PROJECT 3: MATCHED FILTERING

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CSE4214: Digital Communications (Fall 2013)
Computer Science and Engineering, York University

1. PURPOSE

In this lab, you will design optimum filters that maximize the output signal-to-noise ratio (SNR). Referred to as matched filters, such filters are used in many applications, including: (i) Digital communications to detect the presence of one of the two signals representing bits 0 and 1 for binary transmission; (ii) Radar, in which a known signal is sent out, and the signal reflected from the target (backscatter) is examined for unknown elements (range, velocity, and direction) attributed to the target, and; (iii) Image processing to improve SNR for medical images such as X-rays, MRIs, and CT scans. Though several different implementations of matched filters for different scenarios have been designed, you will consider the case of detecting a known, deterministic signal in additive white Gaussian noise. In particular, you will implement the correlator and time reversal implementations of the matched filter.

You will complete four simulations in MATLAB and submit their solutions along with a soft copy of the code in the form of a report.

2. OBJECTIVES

By the end of this project, you will be able to:

1. Design the *correlator implementation* of the matched filter and test it in the Matlab simulation environment to detect transmitted bits in a binary communication system.
2. Design the *time reversal implementation* of the matched filter and test it in the Matlab simulation environment to detect transmitted bits in a binary communication system.
3. Compare the performance of the correlator and time-reversal implementations with each other.

3. REFERENCE

Bernard Sklar text: Sections 3.1 – 3.2. Pages 106 – 136.

4. BINARY SIGNAL DETECTION IN AWGN

In a binary communication system, binary data consisting of 0's and 1's are transmitted by means of two signal waveforms, say $s_0(t)$ and $s_1(t)$. Suppose that the data rate is specified at R bits per second. Then each bit is mapped into a corresponding signal waveform according to the rule

$$0 \longrightarrow s_0(t), \quad 0 \le t \le T \tag{1}$$

$$1 \longrightarrow s_1(t), \quad 0 \le t \le T \tag{2}$$

where $T = 1/R$ is defined as the bit time interval. We assume that the data bits 0 and 1 are equally probable, i.e., each occurs with a probability of 1/2, and are mutually statistically independent. The channel through which the signal is transmitted is assumed to corrupt the signal by the addition of noise, denoted by $n(t)$, which is a sample function of white Gaussian noise with power spectrum $N_0/2$ watts/Hz. Such a channel is called additive white Gaussian noise (AWGN) channel. Consequently, the received waveform is expressed as

$$r(t) = s_i(t) + n(t), \quad i = 0, 1, \quad 0 \leq t \leq T. \tag{3}$$

The task of the receiver is to determine whether a 0 or a 1 was transmitted after observing the received signal $r(t)$ in the interval $0 \leq t \leq T$. The receiver is normally designed to minimize the probability of error. Such a receiver is based on the optimum principle of mnatched filters.

In this project, we will consider the optimum receivers for transmitting binary information through an additive white Gaussian noise (AWGN). The receivers being considered in this project are the signal correlator and the time reversal implementation of the matched filter. We provide a brief introduction to the two receivers without going into the derivations, which can be seen in the text.

5. SIGNAL CORRELATOR IMPLEMENTATION

The signal correlator cross-correlates the received signal $r(t)$ with the two possible transmitted signal $s_0(t)$ and $s_1(t)$. In other words the signal correlator computes the two outputs

$$r_0(t) = \int_0^t r(\tau) s_0(\tau) d\tau \tag{4}$$

$$r_1(t) = \int_0^t r(\tau) s_1(\tau) d\tau \tag{5}$$

within the interval $0 \leq t \leq T$, samples the two outputs at $t = T$, and feeds the sampled outputs to the detector. For equiprobable signals, the detector decides in favour of the signal with the higher correlator output. Thus $s_0(t)$ is selected if $r_0(t) > r_1(t)$, and vice versa.

6. TIME-REVERSAL IMPLEMENTATION

Time reversal (TR) provides an alternative to the signal correlator for demodulating the received signal $r(t)$. A filter that is matched to the signal waveform $s(t)$ and is time-limited to $0 \leq t \leq T$, has an impulse response

$$h(t) = s(T - t), \quad 0 \leq t \leq T. \tag{6}$$

Consequently, the output of the matched filter for the two signals is given by

$$y_0(t) = r(t) \otimes h_0(t) \tag{7}$$

$$y_1(t) = r(t) \otimes h_1(t) \tag{8}$$

where $\otimes$ is the convolution operator and $\{h_0(t), h_1(t)\}$ are the TR implementations of the filter matched to $\{s_0(t), s_1(t)\}$ based on Eq. (6). The detector samples the two outputs $y_i(t)$ at $t = T$ and decides in favour of the signal that has a higher output. It may be noted that the matched filter output at the sampling instant $t = T$ is identical to the output of the signal correlator.

