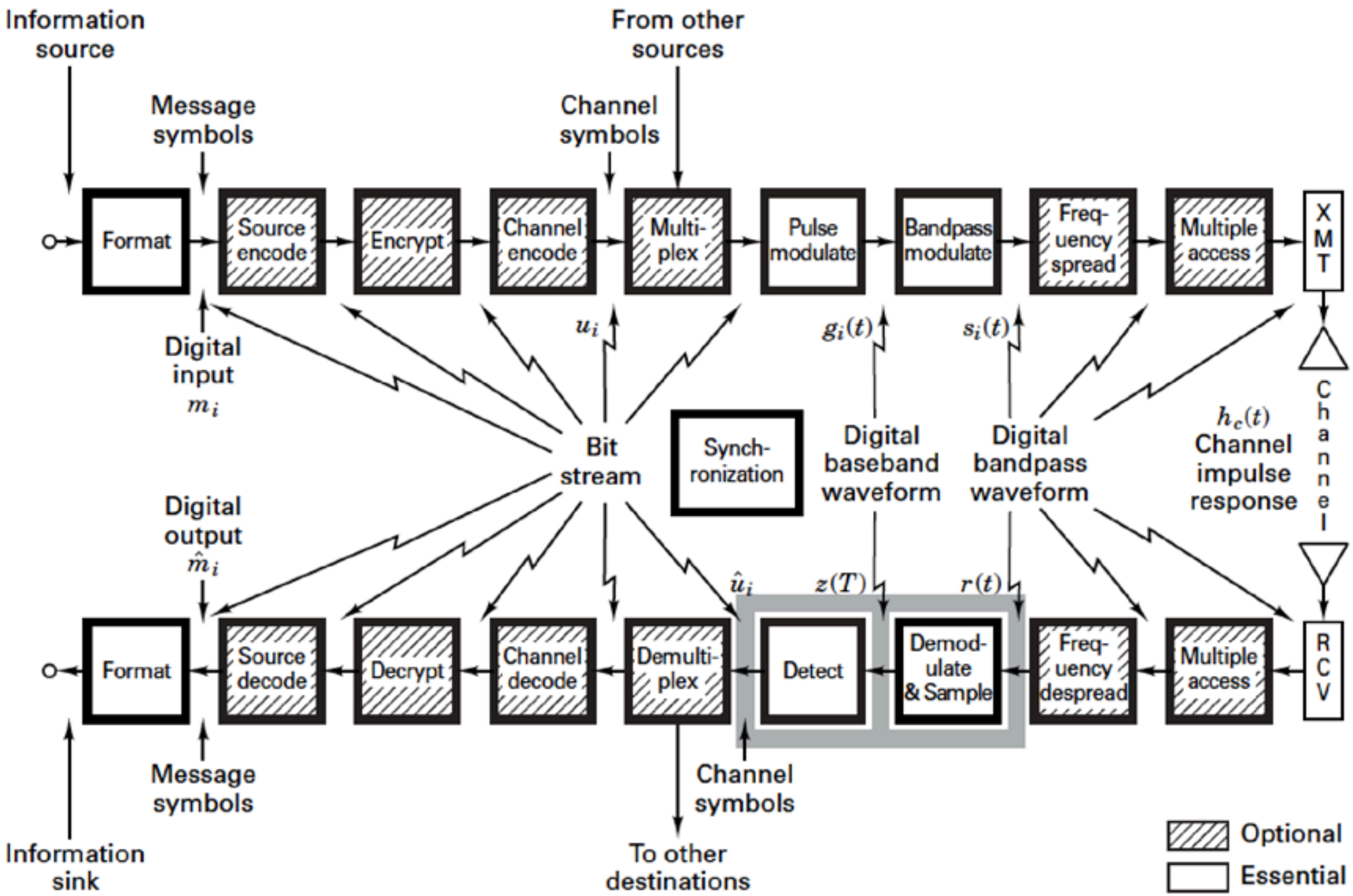
Chapter 3 Part 2

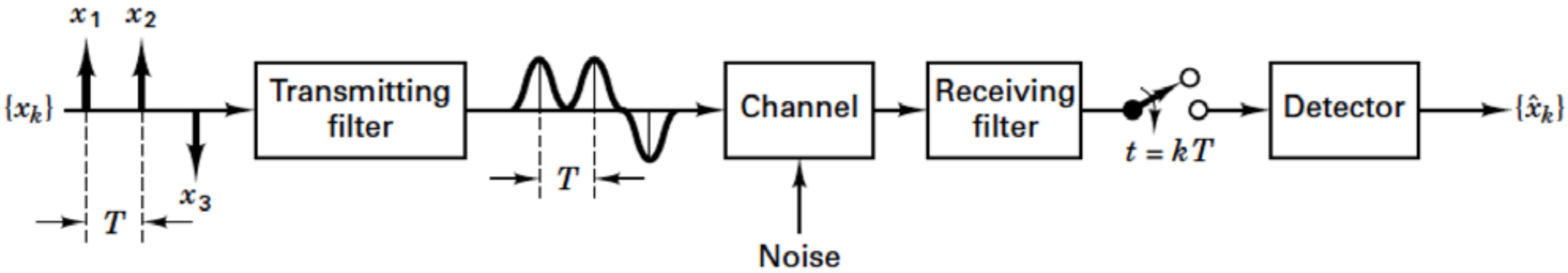
Baseband Demodulation/ Detection

Baseband Demodulation/Detection



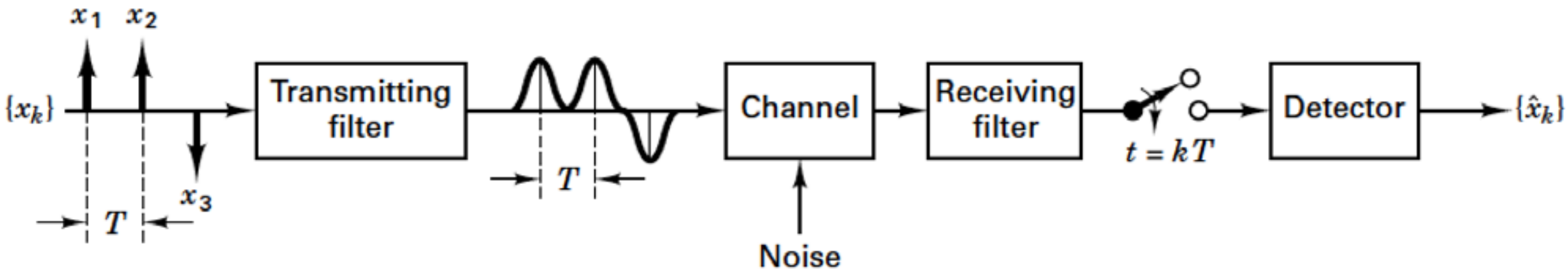
Intersymbol Interference

Filtering Aspect of a Digital Communication System



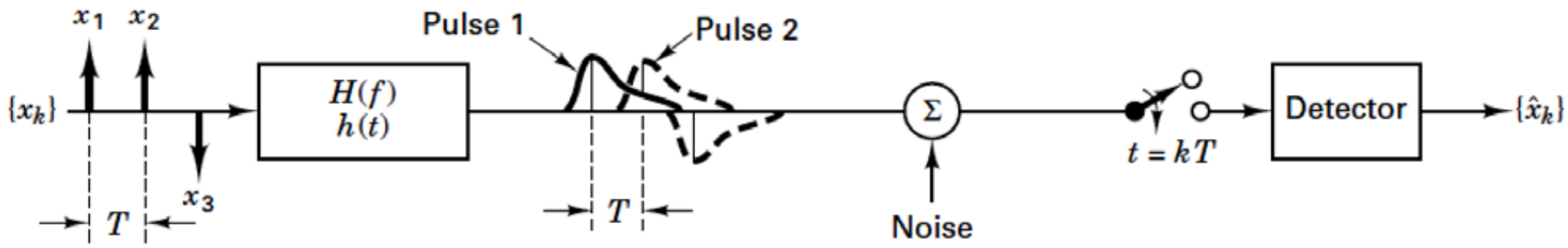
- There are various filters throughout the system: at transmitter, at receiver, and at channel.
  - At the transmitter: transmitting filter $H_t(f)$
  - At the channel, distributed reactances for cable or fading channel for wireless. We use $H_c(f)$ to represent the filtering effect.
  - At the receiver: receiving filter $H_r(f)$

Functions of Filters



1. Transmitting filter $H_t(f)$: The input to the transmitting filter are impulses denoting the information symbols $\{x_k\}$. Bit 1 is represented by a positive impulse and bit 0 by a negative impulse. These impulses modulate rectangular pulses such that bit 1 is now represented by a positive rectangular pulse and bit 0 by a negative rectangular pulse. Since the bandwidth of a rectangular pulse is infinite, the rectangular pulses are band limited by the transmitting filter.
2. Channel $H_c(f)$: The channel is modeled by a LTI system with a transfer function $H_c(f)$. It accounts for distortion produced, for example, by fading or by bandwidth constraints.
3. Receiving filter $H_r(f)$: The receiving filter compensates for distortion resulting from the transmitter and the channel. It models both the receiving and equalizing filters.

