CSE4214 Digital Communications

Chapter 4

Bandpass Modulation and Demodulation/Detection

Bandpass Modulation

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Bandpass Modulation

- Baseband transmission is conducted at low frequencies
- Passband transmission is to send the signal at high frequencies
  - Signal is converted to a sinusoidal waveform, e.g.

$$s(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

    where ω_0 is called carrier frequency is much higher than the highest frequency of the modulating signals, i.e. messages
  - Bits are encoded as a variation of the amplitude, phase, frequency, or some combination of these parameters.

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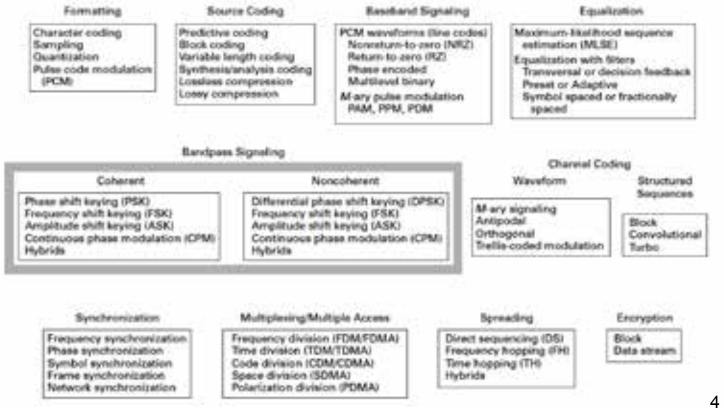
Types of Bandpass Modulation



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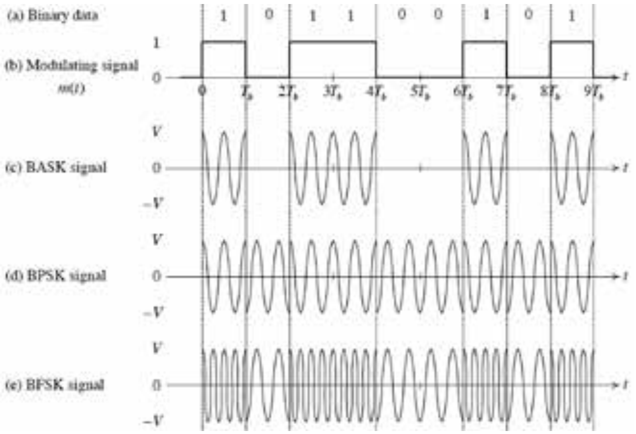
Bandpass Examples



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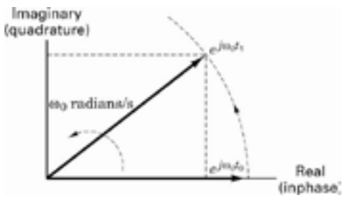
Phasor Representation of a Sinusoid

Phasor Representation of Sinusoidal Signals

- Using Euler identity

$$e^{j\omega_0 t} = \underbrace{\cos\omega_0 t}_{\text{Inphase (I) Component}} + j\underbrace{\sin\omega_0 t}_{\text{Quadrature (Q) Component}}$$

- The unmodulated carrier wave $c(t) = \cos(\omega_0 t)$ is represented as a unit vector rotating in a counter-clockwise direction at a constant rate of ω_0 radians/s.



Amplitude Modulation (AM)

- A double side band, amplitude modulated (DSB-AM) signal is represented by

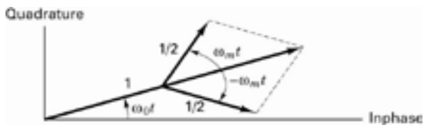
$$s(t) = \cos\omega_0 t \cdot (1 + \cos\omega_m t)$$

where $c(t) = \cos(\omega_0 t)$ is the carrier signal and $x(t) = \cos(\omega_m t)$ is the information bearing signal.

