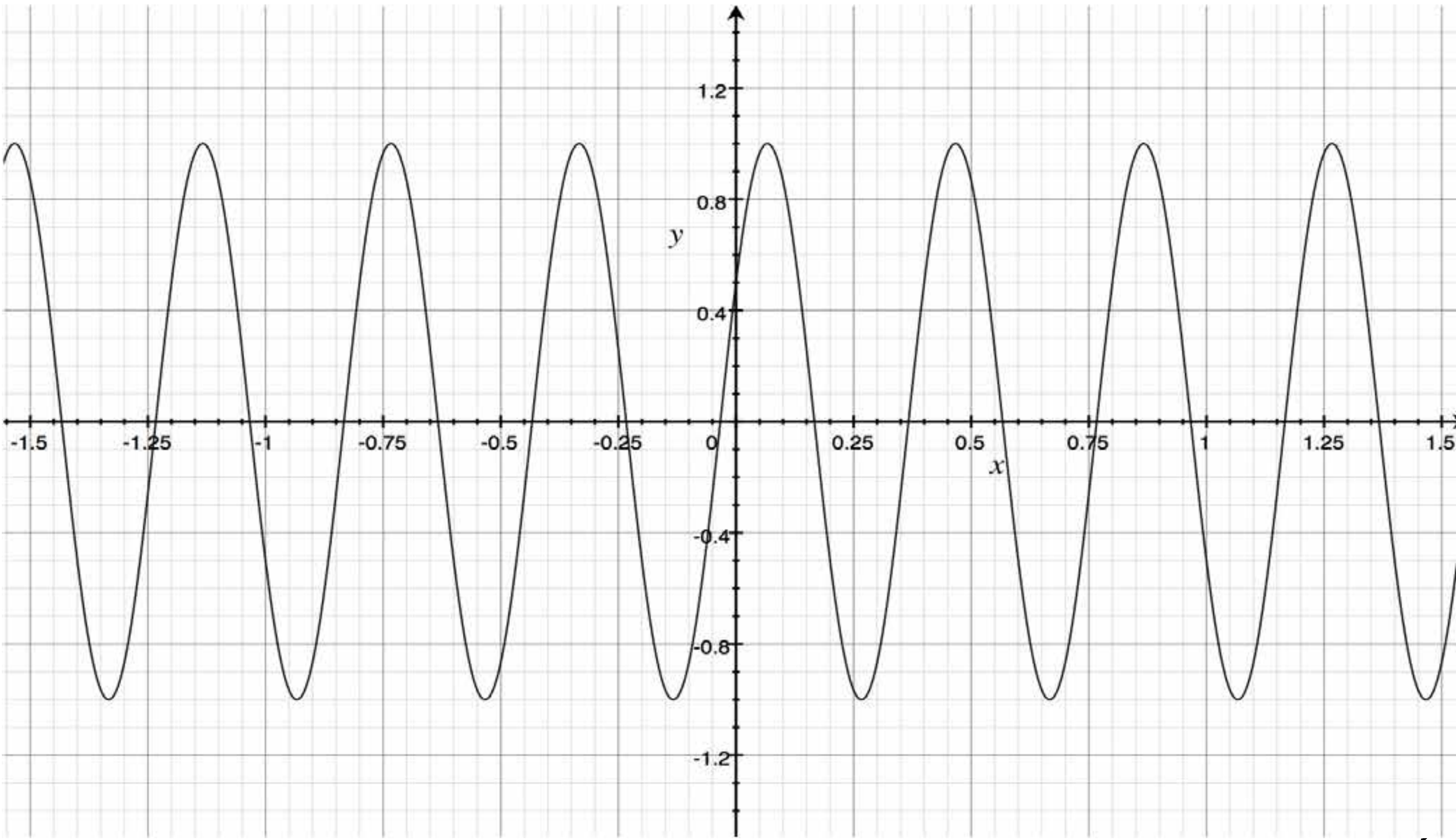
Signals and Spectra

Activity 1

Plot the CT signal $x(t) = \sin(5\pi t + 30^o)$. Discretize the CT signal with an uniform sampling period of $T_s = 0.25$s. Sketch the resulting waveforms.