An Equivalent Model

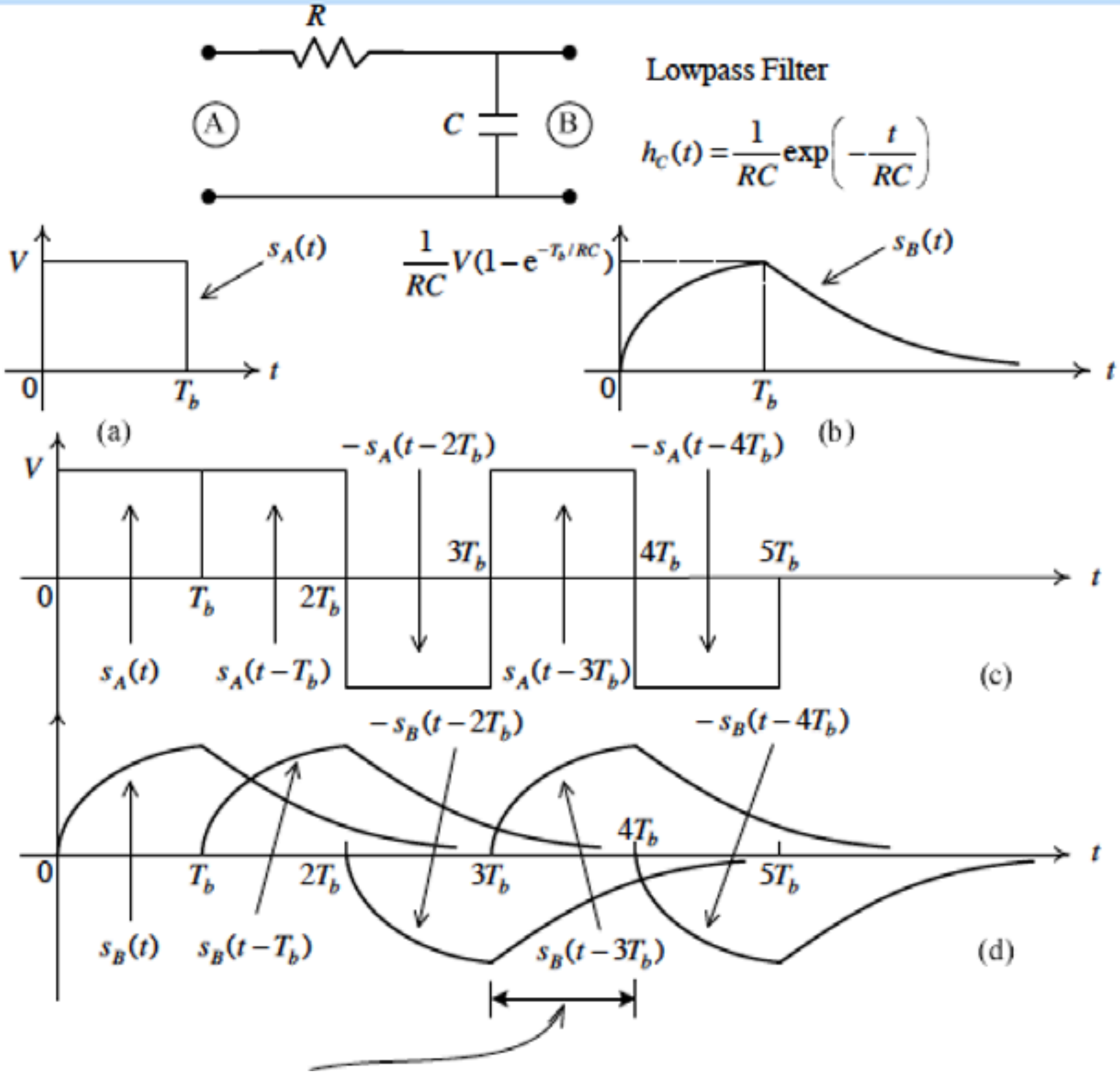


- Lumping all the filtering effects into one overall equivalent system transfer function: $H(f) = H_t(f)\, H_c(f)\, H_r(f)$.
- The pulses at the output of the equivalent filter overlap each other. This effect is called **intersymbol interference** (ISI).
- ISI restricts the transmission rate of a communications system. Clearly if the transmission rate R_s is increased (T is reduced), then the ISI will also increase.

Intersymbol Interference (ISI)

- A form of distortion of a signal in which one symbol interferes with subsequent symbols.
- Mainly caused by multipath propagation (in wireless) or the inherent non-linear response of a channel.
- ISI causes successive symbols to “blur” together.
