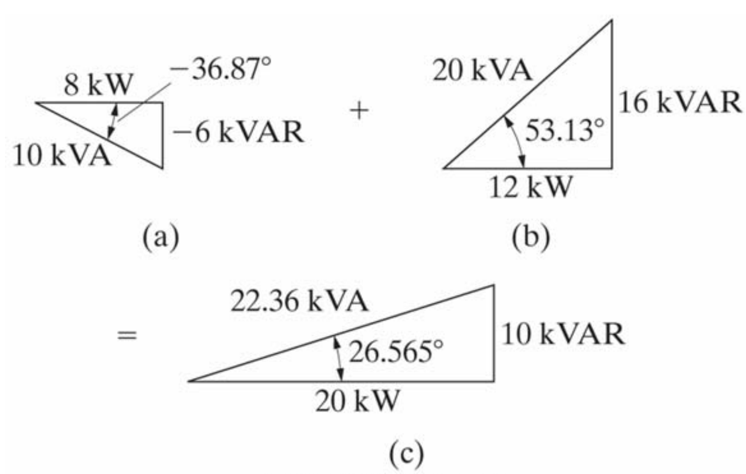


Example

Load 1
8 KW leading pf 0.8
Load 2
20 kVA at lagging pf 0.6



- Find the pf of the 2 loads in parallel
- Find I_s and the apparent power to supply the load
- Assuming 60 Hz, what is the capacitor required to correct the power factor



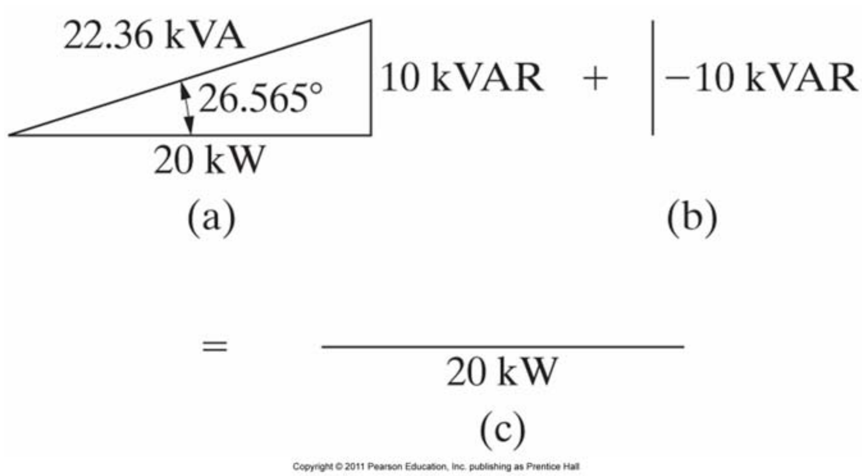
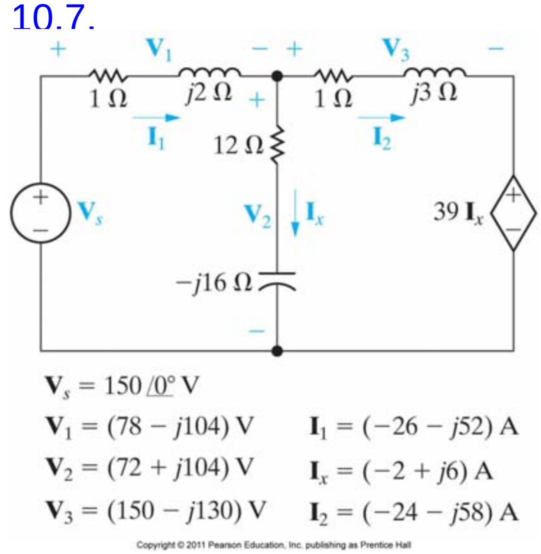
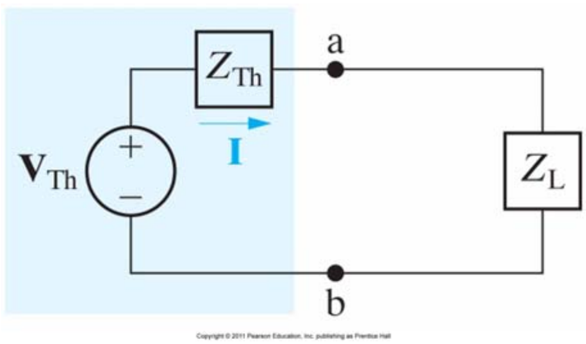


