PROJECT 4: PHASE SHIFT KEYING

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1. PURPOSE

In this lab, you will design a bandpass communication system based on phase shift keying (PSK). Recall that the main reason for using bandpass modulation is to transform the frequency content of the information bearing signals into signals with more convenient, generally higher frequency, spectra. This provides four main advantages:

- Bandpass modulation allows for frequency division multiplexing such that a large number of signals can be transmitted simultaneously over the same communication chnanel.
- By selectively changing the frequency range of the transmitted signals, the available radio spectrum can be allocated to different services on a rational basis and regulated to reduce interference between different services.
- The transmitted signals can be matched to the characteristics of the communication channel.
- Since the length of the antenna is directly propoportional to the frequency content of the signal, the antennas used in bandpass communication are much more efficient and have reasonable dimensions.

You will complete four simulations in MATLAB and submit their solutions along with a soft copy of the code in the form of a report.

2. OBJECTIVES

By the end of this project, you will be able to:

1. Design the PSK *modulator* in Matlab as a representative bandpass communication system.
2. Design the PSK *demodulator* in Matlab and use it to detect the transmitted symbols.
3. Quantify the performance of the PSK communication system in presence of additive white Gaussian noise (AWGN) through Monte Carlo simulations and compare with the theoretical limits derived in the class.

3. REFERENCE

Bernard Sklar text: Sections 4.2 – 4.3 and 4.7. Pages 168 – 188 and 209 – 218.

4. INTRODUCTION:

In phase shift keying (PSK), the information that is transmitted over a communication channel is impressed on the phase of the carrier. Since the range of the carrier phase is $0 \leq \phi \leq 2\pi$, the phase values used to transmit digital information are

$$\phi_i(t) = \frac{2\pi i}{M} \quad \text{for} \quad i = 1, \ldots, M. \tag{1}$$

For binary transmission, the phase values used are therefore $\phi_1 = \pi$ and $\phi_2 = 2\pi$ or 0 radians. The general representation of a set of M carrier-phase-modulated signal waveforms is

$$s_i(t) \quad = \quad \sqrt{\frac{2E}{T}} \cos\big[\omega_o t - \frac{2\pi i}{M}\big], \quad i = 1, \ldots, M, \quad 0 \le t \le T \tag{2}$$

where ω_o is the carrier frequency in radians/s. Note that the transmitted signal $s_i(t)$ with duration of T has a constant envelope and the phase of the carrier changes abruptly at the beginning of each signal interval. Also, it can be shown that E is the energy present in the signal $s_i(t)$.

Problem 1: Using Matlab, generate the constant-envelope PSK signal waveforms $s_i(t)$ as given in (**??**) for $M = 8$. For convenience, the signal energy can be normalized to unity, i.e., $E/T = 1$, and the carrier frequency $\omega_o = 12\pi/T$ with $T = 1$s. Choose a sampling frequency of 30 samples/s.

In this project, we will perform a Monte Carlo simulation of M-ary BPSK communication system. Below we provide a brief description of the important concepts involved in the project.

5. SIGNAL CONSTELLATION

Signal constellations are geometric representations for signals in a vector space. Let us illustrate the concept by drawing the signal constellation diagram for Binary PSK (BPSK). The transmitted signal $s_i(t)$ takes the form

$$\text{Bit 1:} \qquad s_1(t) = \sqrt{\frac{2E}{T}} \cos\big[\omega_o t - \pi\big], \quad 0 \le t \le T \tag{3}$$

$$\text{Bit 0:} \qquad s_2(t) = \sqrt{\frac{2E}{T}} \cos\big[\omega_o t\big], \quad 0 \le t \le T. \tag{4}$$