Plot



Activity 2

For the following sinusoidal signals

(a) $x[k] = \sin(5\pi k)$

(b) $y[k] = \cos(k\pi/3)$

Determine the fundamental period K_0 of the DT signals.

Determine fundamental period

(a) $x[k] = \sin(5\pi k) = \sin(5\pi(k+K_0))$

$5\pi K_0 = 2\pi \rightarrow K_0 = 2/5$

(b) $y[k] = \cos(k\pi/3) = \cos((k+K_0)\pi/3)$

$\pi K_0/3 = 2\pi \rightarrow K_0 = 6$

Activity 3

For the signal

$Y=1-|x-1|$

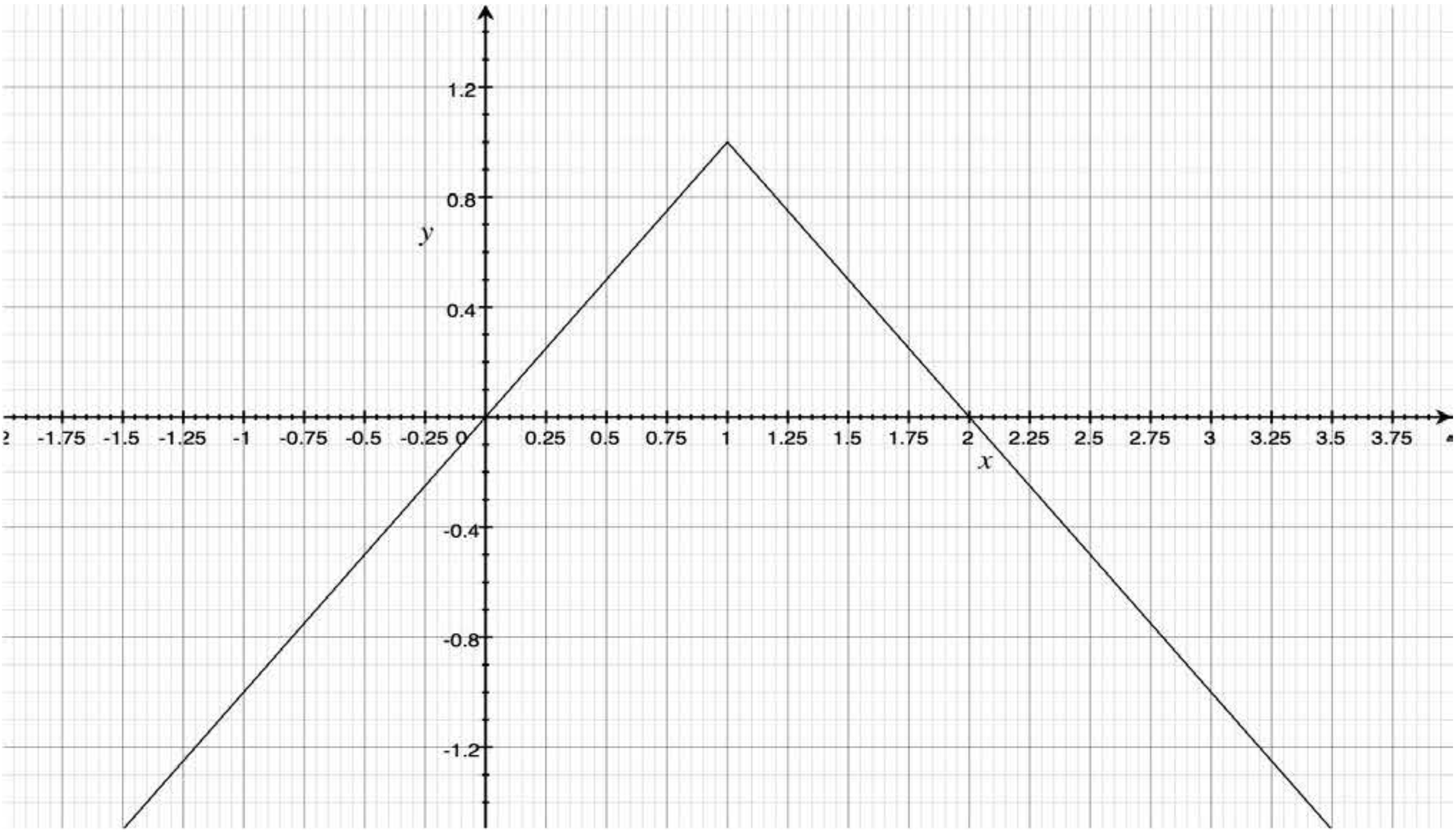
do the following:

(a) sketch the signal

(b) evaluate the odd part of the signal

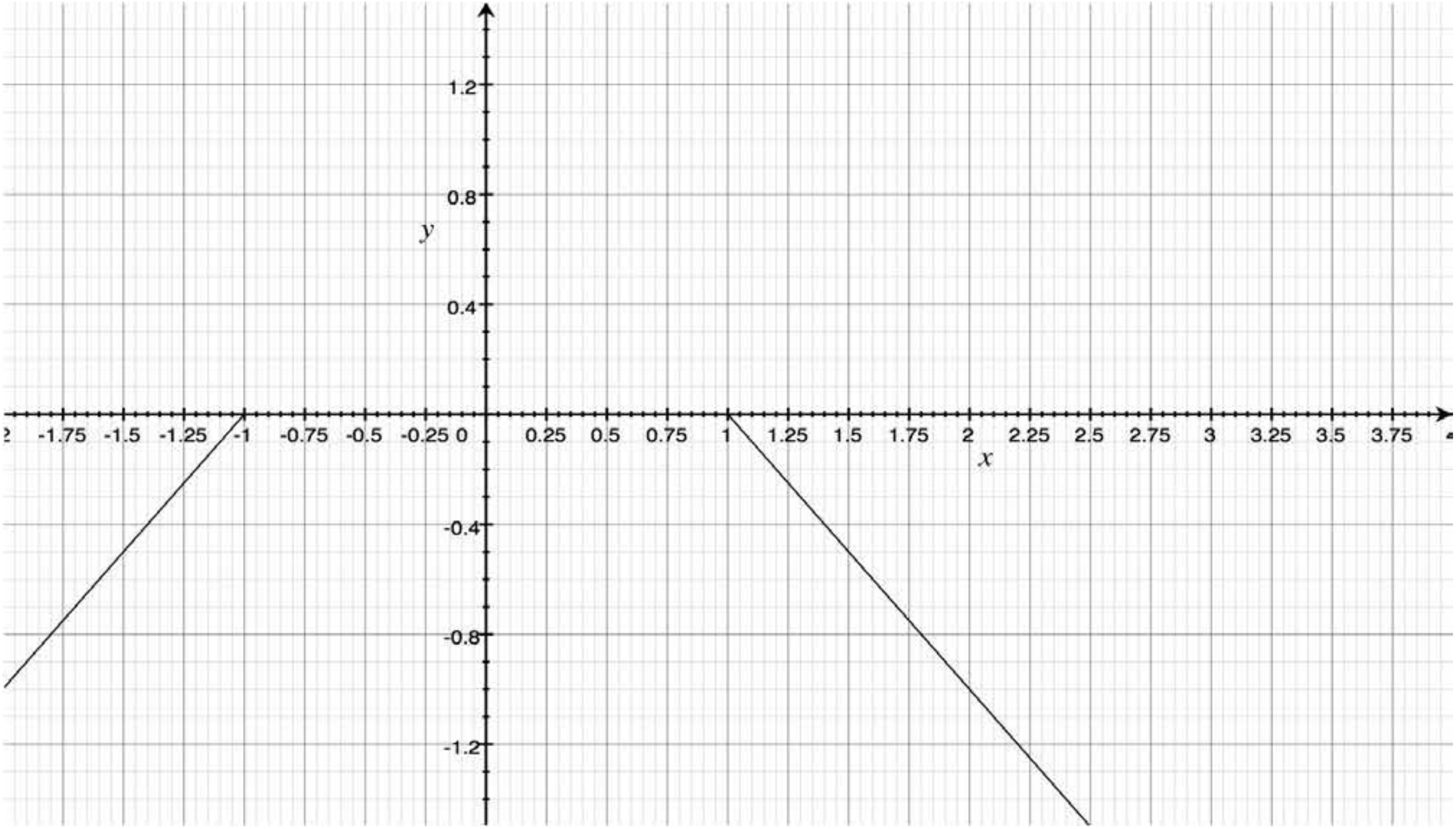
(c) evaluate the even part of the signal.