- ISI occurs even in the absence of noise (due to the effects of filtering and channel-induced distortions)

ISI Example



In this interval: $y(t) = b_0 s_B(t) + b_1 s_B(t-T) + b_2 s_B(t-2T) + b_3 s_B(t-3T) + n_0(t)$

Nyquist Criterion for Zero ISI

$$y(t) = \sum_{k=-\infty}^{\infty} b_k s_R(t - kT) + n_0(t)$$

where $s_R(t) = h_T(t) * h_C(t) * h_R(t)$ is the overall response of the system due to a unit impulse at the input

$$b_k = \begin{cases} V & \text{if the } k^{th} \text{ bit is 1} \\ -V & \text{if the } k^{th} \text{ bit is 0} \end{cases}$$

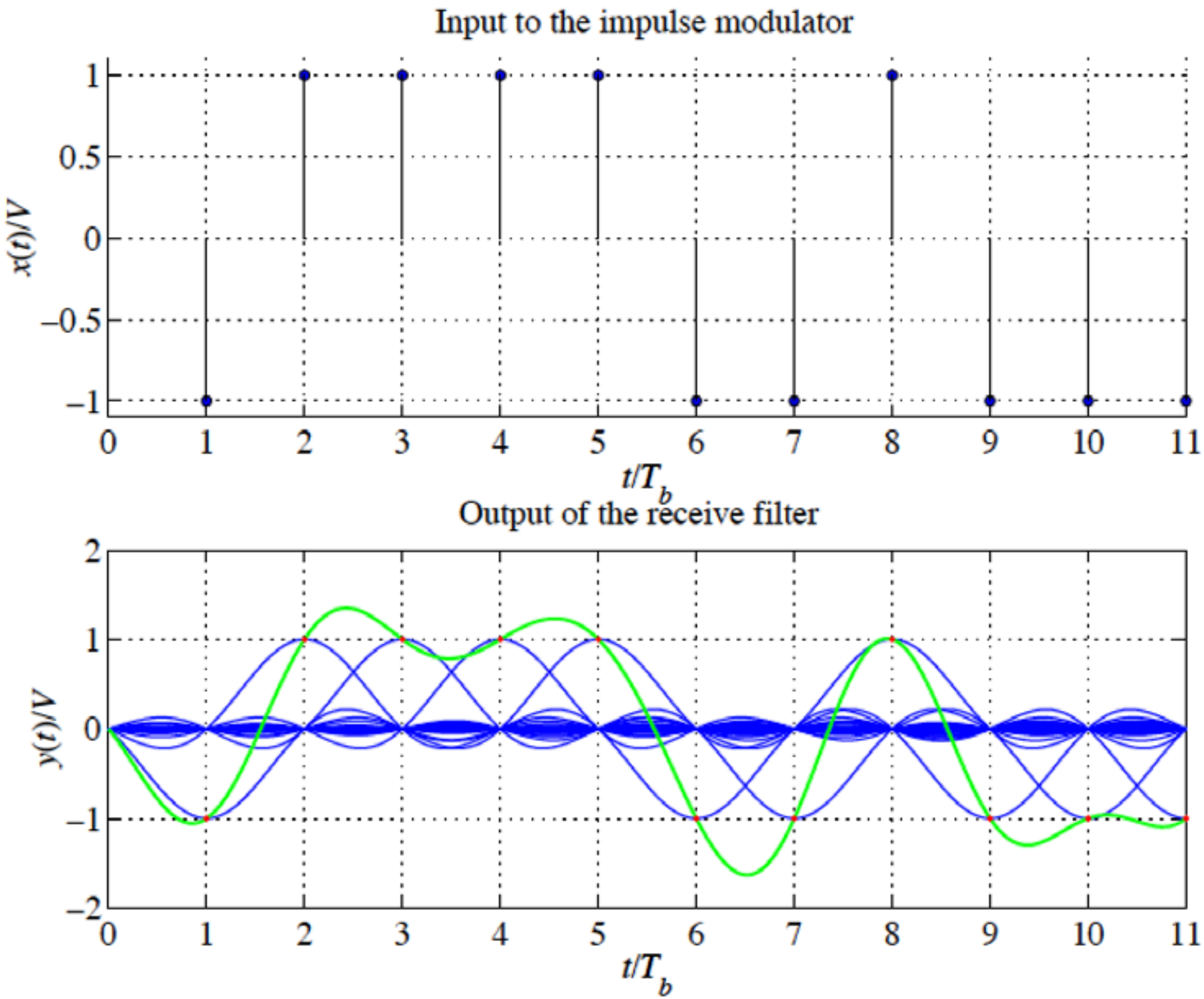
Normalize $s_R(0)=1$, and take at sampling time $t=mT$

$$y(mT) = b_m + \underbrace{\sum_{\substack{k=-\infty \\ k\neq m}}^{\infty} b_k s_R(mT - kT)}_{\text{ISI term}} + n_0(mT)$$