Figure 10.17 The circuit, with solution, for Example 10.7.



Maximum Power Transfer

- Assume the source is replaced by its Thevenin equivalent circuits.
- V_{TH}, Z_{TH} and a load or Z_L is connected



Maximum Power Transfer

$$I = \frac{V_{TH}}{(R_{TH} + R_L) + j(X_{TH} + X_L)}$$

$$P = |I|^2 R_L$$

$$P = \frac{|V_{TH}|^2 R_L}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{\partial P}{\partial X_L} = 0$$

$$X_L = X_{TH}, R_L = \sqrt{R_{TH}^2 + (X_L + X_{TH})^2}$$

$$Z_L = Z_{TH}^*$$

Restriction

- Sometimes, we have restrictions on the load impedance.
- First, set X_L as close as possible to $-X_{TH}$, then calculate R_L as close as possible to $\sqrt{{R_{TH}}^2 + (X_{TH} + X_L)^2}$
- If we can change the magnitude of the load impedance, but not the phase; set the magnitude of the load impedance to the magnitude of Thevenin impedance

Figure 10.18 A circuit describing maximum power transfer.

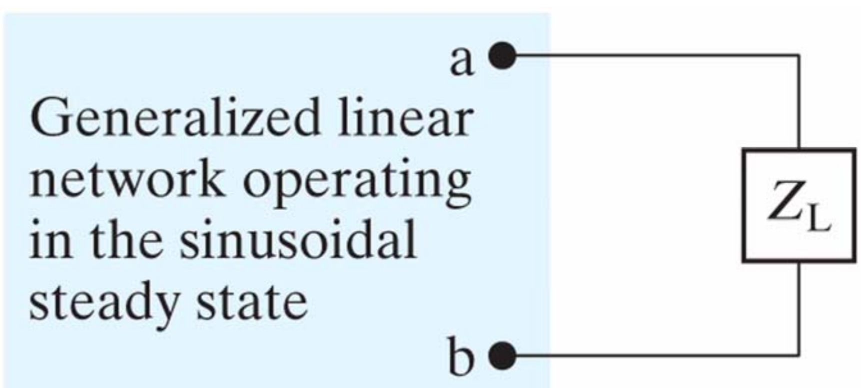


Figure 10.19 The circuit shown in Fig. 10.18, with the network replaced by its Thévenin

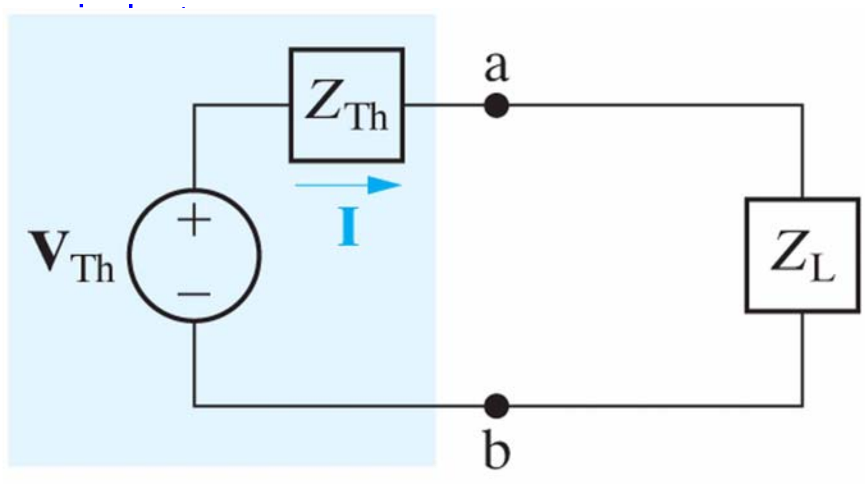


Figure 10.20 The circuit for Example 10.8.