Plot of Y=1-|x-1|

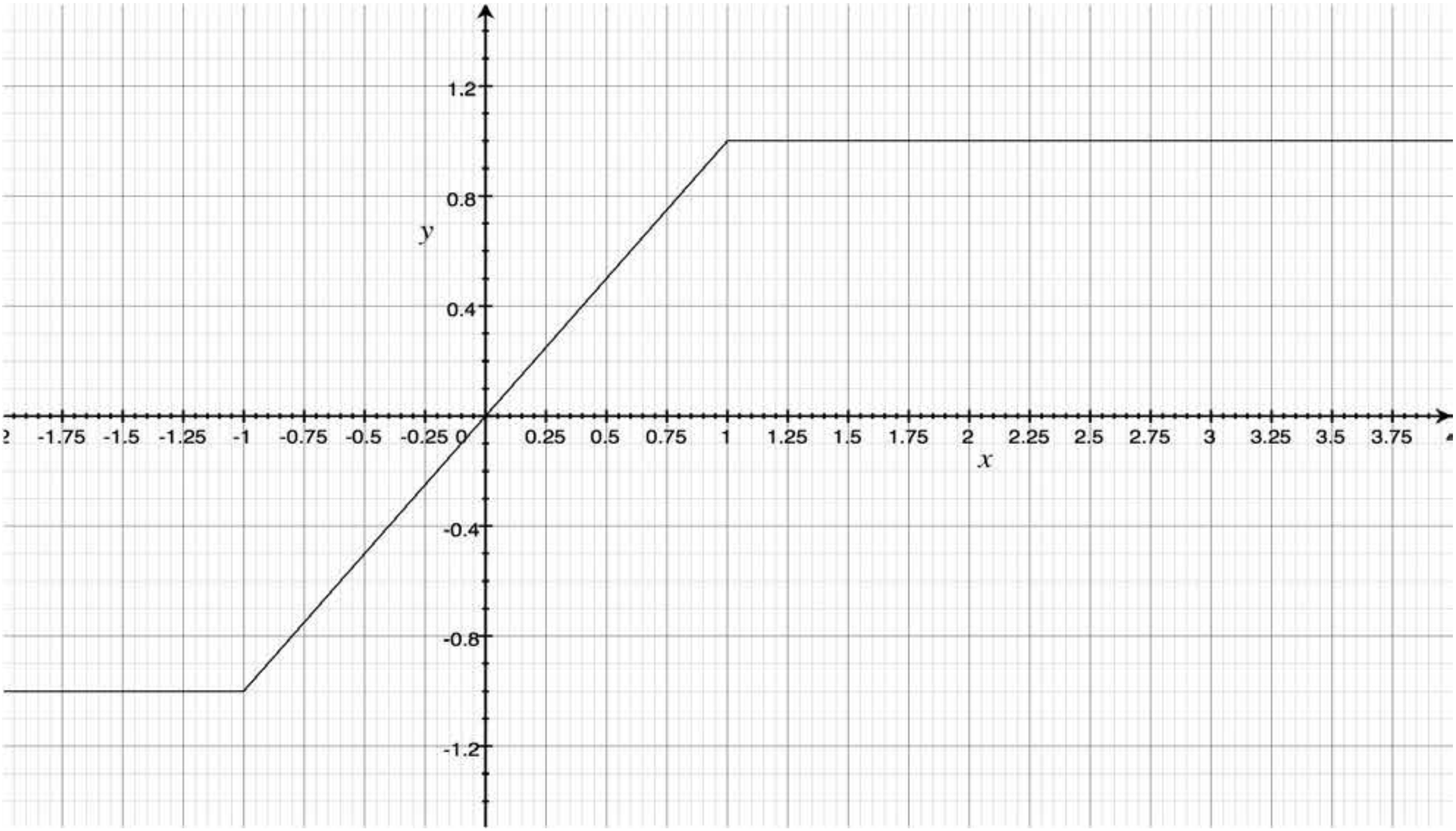


1.2
0.8
y
0.4
-1.75
-1.5
-1.25
-1
-0.75
-0.5
-0.25
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
3.25
3.5
3.75
x
-0.4
-0.8
-1.2