Activity 1

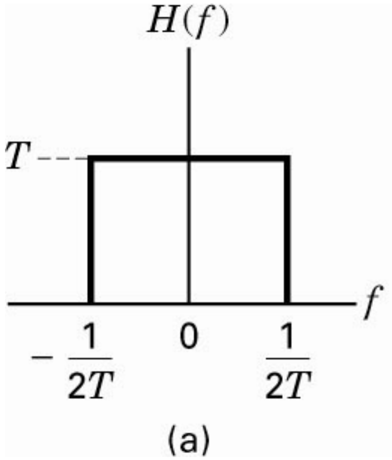
Under what conditions the Nyquist Criterion for zero ISI holds?

Zero ISI Example

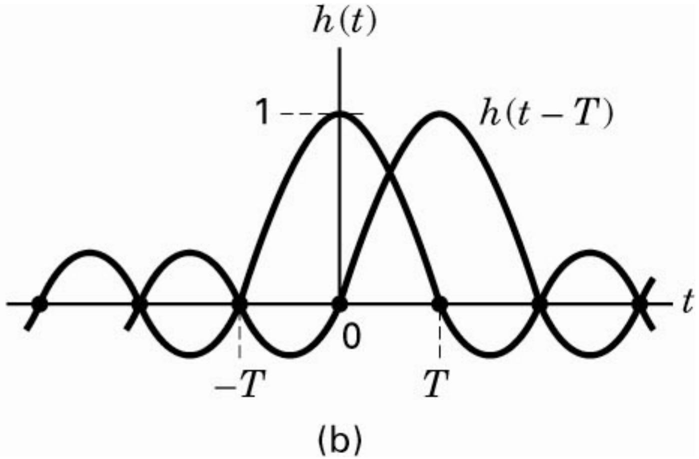


Nyquist Theoretical Minimum System Bandwidth

- The theoretical minimum bandwidth needed in order to detect R_s symbols/s, without ISI, is $R_s/2$ Hz.
- This occurs when the system transfer function is made rectangular.
- For baseband systems, the rectangular bandwidth is called ideal Nyquist filter.
- The sinc(t/T)-shaped pulse is called ideal Nyquist pulse.



Ideal Nyquist filter



Ideal Nyquist pulse

Raised Cosine Filter (1)

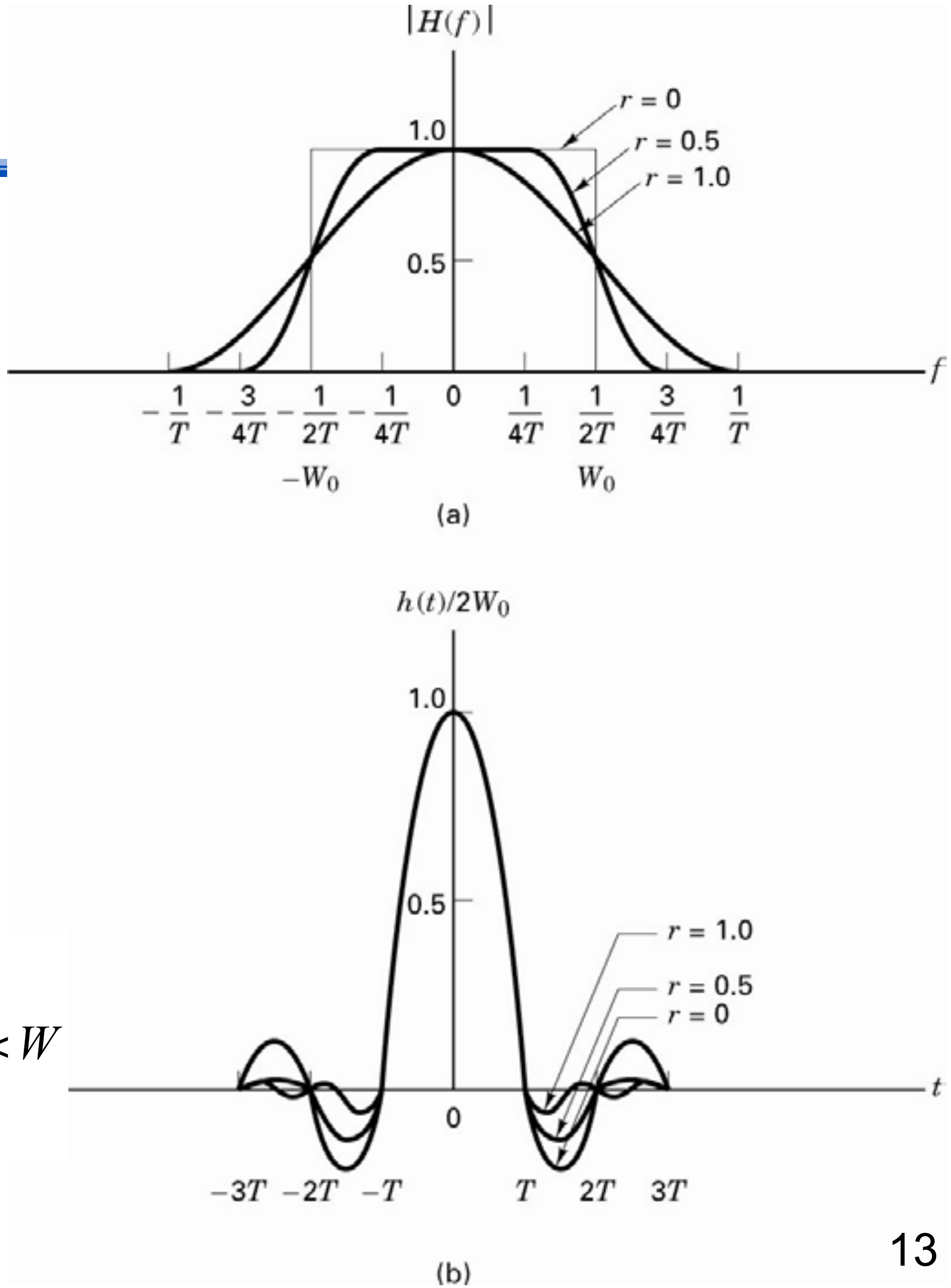
- A system having the overall transfer function *H*(*f*) as a rectangular pulse is difficult to be implemented:

1. The overall amplitude transfer function *H*(*f*) has to be flat over the range $-1/2T < f < 1/2T$ and zero outside the range. This is physically unrealizable because the impulse response is infinitely long and non-causal.
2. The synchronization of the clock in the detector has to be perfect at instants $t = kT$.