- An equivalent representation of DSB-AM signal is given by

$$s(t) = \cos\omega_0 t \cdot [1 + \tfrac{1}{2}(e^{j\omega_m t} + e^{-j\omega_m t})]$$
$$= \mathrm{Re}\left\{e^{j\omega_0 t} \cdot [1 + \tfrac{1}{2}(e^{j\omega_m t} + e^{-j\omega_m t})]\right\}$$

- The phasor representation of the DSB-AM signal is shown as



- The composite signal rotates in a counter-clockwise direction at a constant rate of ω_0 radians/s. However, the vector expands and shrinks depending upon the term $\omega_m t$.

Frequency Modulation (FM)

- A frequency modulated (FM) signal is represented by

$$s(t) = \cos\left[\omega_0 t + k_f \int x(t)dt\right]$$

- Assuming that the information bearing signal $x(t) = \cos(\omega_m t)$, the above expression reduces to

$$\begin{aligned} s(t) &= \cos\left[\omega_0 t + \tfrac{k_f}{\omega_m}\sin(\omega_m t)\right] \\ &= \cos(\omega_0 t)\cos\left(\tfrac{k_f}{\omega_m}\sin(\omega_m t)\right) - \sin(\omega_0 t)\sin\left(\tfrac{k_f}{\omega_m}\sin(\omega_m t)\right) \end{aligned}$$

- For narrow band FM

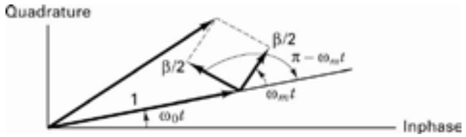
$$\begin{aligned} s(t) &= \cos(\omega_0 t) - \beta\sin(\omega_0 t)\sin(\omega_m t), \quad \beta = \tfrac{k_f}{\omega_m} << 1 \\ &= \mathrm{Re}\left\{e^{j\omega_0 t} - \tfrac{\beta}{2}e^{j\omega_0 t}\left[\tfrac{1}{2}e^{j\omega_m t} - \tfrac{1}{2}e^{-j\omega_m t}\right]\right\} \\ &= \mathrm{Re}\left\{e^{j\omega_0 t}\left[1 + \tfrac{\beta}{2}e^{j\omega_m t} - \tfrac{\beta}{2}e^{-j\omega_m t}\right]\right\} \end{aligned}$$

Frequency Modulation (2)

- The phasor representation of a narrowband FM signal is given by

$$s(t) = \mathrm{Re}\left\{e^{j\omega_0 t}\left[1 + \tfrac{\beta}{2}e^{j\omega_m t} - \tfrac{\beta}{2}e^{-j\omega_m t}\right]\right\}$$

- The phasor diagram of the narrowband FM signal is shown as



- The composite signal speeds up or slows down according to the term $\omega_m t$.

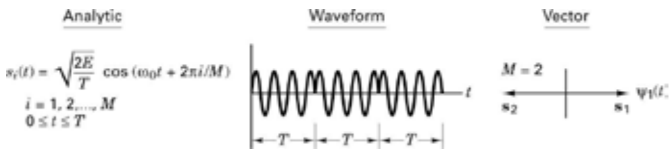
Phase Shift Keying

- The general expression for M-ary PSK is

$$s_i(t) = \sqrt{\tfrac{2E}{T}}\cos\left[\omega_0 t + \phi_i(t)\right] \quad 0 \le t \le T, i = 1, \ldots, M$$

where the phase term $\phi_i(t) = 2\pi i/M$.

- The symbol energy is given by E and T is the duration of the symbol.
- The waveform and phasor representation of the 2-ary PSK (binary PSK) is shown below.



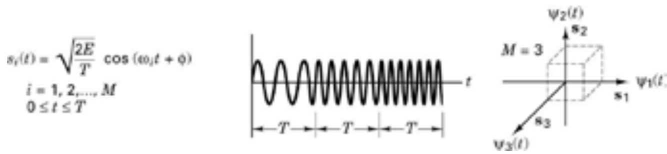
Frequency Shift Keying

- The general expression for MFSK is

$$s_i(t) = \sqrt{\tfrac{2E}{T}}\cos\left[\omega_i t + \phi\right] \quad 0 \le t \le T, i = 1, \ldots, M$$

where the frequency term ω_i has M discrete values and phase ϕ is a constant.