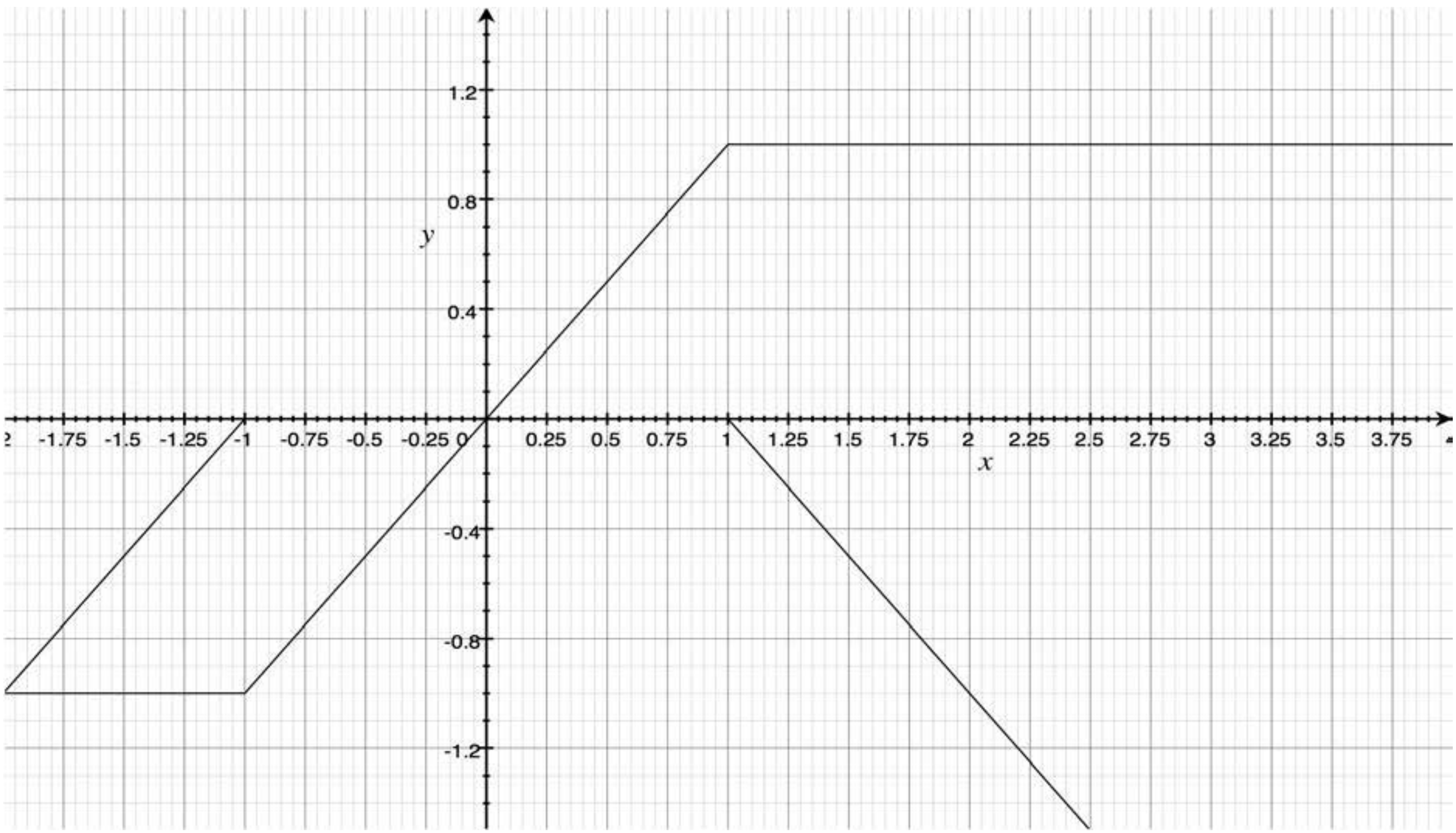
Even part


1.2
0.8
y
0.4
-1.75
-1.5
-1.25
-1
-0.75
-0.5
-0.25
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
3.25
3.5
3.75
x
-0.4
-0.8
-1.2