- An alternative transfer function is the raised-cosine transfer function

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left[\frac{\pi}{4}\frac{|f|+W-2W_0}{W-W_0}\right] & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases}$$

where roll-off factor $r = (W - W_0)/W_0$



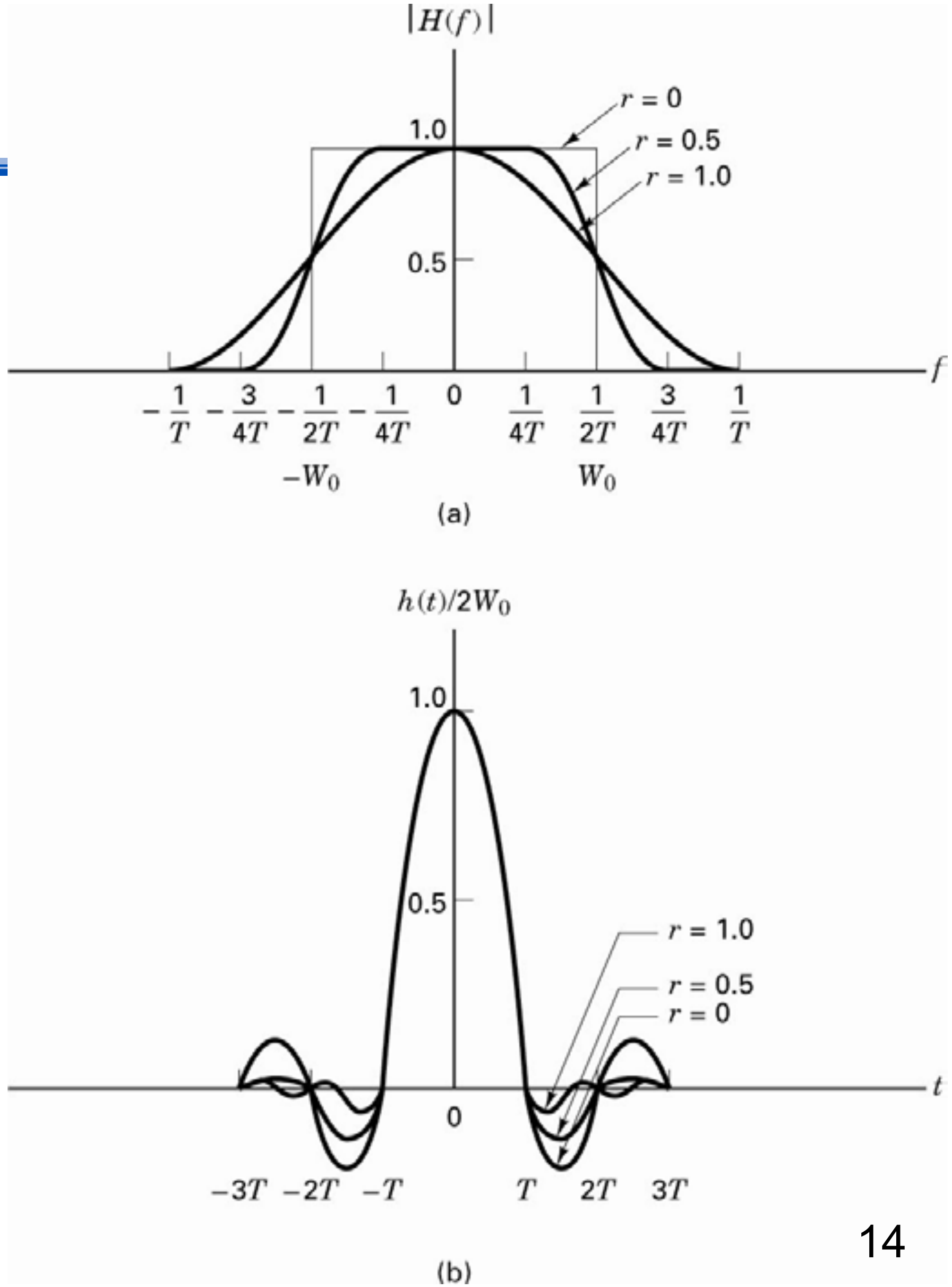
Raised Cosine Filter (2)