- The symbol energy is given by E and T is the duration of the symbol.
- The frequency difference $(\omega_{i+1} - \omega_i)$ is typically assumed to be an integral multiple of π/T.
- The waveform and phasor representation of the 3-ary FSK is shown below.



Amplitude Shift Keying

- The general expression for M-ary ASK is

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos[\omega_0 t + \varphi] \qquad 0 \le t \le T, i = 1,\ldots,M$$

where the amplitude term

$$\sqrt{\frac{2E_i(t)}{T}}$$

has M discrete values and frequency ω_0 and phase ϕ is a constant.

- The waveform and phasor representation of the 2-ary ASK (binary ASK) is shown below.

Amplitude Phase Keying

- The general expression for M-ary APK is

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos[\omega_0 t + \varphi_i(t)] \qquad 0 \le t \le T, i = 1,\ldots,M$$

where both the signal amplitude and phase vary with the symbol.

- The waveform and phasor representation of the 8-ary APK is shown below.

Digital Modulation Summary

PSK	$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[\omega_0 t + \frac{2\pi i}{M}\right] \quad 0 \le t \le T, i = 1,\ldots,M$
FSK	$s_i(t) = \sqrt{\frac{2E}{T}}\cos[\omega_i t + \varphi] \quad 0 \le t \le T, i = 1,\ldots,M$
ASK	$s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos[\omega_0 t + \varphi] \quad 0 \le t \le T, i = 1,\ldots,M$
QAM	$s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos[\omega_0 t + \varphi_i(t)] \quad 0 \le t \le T, i = 1,\ldots,M$

Detection of Signals in Gaussian Noise

Decision Regions:

- Assume that the received signal $r(t)$ is given by

$$r(t) = s_1(t) + n(t) \qquad \text{symbol 1}$$

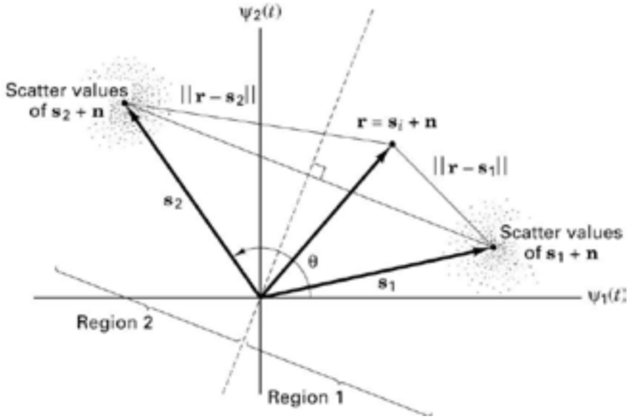
$$r(t) = s_2(t) + n(t) \qquad \text{symbol 2}$$

- The task of the detector is to decide which symbol was transmitted from $r(t)$.
- For equi-probable binary signals corrupted with AWGN, the minimum error decision rule is equivalent to choosing the symbol such that the distance $d(r,s_i) = \|r - s_i\|$ is minimized.

Procedure:

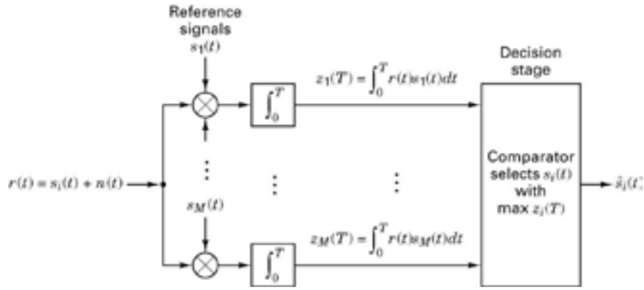
1. Pick an orthonormal basis functions for the signal space.
2. Represent $s_1(t)$ and $s_2(t)$ as vectors in the signal space.
3. Connect tips of vectors representing $s_1(t)$ and $s_2(t)$.
4. Construct a perpendicular bisector of the connecting lines.
5. The perpendicular bisector divides 2D plane in 2 regions.
6. If $r(t)$ is located in R1, choose $s_1(t)$ as transmitted signal
7. If $r(t)$ is located in R2, choose $s_2(t)$ as transmitted signal
8. The figure is referred to as the signal constellation

