CSE4214 Digital Communications

Chapter 5 Part 1

Channel Coding

Channel Coding

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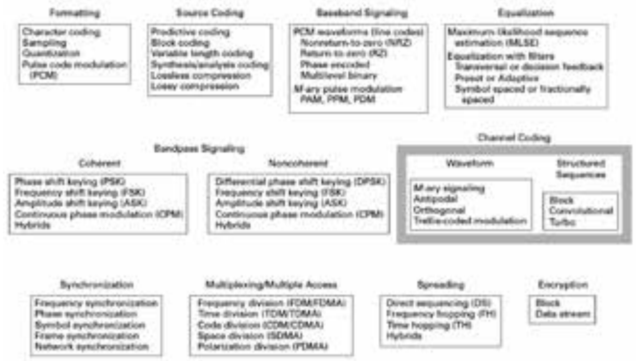
Topics Covered



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What Is Channel Coding?

- Signal transformations designed to improve communications performance by enabling the transmitted signals to better withstand the effects of various channel impairments, such as noise, interference, and fading.
- Two types channel coding
  - Waveform coding
  - Structured sequences

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Types of Channel Coding

- Waveform coding
  - Transforming waveforms into “better waveforms” to make the detection process less subject to errors
- Structured sequences
  - Transforming data sequences into “better sequences”, by having structured redundancy (redundant bits). The redundant bits can then be used for the detection and correction of errors.

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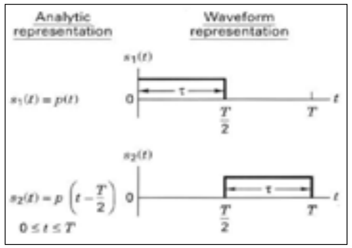
Waveform Coding

Orthogonal Signals

- Orthogonal condition

Signal set $\{\phi_k(t)\}^n$ is an **orthogonal** set if

$$\int_{-\infty}^{\infty} \phi_j(t)\phi_k(t)dt = \begin{cases} 0 & j \neq k \\ c_j & j = k \end{cases}$$



sine and cosine wave are orthogonal waveform set

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Cross Correlation

- A measure of the distance between the signal vectors.
- The smaller the cross-correlation, the more distance are the vectors from each other.
- Cross correlation coefficient of orthogonal signal

$$z_{ij} = \frac{1}{E}\int_0^T s_i(t)s_j(t)\,dt = \begin{cases} c & for\ i = j \\ 0 & otherwise \end{cases}$$

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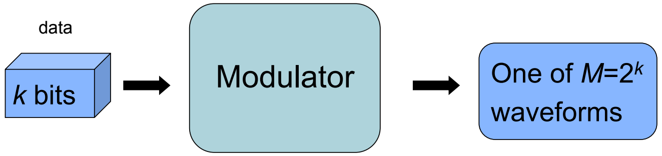
M-ary Signaling

- Accepts k data bits at a time, then produce one of $M=2^k$ waveforms.
- Binary signaling is a special case where k=1



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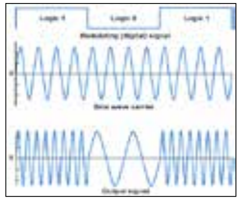
M-ary Signaling

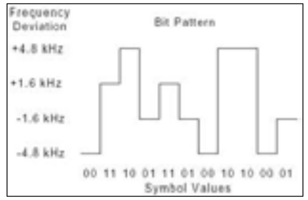
- For example: Multiple Frequency Shift Keying (MFSK)

BFSK



Input: 1 bit
M = 2^1 = 2
Output: one of 2 waveforms

4FSK



Input: 2 bit
M = 2^2 = 4
Output: one of 4 waveforms

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The Goal of Waveform Coding

- Transform a waveform set (representing a message set) into an improved waveform set. The improved waveform set can then be used to provide improved probability of bit error (P_B) compared to the original set.
- The encoding procedure endeavors to make each of the waveforms in the coded signal set as unalike as possible; the goal is to render the cross-correlation coefficient z_{ij} (among all pairs of signals) as small as possible.
- The most popular of waveform codes are referred to as orthogonal and biorthogonal codes.

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Orthogonal Codes

- Orthogonal condition

$$z_{ij} = \frac{1}{E}\int_0^T s_i(t)s_j(t)\,dt = \begin{cases} c & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$$

  - Each waveform in the set [$s_i(t)$] may consist of a sequence of pulses, where each pulse is designated with a level +1 or -1, which in turn represents the binary digit 1 or 0, respectively.
  - When the set is expressed in this way, z_{ij} can be simplified by stating that [$s_i(t)$] constitutes an orthogonal set if and only if

$$z_{ij} = \frac{\text{number of digit agreements} - \text{number of digit disagreements}}{\text{total number of digits in the sequence}} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$$

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Orthogonal Codes

- Orthogonal codes

$$z_{ij} = \frac{\textit{number of digit agreements} - \textit{number of digit disagreements}}{\textit{total number of digits in the sequence}} = \begin{cases} 1 & \textit{for } i = j \\ 0 & \textit{otherwise} \end{cases}$$