For BPSK, the basis consists of a single function that is used to express both $s_1(t)$ and $s_2(t)$. The basis for BPSK is

$$\psi(t) \quad = \quad \sqrt{\frac{2}{T}} \cos(\omega_o t) \tag{5}$$

where coefficient $\sqrt{\frac{2}{T}}$ ensures unit energy in $\psi(t)$. Expressed in terms of $\psi(t)$, the BPSK signals $s_1(t)$ and $s_2(t)$ are

$$s_1(t) \quad = \quad -\sqrt{E}\,\psi(t) \tag{6}$$

$$s_2(t) \quad = \quad \sqrt{E}\,\psi(t) \tag{7}$$

The signal constellation for BPSK is obtained by representing the basis functions $\psi(t)$ as the x-axis and plotting the coefficients in equations (**??**–**??**) as coordinates with respect to the axis. For BPSK, we get the constellation shown in figure 1. Note that the vectors $\bar{s}_1$ and $\bar{s}_2$ with coordinates

$$\bar{s}_1 \quad = \quad (-\sqrt{E}, 0)$$

$$\bar{s}_2 \quad = \quad (\ \sqrt{E}, 0)$$

are vector representations for the signals $s_1(t)$ and $s_2(t)$ respectively. Any received signal $r(t)$ can be similarly represented by a vector $\bar{r}$ in the vector space illustrated in figure 1. A decision on which of the two signals $s_1(t)$ and $s_2(t)$ was transmitted given that $r(t)$ is received, is made by measuring the distance between the vectors $\bar{r}$ and $\bar{s}_1$, i.e., $||\bar{r} - \bar{s}_1||$, and the vectors $\bar{r}$ and $\bar{s}_2$, i.e., $||\bar{r} - \bar{s}_2||$. The signal corresponding to the smaller distance is selected. This is equivalent to finding the location of $\bar{r}$ in the signal space with respect to regions 1 and 2 as illustrated in figure 1. Signal $s_1(t)$ is selected as the transmitted signal if $\bar{r}$ lies in region 1 while signal $s_2(t)$ is selected if $\bar{r}$ lies in region 2.

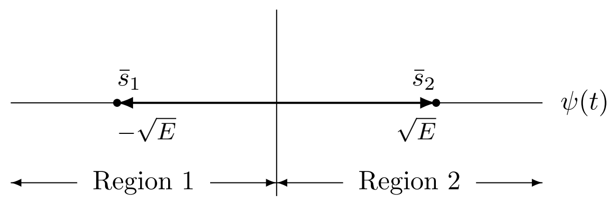

Figure 1: 2-signal point constellation for BPSK.

6. COHERENT DETECTION OF PSK

For typical coherent M-ary PSK (MPSK) systems, the transmitted signal $s_i(t)$ is given in (**??**), repeated below for convenience

$$s_i(t) \quad = \quad \sqrt{\frac{2E}{T}} \cos\left[\omega_o t - \frac{2\pi i}{M}\right], \quad i = 1, \ldots, M, \quad 0 \le t \le T. \tag{8}$$

which can be expanded as

$$s_i(t) \quad = \quad \sqrt{\frac{2E}{T}} \left(\cos(\omega_o t) \cos\left(\frac{2\pi i}{M}\right) + \sin(\omega_o t) \sin\left(\frac{2\pi i}{M}\right) \right) \quad i = 1, \ldots, M, \quad 0 \le t \le T. \tag{9}$$

The basis functions for (**??**) are therefore given by

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_o t) \qquad \text{and} \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin(\omega_o t) \tag{10}$$

where it can be proven that $\psi_1(t)$ and $\psi_2(t)$ is an orthonormal pair of functions.

Theoretical Problem 2: Show that the basis functions selected in (**??**) is an orthonormal pair of functions.

Theoretical Problem 3: The procedure for constructing the signal constellation (figure 1) can be extended to any M-ary shift keying scheme. Draw the signal constellation for an 8-ary PSK.

The optimum MPSK detector correlates the received signal $r(t)$ with the basis functions $\psi_1(t)$ and $\psi_2(t)$, plots the output of the correlators in the signal space, and selects the signal $s_i(t)$ depending on the region where the output of the correlator lies.

Theoretical Problem 4: In the signal constellation plot that you drew in problem 3, mark the decision regions for an 8-ary PSK communication system.