Detection of Signals in Gaussian Noise (2)



Correlator Receiver for M-ary Transmission (1)

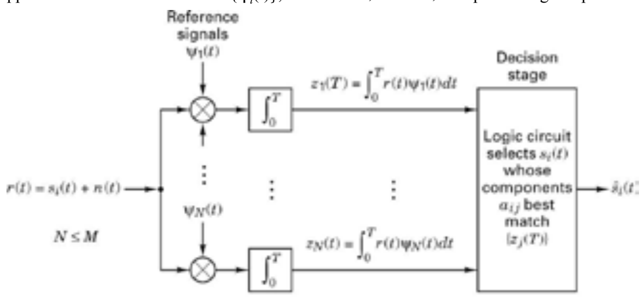
Approach 1: Use correlator implementation of matched filter.



Decision Rule: Use signal $s_i(t)$ that results in the highest value of $z_i(t)$.

Correlator Receiver for M-ary Transmission (2)

Approach 2: Use Basis functions $\{\psi_i(t)\}$, $1 <= i <= N$, $N <= M$, to represent signal space



Each signal $s_i(t)$ is represented as a linear combination of the basis functions

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) + \cdots + a_{iN}\psi_N(t), \quad 1 \le i \le M$$

Decision Rule: Pick signal $s_i(t)$ whose coefficient a_{ij} best match $z_j(T)$.

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Coherent & Non-Coherent Detection

Definitions

- Coherent detection – the receiver exploits knowledge of the carrier's phase to detect the signal
  - Require expensive and complex carrier recovery circuit
  - Better bit error rate of detection
- Non-coherent detection – the receiver does not utilize phase reference information
  - Do not require expensive and complex carrier recovery circuit
  - Poorer bit error rate of detection
  - Differential systems have important advantages and are widely used in practice

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Coherent Receiver

- Carrier recovery for demodulation
  - Received signal $r(t) = A\cos(\omega_c t + \varphi) + n(t)$
  - Local carrier $\cos(\omega_c t + \hat{\varphi})$
  - Carrier recovery – phase lock loop circuit

$$\Delta\varphi = \varphi - \hat{\varphi} \rightarrow 0$$

  - Demodulation leads to recovered baseband signal

$$Y(t) = s(t + \tau) + n(t)$$

- Timing recovery for sampling
  - Align receiver clock with transmitter clock, so that sampling → no ISI

$$Y_k = s_k + n_k$$

Non-Coherent Receiver

- No carrier recovery for demodulation
  - Received signal $r(t) = A\cos(\omega_c t + \varphi) + n(t)$
  - Local carrier $\cos(\omega_c t + \hat{\varphi})$
  - No carrier recovery

$$\Delta\varphi = \phi = \varphi - \hat{\varphi} \neq 0$$

  - Demodulation leads to recovered baseband signal

$$Y(t) = s(t + \tau)e^{j\phi} + n(t)$$

- Timing recovery for sampling
  - Align receiver clock with transmitter clock, sampling results in

$$Y_k = s_k e^{j\phi} + n_k$$

could not recover transmitted symbols properly from Y_k

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Coherent Detection

Binary PSK (1)

In coherent detection, exact frequency and phase of the carrier signal is known.

Binary PSK:

1. The transmitted signals are given by

$$s_1(t)=\sqrt{\tfrac{2E}{T}}\cos[\omega_0 t+\varphi],\qquad 0\le t\le T$$
$$s_2(t)=\sqrt{\tfrac{2E}{T}}\cos[\omega_0 t+\varphi+\pi],\quad 0\le t\le T$$
$$=-\sqrt{\tfrac{2E}{T}}\cos[\omega_0 t+\varphi],\qquad 0\le t\le T$$

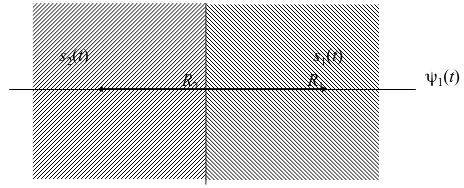
2. Pick the basis function
$$\psi_1(t)=\sqrt{\tfrac{2}{T}}\cos[\omega_0 t+\phi],\ 0\le t\le T$$

3. Represent the transmitted signals in terms of the basis function

$$s_1(t)=\sqrt{E}\psi_1(t),$$
$$s_2(t)=-\sqrt{E}\psi_1(t),$$

Binary PSK (2)

4. Draw the signal constellation for binary PSK



5. Divide the signal space into two regions by the perpendicular to the connecting line between tips of vectors $s1$ and $s2$.
6. The location of the received signal determines the transmitted signal.