Odd part


1.2
0.8
y
0.4
-1.75 -1.5 -1.25 -1 -0.75 -0.5 -0.25 0 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75
x
-0.4
-0.8
-1.2

Both even and odd



Activity 4

- Solve the integral

$$\int_{-\infty}^{\infty} (t^3 + 5t^2 + 5t + 25)\delta(t + 5)dt = x(t_o)$$

=x(-5)

Random Signals

Activity 1

Consider a random process

$$X(t) = A\sin(2\pi f_0 t + \varphi)$$

where A and f_0 are constants, while Φ is a uniformly distributed RV over [0, 2π]. Calculate the mean and autocorrelation for the aforementioned process.

Activity 1

Given *Φ* is a uniformly distributed RV over [0, 2π], we have:

$$p_\phi(\varphi) = \begin{cases} \dfrac{1}{2\pi} & [0, 2\pi] \\ 0 & else \end{cases}$$

$$m_x = E\left[X(t)\right] = E\left[A\sin(2\pi f_o t + \varphi)\right] = AE\left[\sin(2\pi f_o t + \varphi)\right]$$

$$= A\int_0^{2\pi} \sin(2\pi f_o t + \varphi) p_\phi(\varphi) d\varphi = \frac{A}{2\pi}\int_0^{2\pi} \left[\sin(2\pi f_o t + \varphi)\right] d\varphi = 0$$

$$R_{XX}\left(t_1, t_2\right) = E\left[X(t_1)X(t_2)\right]$$

$$= E\left[A^2 \sin\left(2\pi f_0 t_1 + \varphi\right)\sin\left(2\pi f_0 t_2 + \varphi\right)\right]$$

$$= \frac{A^2}{2} E\left[\cos 2\pi f_0 (t_2 - t_1)\right] - \frac{A^2}{2} E\left[\cos 2\pi f_0 (t_2 + t_1 + 2\varphi)\right]$$

$$= \frac{A^2}{2} \cos 2\pi f_0 (t_2 - t_1)$$

Activity 2

Suppose we form a random process Y(t) by modulating a carrier with another random process X(t), i.e.

$$Y(t) = X(t) A \sin(2\pi f_0 t + \varphi)$$

Where *Φ* is a uniformly distributed RV over [0, 2π] and independent of X(t). Under what condition is Y(t) WSS?

Activity 2

For a WSS process, m_Y=constant, $R_{YY}(t_1, t_2)=R_{YY}(t_1-t_2)$. Let's find m_x and $R_{YY}(t_1, t_2)$

$$m_Y = E\left[X(t)\sin(2\pi f_o t+\varphi)\right] = E\left[X(t)\right]E\left[\sin(2\pi f_o t+\varphi)\right] = 0$$

$$R_{YY}\left(t, t+\tau\right) = E\left[X(t)X(t+\tau)\sin\left(2\pi f_0 t+\varphi\right)\sin\left(2\pi f_0(t+\tau)+\varphi\right)\right]$$

$$= E\left[X(t)X(t+\tau)\right]E\left[\cos(2\pi f_0\tau)+\cos(2\pi f_0(2t+\tau))+2\varphi\right]$$

$$= \frac{1}{2}R_{XX}\left(t, t+\tau\right)\cos 2\pi f_0(t_2-t_1)+\frac{1}{2}R_{XX}\left(t, t+\tau\right)E\left[\cos 2\pi f_0(t_2+t_1+2\varphi)\right]$$

$$= \frac{1}{2}R_{XX}\left(t, t+\tau\right)\cos 2\pi f_0(t_2-t_1)$$

It is clear m_Y=constant, $R_{YY}(t_1, t_2)=R_{YY}(t_1-t_2)$ only if X(t) is a WSS process.