Problem 1. Suppose the two orthogonal signals shown in fig. 1 are used to transmit binary information through an AWGN channel. The received signal in each bit interval of duration T given by eq. (3), is sampled at a rate of $10/T$, i.e., at ten samples per bit interval. Hence, in discrete time and in the absence of noise, the signal waveform $s_0(t)$ with amplitude A is represented by ten samples $(A, A, \ldots A)$ and the signal waveform $s_1(t)$ is represented by ten samples $(A, A, A, A, A, -A, -A, -A, -A, -A)$. Consequently, the sampled version of the received sequence when $s_0(t)$ is transmitted, is

$$r_k = A + n_k, \quad k = 1, \ldots, 10 \tag{9}$$

and when $s_1(t)$ is transmitted, is

$$r_k = \begin{cases} A + n_k, & 1 \leq k \leq 5 \\ -A + n_k, & 6 \leq k \leq 10 \end{cases} \tag{10}$$

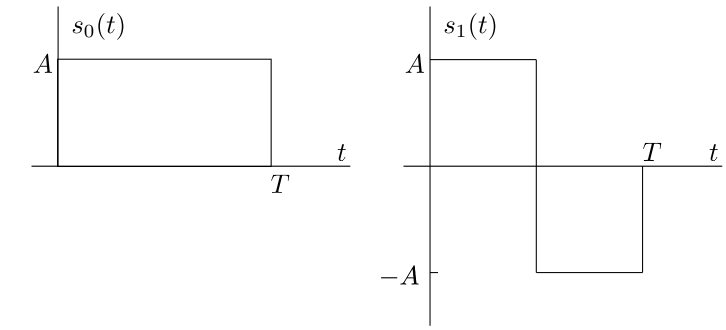

Figure 1: Orthogonal signal waveforms $s_0(t)$ and $s_1(t)$ for problem 1.

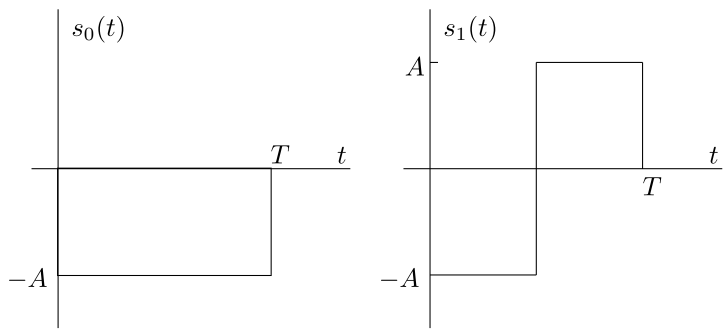

Figure 2: Orthogonal signal waveforms $s_0(t)$ and $s_1(t)$ for problem 2.

where the sequence $\{n_k\}$ is iid, zero mean, Gaussian with each random variable having the variance σ^2. Write a Matlab routine that generates the sequence $\{r_k\}$ for each of the possible received signals, and perform a discrete-time correlation of the sequence $\{r_k\}$ with each of the possible signals $s_0(t)$ and $s_1(t)$ represented by their samples versions for different values of the AWGN variance $\sigma^2 = 0$, $\sigma^2 = 0.1$, $\sigma^2 = 1$, and $\sigma^2 = 2$. The signal amplitude may be normalized to $A = 1$. Plot the correlator outputs at time instant $k = 1, 2, 3, \ldots 10$. Comment on the results.

Problem 2. Repeat problem 1 for the two signal waveforms $s_0(t)$ and $s_1(t)$ illustrated in fig. 2. Describe the similarities and differences between this set of two signals and those illustrated in fig. 1. Is one set better than the other from the viewpoint of transmitting a sequence of binary information signals.

Problem 3. In this problem, the objective is to substitute two matched filters in place of two correlators in problem 1. The condition for generating signals is identical to problem 1.

Write a Matlab function that generates the sequence $\{r_k\}$ for each of the two possible received signals, and perform the discrete-time matched filtering of the sequence $\{r_k\}$with each of the two possible signals $s_0(t)$ and $s_1(t)$, represented by their sampled versions, for different values of the AWGN variance $\sigma^2 = 0$, $\sigma^2 = 0.1$, $\sigma^2 = 1$, and $\sigma^2 = 2$. Plot the correlator outputs at time instants corresponsing to $k = 1, \ldots 10$.

Problem 4. Repeat problem 3 for the signal waveforms shown in fig. 2.

Summarize your observations and explain if and when the observations agree to your intuition. Explain where and why the two do not agree.