- The impulse response of the raised cosine filter is given by

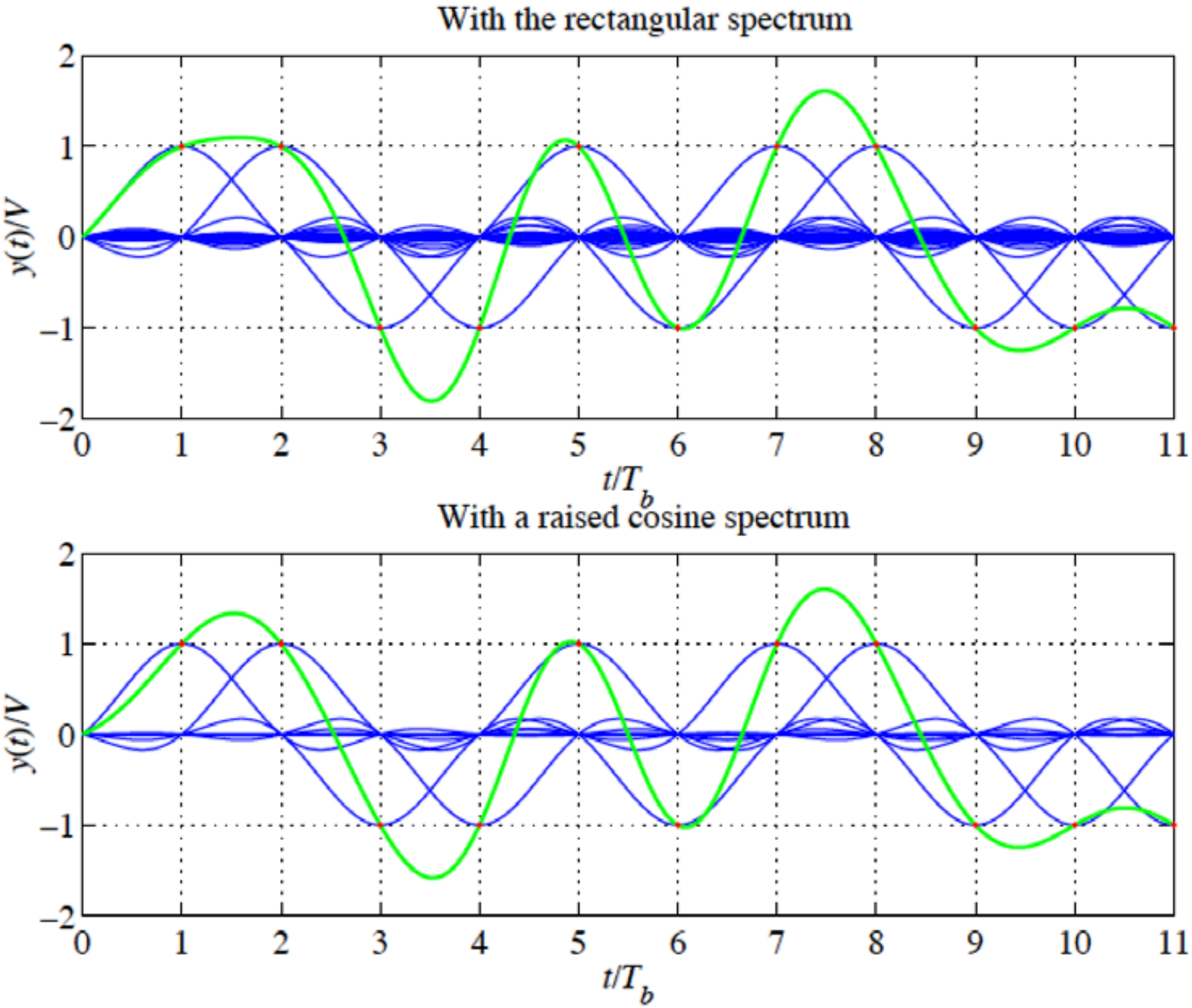
$$h(t) = 2W\left(\text{sinc}(2W_0 t)\right)\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

- The minimum system bandwidth required for a symbol rate of R_s with the raised cosine filter is given by

$$W = \tfrac{1}{2}(1 + r)R_s$$



Zero ISI - Raised Cosine Example

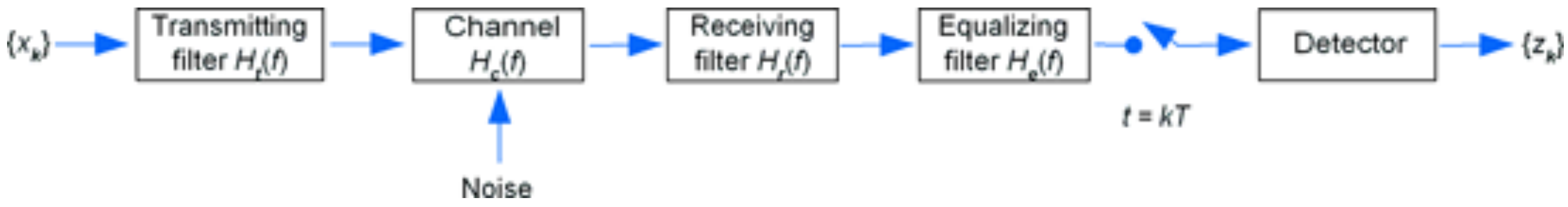


Activity 2

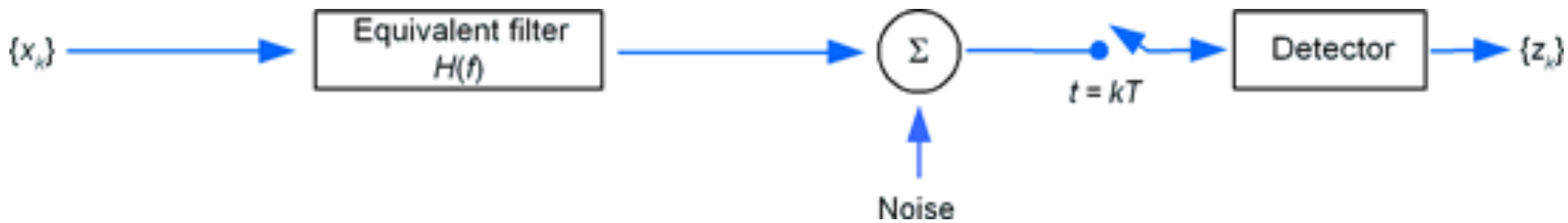
Find the minimum required bandwidth for the baseband transmission of a 4-level PAM pulse sequence having a date rate of R = 2400 bit/s if the system transmission characteristic consist of a raised cosine spectrum with 100% excess bandwidth ($r = 1$).

Equalization

Channel Characterization



- The baseband digital communication system is modified such that the equalizing and receiving filters are considered as separate blocks.