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Orthogonal codeword set

1 bit data set

0 → [0 0] (i)

1 → [0 1] (j)

z_{ii} = (2 – 0) / 2 = 1

z_{ij} = (1 – 1) / 2 = 0

$$H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

H is orthogonal codeword set

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Orthogonal Codes

- 2 bit data set

Data set	Orthogonal codeword set
0 0	
0 1	
1 0	
1 1	

$$\mathbf{H}_2 = \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right] = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{H}_1 & \overline{\mathbf{H}}_1 \end{bmatrix}$$

- H – Hadamard matrix

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Hadamard Matrix

- For a *k*-bit data set from the $\mathbf{H}_{k-1}$ matrix, as follows:

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_{k-1} & \mathbf{H}_{k-1} \\ \mathbf{H}_{k-1} & \overline{\mathbf{H}}_{k-1} \end{bmatrix}$$

Orthogonal codeword set

k: number of data bits per codeword

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Error Performance: P_E

- Waveform coding with orthogonal codes improves probability of bit error (P_B) performance
  - Probability of codeword error (P_E) can be upper bounded as

$$P_E(M) \le (M-1)\,Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

M: codeword set = 2^k

(*k*: number of data bits per codeword)

E_s: energy per codeword = $k.E_b$

E_b: bit energy

N_0: noise power spectral density

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Error Performance: P_B

- Waveform coding with orthogonal codes improves probability of bit error (P_B) performance

Knowing that

$$\frac{P_B(k)}{P_E(k)} = \frac{2^{k-1}}{2^k - 1} \quad or \quad \frac{P_B(M)}{P_E(M)} = \frac{M/2}{(M-1)}$$

Probability of bit error (P_B) can be upper bounded as

$$P_B(k) \le (2^{k-1})\, Q\left(\sqrt{\frac{kE_b}{N_0}}\right) \quad or$$

$$P_B(M) \le \frac{M}{2}\, Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

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Biorthogonal Codes

Biorthogonal Codes

3 bit data set

Data Set

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Orthogonal codeword set

$$\mathbf{H}_3 = \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} \mathbf{H}_2 & \mathbf{H}_2 \\ \mathbf{H}_2 & \overline{\mathbf{H}}_2 \end{bmatrix}$$

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Biorthogonal Codes

3 bit data set

Requires one-half code compared to orthogonal code → half bandwidth

Data Set

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Biorthogonal codeword set

$$\mathbf{B}_3 = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right] \qquad \mathbf{B}_k = \begin{bmatrix} \mathbf{H}_{k-1} \\ \overline{\mathbf{H}}_{k-1} \end{bmatrix}$$

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Biorthogonal Codes

- Biorthognal set is really two sets of orthogonal codes such that each codeword in one set has its antipodal codeword in the other set.
- Biorthognal set consists of a combination of orthogonal and antipodal signals

$$z_{ij} = \begin{cases} 1 & for\, i = j \\ -1 & for\, i \neq j, |i-j| = M/2 \\ 0 & for\, i \neq j, |i-j| \neq M/2 \end{cases}$$

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Error Performance

- Waveform coding with biorthogonal codes
  - Probability of codeword error (P_E) can be upper bounded as

$$P_E(M) \leq (M-2)Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

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Error Performance

- Waveform coding with biorthogonal codes
  - Probability of codeword error (P_E) can be upper bounded as

$$P_E(M) \leq (M-2)Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

  - We can approximate $P_B(M)$ by:

$$P_B(M) \leq \frac{1}{2}\left[(M-2)Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q\left(\sqrt{\frac{2E_s}{N_0}}\right)\right]$$

- Compared to orthogonal code, biorthogonal code has improved P_B performance and requires only half the bandwidth.

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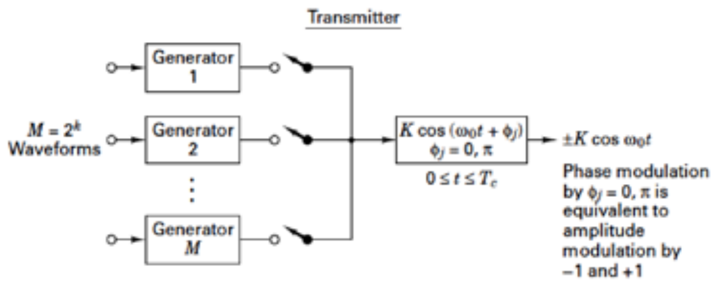
Waveform Coding System



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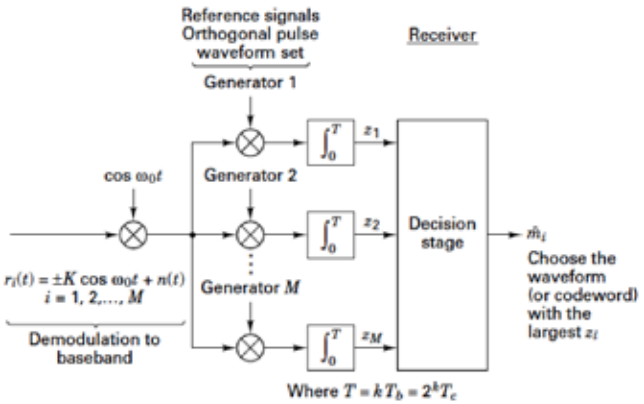
Waveform Coding System



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Error Control

Types of Error Control

Two basic ways that redundancy is used for controlling errors

Error detection and retransmission

Utilizes parity bits to detect error. The receiving terminal does not attempt to correct the error; it simply requests that the transmitter retransmit the data.