M-ary PSK (1)

M-ary PSK:

1. The transmitted signals are given by

$$s_i(t)=\sqrt{\tfrac{2E}{T}}\cos[\omega_0 t+\tfrac{2\pi i}{M}],\quad 0\le t\le T, i=1,\ldots,M$$

2. Pick the basis function

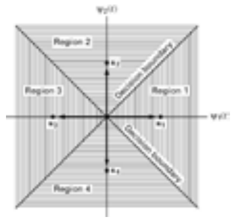
$$\psi_1(t)=\sqrt{\tfrac{2}{T}}\cos[\omega_0 t],\ 0\le t\le T$$
$$\psi_2(t)=\sqrt{\tfrac{2}{T}}\sin[\omega_0 t],\ 0\le t\le T$$

3. Represent the transmitted signals in terms of the basis function

$$s_i(t)=a_{i1}\psi_1(t)+a_{i2}\psi_2(t),\qquad i=1,\ldots,M$$
$$=\sqrt{E}\cos\left(\tfrac{2\pi i}{M}\right)\psi_1(t)+\sqrt{E}\sin\left(\tfrac{2\pi i}{M}\right)\psi_2(t),$$

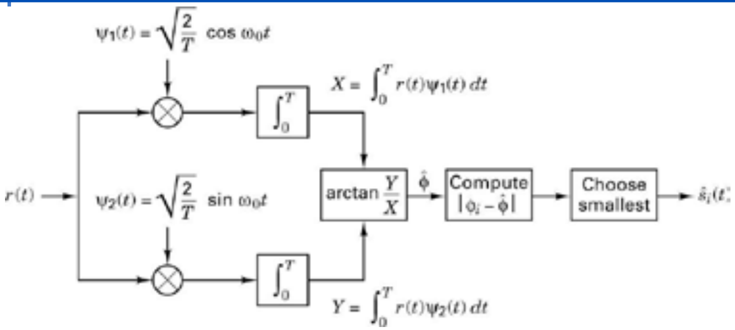
M-ary PSK (2)

4. Draw the signal constellation for MPSK. The following illustrates the signal constellation for $M=4$.

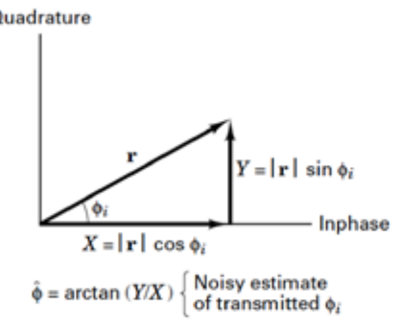


5. Divide the signal space into two regions by the perpendicular to the connecting line between tips of signals vectors.
6. The location of the received signal determines the transmitted signal.
7. Note that the decision region can also be specified in terms of the angle that the received vector makes with the horizontal axis.

M-ary PSK (3)



M-ary PSK (4)



FSK

- A typical set of FSK is described by:

$$s_i(t) = \sqrt{\tfrac{2E}{T}} \cos\left[\omega_i t + \varphi\right] \qquad 0 \le t \le T, i = 1, \ldots, M$$

E is the energy content of $s_i(t)$ over each symbol duration T, and $(\omega_{i+1}-\omega_i)$ is typically assumed to be an integral multiple of π/T. The phase term is an arbitrary constant and can be set equal to zero.

