CSE4214 Digital Communications

Chapter 4

Activities

Activity 1

Consider two waveforms $\cos(2\pi f_1 t + \phi)$ and $\cos(2\pi f_2 t)$ to be used for non-coherent FSK-signaling, where $f_1 > f_2$. The symbol rate is equal to $1/T$ symbols/s, where T is the symbol duration and ϕ is a constant arbitrary angle from 0 to 2π.

Prove that the minimum tone spacing for non-coherent detected orthogonal FSK signaling is $1/T$.

2

Activity 1

For two waveforms to be orthogonal, they must fulfill the orthogonality constraint, i.e.

$$\int_0^T \cos(2\pi f_1 t + \phi)\cos(2\pi f_2 t)dt = 0$$

$$\cos\phi \int_0^T \cos 2\pi f_1 t \cos 2\pi f_2 t dt - \sin\phi \int_0^T \sin 2\pi f_1 t \cos 2\pi f_2 t dt = 0$$

$$\frac{\cos\phi}{2} \int_0^T [\cos 2\pi(f_1 + f_2)t + \cos 2\pi(f_1 - f_2)t]dt$$

$$-\frac{\sin\phi}{2} \int_0^T [\sin 2\pi(f_1 + f_2)t + \sin 2\pi(f_1 - f_2)t]dt = 0$$

3

Activity 1

which yields,

$$\cos\phi\left[\frac{\sin 2\pi(f_1+f_2)t}{2\pi(f_1+f_2)} + \frac{\sin 2\pi(f_1-f_2)t}{2\pi(f_1-f_2)}\right]_0^T + \sin\phi\left[\frac{\cos 2\pi(f_1+f_2)t}{2\pi(f_1+f_2)} + \frac{\cos 2\pi(f_1-f_2)t}{2\pi(f_1-f_2)}\right]_0^T = 0$$

$$\cos\phi\left[\frac{\sin 2\pi(f_1+f_2)T}{2\pi(f_1+f_2)} + \frac{\sin 2\pi(f_1-f_2)T}{2\pi(f_1-f_2)}\right] + \sin\phi\left[\frac{\cos 2\pi(f_1+f_2)T - 1}{2\pi(f_1+f_2)} + \frac{\cos 2\pi(f_1-f_2)T - 1}{2\pi(f_1-f_2)}\right] = 0$$

Assume that $(f_1+f_2)>>1$, we have the following approximation:

$$\frac{\sin 2\pi(f_1+f_2)T}{2\pi(f_1+f_2)} \approx \frac{\cos 2\pi(f_1+f_2)T}{2\pi(f_1+f_2)} \approx 0$$

We have:

$$\cos\phi \sin 2\pi(f_1-f_2)T + \sin\phi[\cos 2\pi(f_1-f_2)T - 1] \approx 0$$

4

Activity 1

For arbitrary ϕ, to let $\cos\phi \sin 2\pi(f_1 - f_2)T + \sin\phi[\cos 2\pi(f_1 - f_2)T - 1] \approx 0$

We must let $\sin 2\pi(f_1 - f_2)T = 0$ at the same time $\cos 2\pi(f_1 - f_2)T = 1$

We know:

$\sin x = 0$ for $x = n\pi$

$\cos x = 1$ for $x = 2k\pi$

If $n=2k$, the $\sin x = 0$ and $\cos x = 1$. We can write:

$$2\pi(f_1 - f_2)T = 2k\pi$$

or

$$f_1 - f_2 = \frac{k}{T}$$

So the minimum spacing occurs for k=1, in which $(f_1 - f_2) = 1/T$

5

Activity 2

Find the bit error probability for a BPSK system with a bit rate of 1Mbit/s. The received waveforms $s_1(t) = A\cos\omega_0 t$ and $s_2(t) = -A\cos\omega_0 t$ are coherently detected with a matched filter. The value of A is 10mV. Assume that the single-sided noise power spectral density is $N_0 = 10^{-11}$ W/Hz and that signal power and energy per bit are normalized relative to a 1 ohm load.

6

Activity 2

Solution:

$$\because A = \sqrt{\frac{2E_b}{T}} = 0.01V,\ T = \frac{1}{R} = 10^{-6}s$$

$$\therefore E_b = \frac{A^2}{2}T = 5 \times 10^{-11} J \quad \text{and} \quad \sqrt{\frac{2E_b}{N_0}} = 3.16$$

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(3.16)$$

Using Table B.1, we obtain P_B=8X10^{-4}

7