CSE4214 Digital Communications

Chapter 5

Activities

CSE4214 Digital Communications

Part 2

Activity 1

Configure a (4,3) even-parity error-detection code such that the parity symbol appears as the leftmost symbol of the codeword. Which error patterns can the code detect?

Compute the probability of an undetected message error, assume that all symbol error are independent events and that the probability of a channel symbol error is $p = 10^{-3}$.

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Activity 1 Solution

Configure a (4,3) even-parity error-detection code such that the parity symbol appears as the leftmost symbol of the codeword. Which error patterns can the code detect?

The code is capable of detecting all single and triple error patterns.

Message	Parity	Code Word	
000	0	0	000
100	1	1	100
010	1	1	010
110	0	0	110
001	1	1	001
101	0	0	101
011	0	0	011
111	1	1	111
		parity	message

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Activity 1 Solution

Compute the probability of an undetected message error assume that all symbol error are independent events and that the probability of a channel symbol error is $p = 10^{-3}$.

Given:

$$P_{nd} = \sum_{j=1}^{\substack{n/2(\text{for n even}) \\ (n-1)/2(\text{for n odd})}} \binom{n}{2j} p^{2j}(1-p)^{n-2j}$$

The probability of an undetected error is equal to the probability that 2 or 4 errors occur anywhere in a codeword

$$P_{nd} = \binom{4}{2} p^2 (1-p)^2 + \binom{4}{4} p^4$$
$$= 6p^2(1-p)^2 + p^4 = 6p^2 - 12p^3 + 7p^4$$
$$= 6 \times (10^{-3})^2 - 12 \times (10^{-3})^3 + 7 \times (10^{-3})^4 \approx 6 \times 10^{-6}$$

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Activity 2

Generate a codeword for message vector [1,1,0] (U_4) in a (6,3) code if the generator matrix G is given by

$$G = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

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Activity 2 Solution

Generate a codeword for message vector [1,1,0] (U_4) in a (6,3) code if the generator matrix G is given by

$$G = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$U = mG$$

$$U_4 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 1 \cdot V_1 + 1 \cdot V_2 + 1 \cdot V_3$$

$$= 110100 + 011010 + 000000$$

$$= 101110$$

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Activity 3

For the (6,3) code, the codewords are described as follows:

$$U = [m_1, m_2 m_3] \times \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find u_1, u_2, …, u_6.

Activity 3 Solution

Given:

$$U = [m_1, m_2 m_3] \times \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

We have:

$$U = [m_1, m_2 m_3] \times \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [m_1 + m_3, m_1 + m_2, m_2 + m_3, m_1, m_2, m_3]$$

Activity 4

Suppose that codeword U=101110 from the (6,3) code in Activity 3 is transmitted and the vector **r**=001110 is received; i.e. the leftmost bit is received in error. Find the syndrome $S = eH^T$ vector value and verify that it is equal to **eH**^{T}.

Activity 4 Solution

$$S = rH^T = [001110]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [1, 1+1, 1+1]$$

$= [\ 1 \quad 0 \quad 0]$ (syndrome of corrupted code vector)

$S = eH^T = [\ 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]\ \ H^T$

$= [\ 1 \quad 0 \quad 0]$ (syndrome of error pattern)

Activity 5

Assume that codeword **U**=1 0 1 1 1 0, from (6,3) code in Activity 3, is transmitted, and the vector **r**= 0 0 1 1 1 0 is received. Show how a decoder can correct the error (by using syndrome look-up table)

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Activity 5 Solution

Assume that codeword U=1 0 1 1 1 0, from (6,3) code in Activity 3, is transmitted, and the vector r= 0 0 1 1 1 0 is received. Show how a decoder can correct the error (by using syndrome look-up table)

The syndrome of r is computed as:

$$S = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} H^T = [\,1 \quad 0 \quad 0\,]$$

Using the Lookup Table, the error pattern corresponding to the syndrome above is estimated to be

$$\hat{e} = 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

The corrected vector is then estimated by

$$\hat{U} = r + \hat{e}$$

$$=001110+100000$$

$$=101110$$

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Part 3

Activity 1

Consider the codeword set of (6,3), suppose the codeword 110011 was transmitted and that two leftmost digits were declared by the receiver to be erasures. Verify that the received flawed sequence xx0011 can be corrected.

Message vector	Codeword
000	000000
100	110100
010	011010
110	101110
001	101001
101	011101
011	110011
111	000111

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Activity 1 Solution

Since $d_{min}=p+1=3$, $p=2$, the code can correct as many as 2 erasures.

Compare the rightmost four digits of received word xx0011 with the codeword in the Table, the codeword that was actually transmitted is cloest in Hamming distance to the flawed sequence.

Message vector	Codeword
000	000000
100	110100
010	011010
110	101110
001	101001
101	011101
011	110011
111	000111

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