- Assume that basis functions form an orthonormal set, i.e.

$$\psi_i(t) = \sqrt{\tfrac{2}{T}} \cos \omega_j t \qquad j = 1, \ldots, N$$

$$a_{ij}(t) = \int_0^T \sqrt{\frac{2E}{T}} \cos(\omega_i t) \sqrt{\frac{2}{T}} \cos(\omega_j t) dt = \begin{cases} \sqrt{E} & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$$

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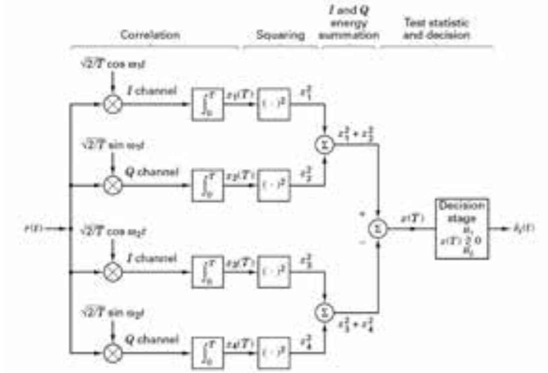
Non-coherent Detection

Binary FSK – Quadrature Receiver

- Implemented with correlators, but based on energy detector without exploiting phase information



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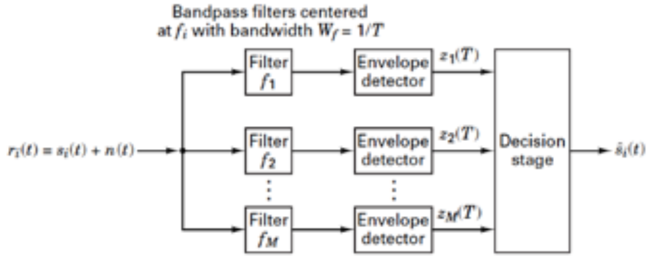
FSK – Envelope Detector

- Implemented with bandpass filters followed by envelope detectors.
- Envelope detector consists of a rectifier and a lowpass filter



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Minimum Tone Spacing for Orthogonal FSK

- FSK is usually implemented as orthogonal signaling.
- Not all FSK signaling is orthogonal, how can we tell if the tone in a signaling set form an orthogonal set?
  - To form an orthogonal set, they must be uncorrelated over a symbol time T
- Minimum tone spacing for orthogonal FSK:
  - *Any pair of tones in the set must have a frequency separation that is a multiple of 1/T hertz*

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Activity 1

Consider two waveforms $\cos(2\pi f_1 t + \phi)$ and $\cos(2\pi f_2 t)$ to be used for non-coherent FSK-signaling, where $f_1 > f_2$. The symbol rate is equal to $1/T$ symbols/s, where T is the symbol duration and ϕ is a constant arbitrary angle from 0 to 2π.

Prove that the minimum tone spacing for non-coherent detected orthogonal FSK signaling is $1/T$.

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Error Performance for Binary Systems

Probability of Bit Error for Coherently Detected BPSK

- For BPSK, the symbol error probability is the bit error probability.
- Assume
  - For transmitting $s_i(t)$ (i=1,2), the received signal is $r(t)=s_i(t)+n(t)$ where $n(t)$ is an AWGN process.
  - Any degradation effects due to channel-induced ISI or circuit-induced ISI have been neglected.
- The antipodal signals are:

$$\left.\begin{aligned} s_1(t) &= \sqrt{E}\psi_1(t) \\ s_2(t) &= -\sqrt{E}\psi_1(t) \end{aligned}\right\} 0 \le t \le T$$

- The decision rule are error probability are:

$$\begin{aligned} &s_1(t) \quad \text{if } z(T) > \gamma_0 = 0 \\ &s_2(t) \quad \text{otherwise} \end{aligned} \qquad P_B = Q\left(\sqrt{\frac{(1-\rho)E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Activity 2

Find the bit error probability for a BPSK system with a bit rate of 1Mbit/s. The received waveforms $s_1(t) = A\cos\omega_0 t$ and $s_2(t) = -A\cos\omega_0 t$ are coherently detected with a matched filter. The value of A is 10mV. Assume that the single-sided noise power spectral density is $N_0=10^{-11}$ W/Hz and that signal power and energy per bit are normalized relative to a 1 ohm load.