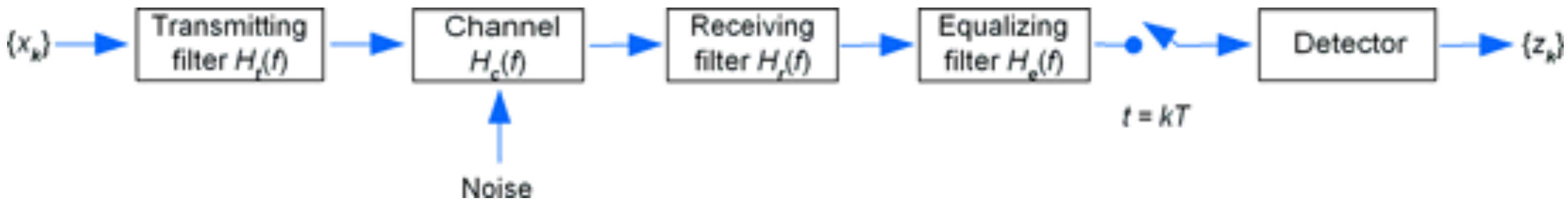
- The equivalent transfer function is obtained by lumping all of the transfer functions in one system as

$$H(f) = H_t(f)H_c(f)H_r(f)H_e(f).$$

- In practice, the transmitting and receiving filters are chosen so that the product is equal to the Nyquist raised cosine transfer function. For example,

$$H_t(f)H_r(f) = H_{RC}(f).$$

Equalization



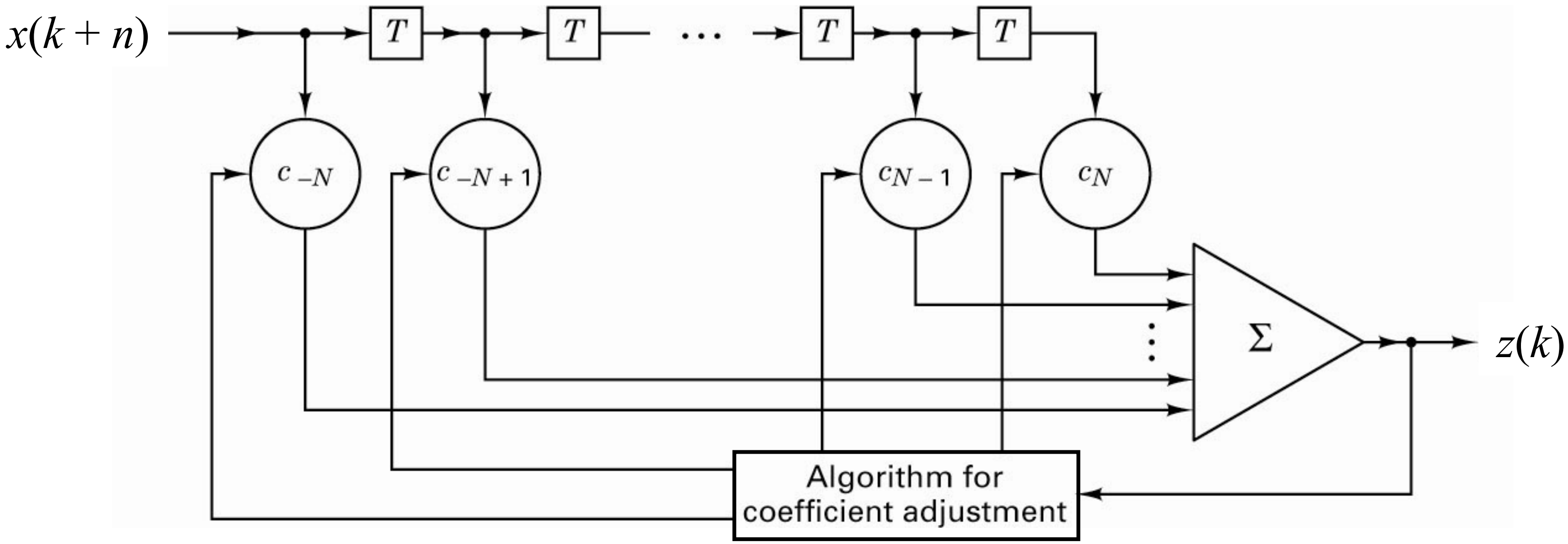
— The equalizing filter attempts to eliminate any distortion produced by the channel. If we model the channel as

$$H_c(f) = |H_c(f)|\, e^{j\theta_c(f)}$$

where $|H_c(f)|$ represents the magnitude and $\theta_c(f)$ represents the phase. To compensate for the channel distortion, the equalizing filter is implemented such as

$$H_e(f) = \frac{1}{H_c(f)} = \frac{1}{|H_c(f)|} e^{-j\theta_c(f)}$$

Equalizer Filter – Transversal Equalizer



- The transversal filter shown above is commonly used for equalization.
- The output of the transversal filter is given by convolving the input samples and tap weights:

$$z[k] = \sum_{n=-N}^{N} x[k-n]c_n \quad \text{for } k = -2N,\ldots,2N \text{ and } n = -N,\ldots,N$$