An alternative approach for coherent detection of PSK is illustrated in figure 4.12 of the Sklar text, where the correlators compute the terms X and Y

$$X = \int_0^T r(t)\psi_1(t)\, dt \qquad \text{and} \qquad Y = \int_0^T r(t)\psi_2(t)\, dt \tag{11}$$

and then derive the phase $\widehat{\phi}$ of the received signal $r(t)$ using

$$\widehat{\phi} = \arctan\left(\frac{Y}{X}\right) \tag{12}$$

and then selecting the signal $s_i(t)$ whose phase is closest to $\widehat{\phi}$. For $M = 4$, the decision criterion is:

$$\begin{array}{lrcccc} s_1(t): & \frac{\pi}{4} & \le & \widehat{\phi} & \le & \frac{3\pi}{4} \\ s_2(t): & \frac{3\pi}{4} & \le & \widehat{\phi} & \le & \frac{5\pi}{4} \\ s_3(t): & \frac{5\pi}{4} & \le & \widehat{\phi} & \le & \frac{7\pi}{4} \\ s_4(t): & \frac{-\pi}{4} & \le & \widehat{\phi} & \le & \frac{\pi}{4} \end{array} \tag{13}$$

7. PROBABILITY OF ERROR

The derivation of the expression for probability of bit error P_B at the detector for BPSK is given in the Sklar text. Here we reproduce the expression

$$P_B = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) \tag{14}$$

where E_b is the energy per bit and N_o is the power spectral density of AWGN at the input of the receiver. An expression for probability of symbol error P_E for M-ary PSK is difficult to evaluate but a good approximation is

$$\frac{P_B}{P_E} = \frac{M/2}{M-1}. \tag{15}$$

In this project, we will perform a Monte Carlo Simulation of an $M = 4$ PSK communication system that models the detector shown in figure 4.12 of the Sklar text. Each step in the Mote Carlo simulation is posed as a problem.

Problem 5: Write a function `myinput` that simulates the transmission of 1000 symbols using the random number generator. To accomplish this task, we use Matlab function `rand` to generate 1000 random numbers in the range (0,1). This range is divided into four equal intervals, (0, 0.25), (0.25, 0.5), (0.5, 0.75), and (0.75, 1) where the subintervals correspond to the symbols a_0, a_1, a_2, and a_3 respectively. If the random number is a boundary number, say 0.25, you have the discretion of selecting any of the two symbols a_0 or a_1. Save the sequence of transmitted symbols as you will be required to compare the sequence with the detected sequence at the receiver.

Problem 6: The symbols, a_i's, are used to select the signal $s_i(t)$. For each of the alphabets, a_i, $1 \leq i \leq 4$, write a function `mytransmit` that generates the constant-envelope PSK signal waveforms $s_i(t)$ as given in (**??**) but for $M = 4$. As in problem 1, the signal energy can be normalized to unity, i.e., $E/T = 1$. The carrier frequency $\omega_o = 6\pi/T$ with $T = 1$. Choose a sampling frequency of 1/30s. Plot the signals $s_i(t)$ for $i = 1, 2, 3$, and 4.

Problem 7: Using the functions `myinput` and `mytransmit`, generate the transmitted output waveform for the 1000 symbols a_i simulated in problem 5. Plot the waveform for the first 5 symbols.

Problem 8: Add AWGN with a PSD of $N_o = 2$ to the waveform generated in problem 7. This represents the received waveform. Plot the waveform for the first 5 received symbols and compare with your plots obtained in problem 7.

Problem 9: Correlate the received waveform with the basis functions $\psi_1(t)$ and $\psi_2(t)$ using expression (**??**). In other words, compute X and Y for each sampled alphabet. Based on the values of X and Y, detect the sequence of alphabets. Compare the detected sequence with the transmitted sequence and calculate the symbol-error rate P_E. Run the simulation (from problems 5 to 9) 3 to 4 times and average the symbol-error rate P_E obtained over different simulations.

Problem 10: Repeat problems 5 to 10 for different values of PSD N_o = 4, 8, 16, and 32. Plot the symbol-error rate P_E versus the E_b/N_o ratio. Compare with the theoretical values obtained from (14) and (15).