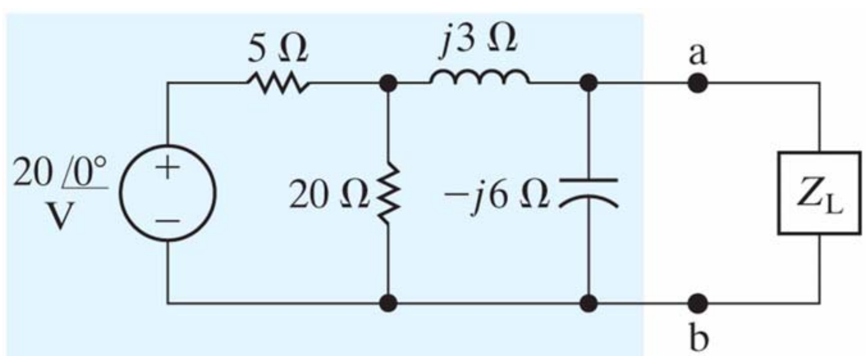


Figure 10.21 A simplification of Fig. 10.20 by source transformations.

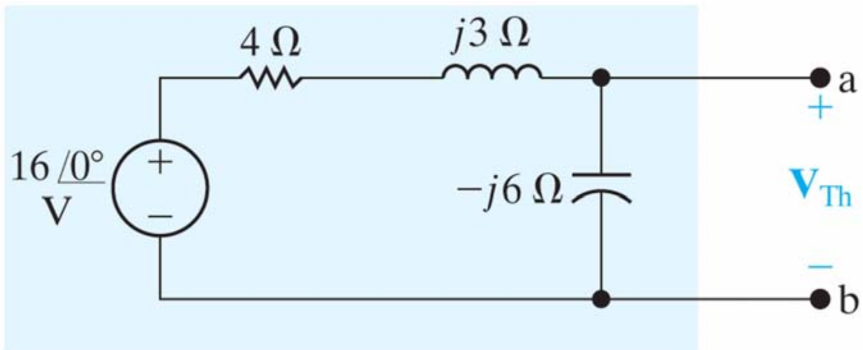


Figure 10.22 The circuit shown in Fig. 10.20, with the original network replaced by its Thévenin equivalent.

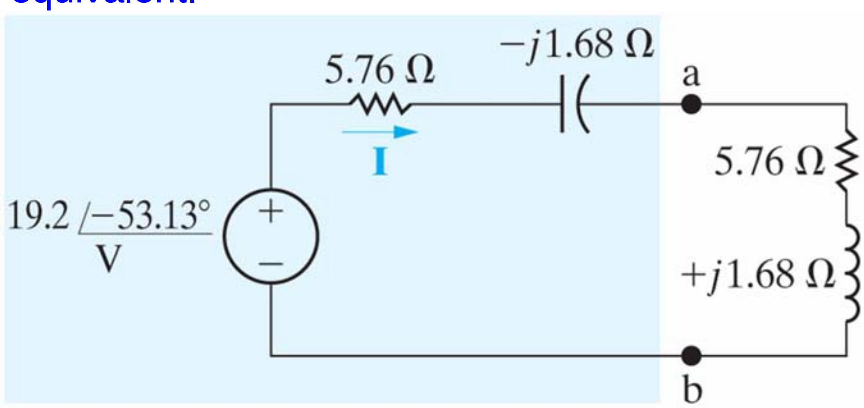


Figure 10.23 The circuit for Examples 10.9 and 10.10.