Error detection and correction

The parity bits are designed for both the detection and correction of errors. (Forward error correction (FEC))

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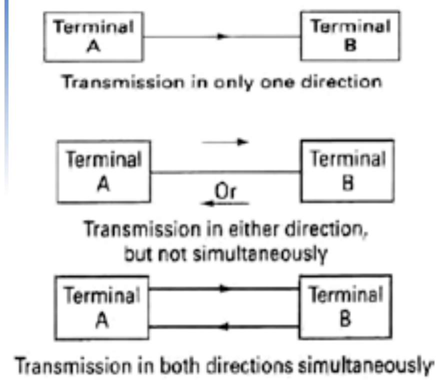
Terminal Connectivity



simplex

Half-duplex

Full-duplex

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Error Detection and Retransmission

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Automatic Repeat Request

- Three most popular automatic repeat request (ARQ) procedures
  - Stop-and-wait ARQ
  - Continuous ARQ with pullback (full-duplex)
  - Continuous ARQ with selective repeat (full-duplex)

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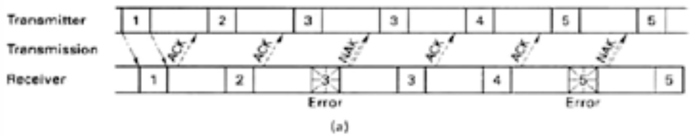
Stop-and-Wait ARQ



The transmitter waits for an ACK of each transmission before it proceeds with the next transmission.
→Requires a half-duplex connection only
(*) The third transmission block is received in error, therefore the receiver responds with a NAK, and the transmitter retransmits this third message block before transmitting the next in the sequence.

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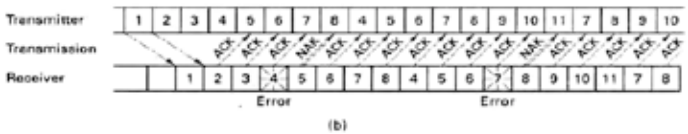
Continuous ARQ with Pullback



Both terminals are transmitting simultaneously: the transmitter is sending message data and the receiver is sending ACK/NAK.
→ A full-duplex connection is necessary

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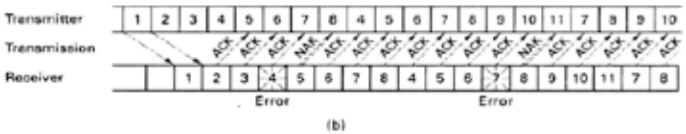
Continuous ARQ with Pullback

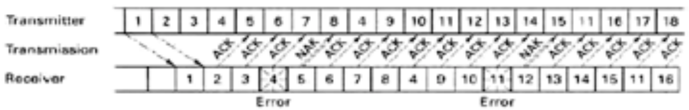


(*) When message 8 is being sent, a NAK corresponding to the corrupted message 4 is being received. The transmitter "pulls back" to the message in error and retransmits all message data, starting with the corrupted message

Continuous ARQ with Pullback

- A sequence number has to be assigned to each block of data. Also, the ACK/NAK need to reference to the sequence numbers (or else there needs to be a priori knowledge of the propagation delays, so that the transmitter knows which messages are associated with which acknowledgments)

Continuous ARQ with Selected Repeat



Both terminals are transmitting simultaneously
→a full-duplex connection is needed.

Only the corrupted message is repeated; then, the transmitter continues the transmission sequence where it had left off instead of repeating any subsequent correctly received messages.

Which ARQ to Use?

Stop-and-wait ARQ

Continuous ARQ with pullback

Continuous ARQ with selective repeat

The choice of which ARQ procedure to choose is a trade-off between the requirements for efficient utilization of the communications resource and the need to provide full-duplex connectivity.

e.g. stop-and-wait ARQ
(*) Half-duplex connectivity is less costly than fullduplex
(*) The associated inefficiency can be measured by the blank time slots.

ARQ and FEC

ARQ

The major advantages of ARQ over FEC

- Error detection requires much simpler decoding equipment
- Error detection requires much less redundancy
- ARQ is adaptive in the sense. that information is retransmitted only when errors occur.

FEC

FEC may be desirable for any of the following reasons:

- A reverse-channel is not available or the delay with ARQ would be excessive
- The retransmission strategy is not conveniently implemented.
- The expected number of errors, without corrections, would require excessive retransmissions.

Summary

- Waveform coding
  - Orthogonal code
  - Biorthogonal code
- Type of error control
  - Error detection and retransmission
  - Error detection and correction (FEC)
  - Three types of ARQ