Probability of Bit Error for Coherently Detected BFSK

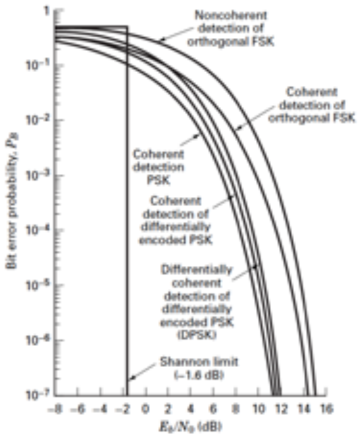
- For BFSK, the symbol error probability is the bit error probability.
- Assume
  - For transmitting $s_i(t)$ (i=1,2), the received signal is $r(t)=s_i(t)+n(t)$ where $n(t)$ is an AWGN process.
  - Any degradation effects due to channel-induced ISI or circuit-induced ISI have been neglected.
- For orthogonal signals are:

$$\left.\begin{aligned} s_1(t) &= A\cos\omega_0 t \\ s_2(t) &= A\cos\omega_1 t \end{aligned}\right\} 0 \le t \le T$$

- The error probability is:

$$P_B = Q\left(\sqrt{\frac{(1-\rho)E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Bit Error Probability for Several Binary Systems



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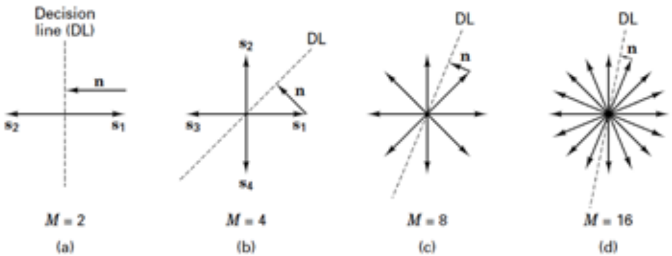
Error Performance for M-ary Systems

M-ary Signaling

- Modulator produces one of $M=2^k$ waveforms
  - Binary signaling is the special case where k=1
- Vectorial view of MPSK signaling



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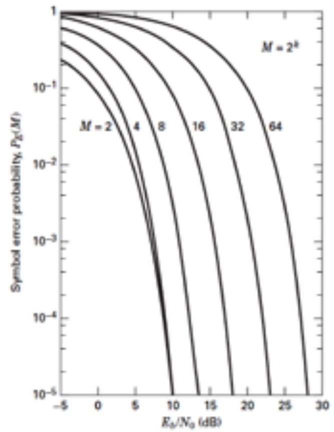
Symbol Error Performance for M-ary Systems (M>2)

- For large energy-to-noise ratios, the symbol error performance $P_E(M)$, for equally likely, coherently detected M-ary PSK signaling:

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$$

where $E_s = E_b(\log_2 M)$ is the energy per symbol, and $M = 2^k$

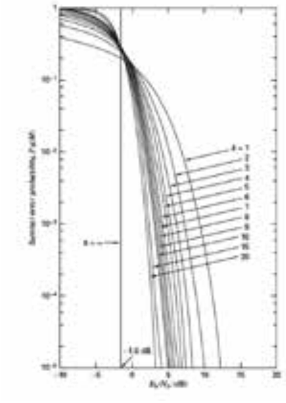


Symbol Error Performance for MFSK

- The symbol error performance $P_E(M)$, for equally likely, coherently detected M-ary orthogonal signaling can be upper bounded as:

$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

where $E_s = E_b(\log_2 M)$ is the energy per symbol, and M is the size of the symbol set.



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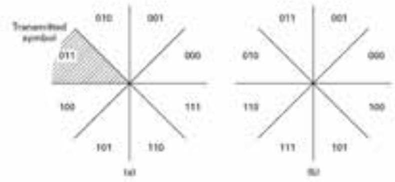
Bit Error Probability versus Symbol Error Probability

- For multiple phase signaling and utilizing Gray code assignment,

$$P_B \approx \frac{P_E}{\log_2 M}$$



Binary assignment Gray code assignment

- For orthogonal signaling,

$$\frac{P_B}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1}$$

$$\lim_{k \to \infty} \frac{P_B}{P_E} = \frac{1}{2}$$

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