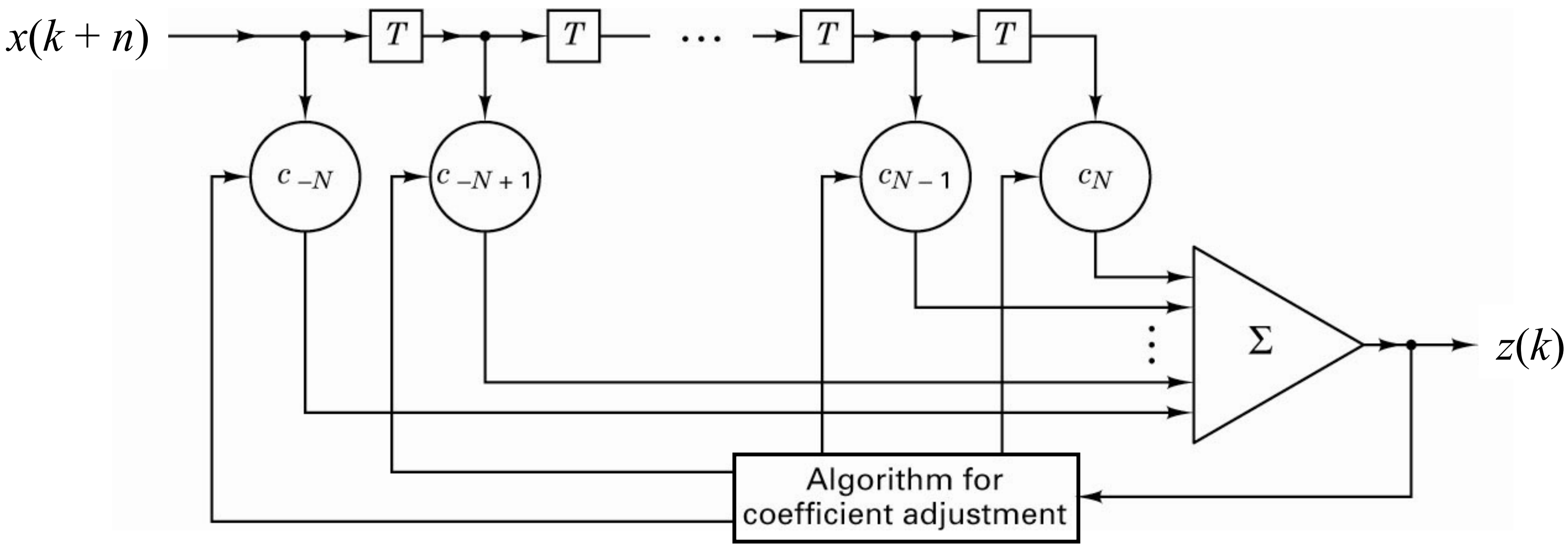
Transversal Equalizer (2)

- The convolution can be expressed as:

$$\underbrace{\begin{bmatrix} z[-2N] \\ z[-2N+1] \\ \vdots \\ z[-1] \\ z[0] \\ z[1] \\ \vdots \\ z[2N-1] \\ z[2N] \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} x[-N] & 0 & 0 & \cdots & 0 \\ x[-N+1] & x[-N] & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 \\ x[N-1] & x[N-2] & \cdots & x[-N] & 0 \\ x[N] & x[N-1] & \cdots & & x[-N] \\ 0 & x[N] & \cdots & & x[-N+1] \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & x[N] & x[N-1] \\ 0 & \cdots & & 0 & x[N] \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} c_{-N} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix}}_{c}$$

- Or a more compact form as: **z**=**xc**
- Solving $\boldsymbol{c} = \boldsymbol{x}^{-1}\boldsymbol{z}$ is possible only if x is a square matrix.
- Note: the size of the vector **z** and the number of rows in the matrix **x** may be chosen to be any value. It depends on the number of ISI points interested

Transversal Equalizer (3)



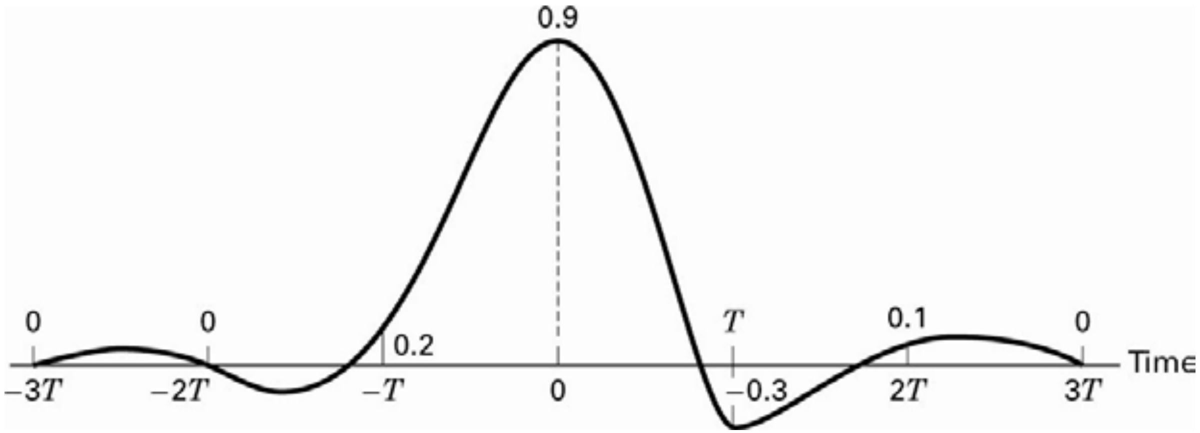
— Solving $\mathbf{c} = \mathbf{x}^{-1}\mathbf{z}$ is possible only if x is a square matrix.

— Alternatively, the zero-forcing solution is obtained by disposing the top N and bottom N rows of the matrix $\mathbf{x}$ and vector $\mathbf{z}$.

— To avoid ISI, the value of $\mathbf{z}$ is assumed to be

$$z[k] = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \ldots, \pm N \end{cases}$$

Activity 5



1. A single impulse is transmitted through a digital communication system with the above waveform received, i.e. $[x(k)]$=[0.0, 0.0, 0.2, 0.9,-0.3, 0.1,0.0]. Use a zero-forcing solution to find the weights $\{c_k\}$ (k=-1,0.1) of a 3-tap transversal equalizer that reduce the ISI so that the equalized pulse samples $z(k)$ have the values [$z(-1)=0$, $z(0)=1$, $z(1)=0$].
2. Using the weights obtained in (1), determine the ISI values of the equalized pulse at sample times $k = \pm1$, ±2, and ±3.
3. What is the largest magnitude sample contributing to the ISI and what is the sum of all of the ISI magnitudes?

Measuring ISI - Eye Pattern

- Eye patterns are obtained by applying the communication system's response to the vertical plates of an oscilloscope.
- A sawtooth wave with fundamental frequency equal to the symbol rate is applied to the horizontal plates of the oscilloscope.
- The setup superimposes the communication system's response within each signaling interval on top of each other within a single interval $(0,T)$.
- The maximum eye opening (M_N) provides a good estimate of the optimal sampling instant.
- The width (S_T) of the eye opening is the range over which the sampling may be performed.
- The range (D_A) of the amplitude variations provide an estimate of the ISI.
- The range (J_T) of the time differences of the zero crossings provide an estimate of jitter.