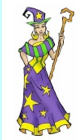
CSE4403 3.0 & CSE6002E - Soft Computing
Fall Semester, 2013



THE SMALL ASSIGNMENT (answers)

Logic & Sets

1. Determine the elements of the set A = {x | $x^2 = 11x - 30$ or $4 - x > 0$ when the universal set U is:

   a) the set of real numbers,
   A consists of all real numbers less than 4, together with the numbers 5 and 6.

   b) the set of rational numbers,
   A consists of all rational numbers less than 4, together with the numbers 5 and 6.

   c) the set of integers,
   A consists of all integers less than 4, together with the numbers 5 and 6.

   d) the set of positive integers,
   A = {1,2,3,5,6} (include 0 if you consider thispositiver)

   e) the set of negative integers,
   A = U

   f) the set of odd integers,
   A consists of all odd integers less than or equal to 5.

   g) the set of even integers,
   A consists of all even integers less than or equal to 6, except 4.

   h) the set of integers greater than 10,
   A = Θ

2. For each of the following, determine the relative complement $A - B$.

   a) A = {1, 4, 7, 10} B = {1, 2, 5}
   {4, 7, 10}

   b) A = {1, 2, 5} B = {1, 4, 7, 10}
   {2, 5}

   c) A = {a, e, i} B = {e, a, i}
   Θ

   d) A = {a,), 17} B = {()}
   {a, 17}

   e) A = {1, 5, 6, a} B = Θ
   A

   f) A = {2, 4} B = {1, 2, 4, 7}
   Θ

   g) A = Θ B = {a, b, 7}
   Θ

3. Determine all subsets of the set $A = \{0, a, \#, 2\}$

   Θ, {0}, {a}, {#}, {2}, {0, a}, {0, #}, {0, 2}, {a, #}, {a, 2}, {#, 2}, {0, a, #}, {0, a, 2}, {0, #, 2}, {a, #, 2}, *A*

4. How many subsets does a set of 10 elements have?

   2^{10} or 1024

5. Expand each of the following into well formed formulas.

   a) $p \equiv q \equiv r$

      $((p) \equiv (q)) \equiv (r)$

   b) $\sim p \lor r \equiv p \supset q \land \sim q \supset r$

      $((\sim p)) \lor (r)) \equiv (((p) \supset ((q) \land (\sim q)))) \supset (r))$

   c) $\sim p \lor r \equiv (p \supset q) \land (\sim q \supset r)$

      $((\sim p)) \lor (r)) \equiv (((p) \supset (q)) \land ((\sim q)) \supset (r)))$

   d) $\sim (p \land q \lor r) \supset p \equiv q$

      $((\sim(((p) \land (q)) \lor (r))) \supset (p)) \equiv (q)$

   e) $p \lor r \supset p \lor q \supset q \lor r$

      $(((p) \lor (r)) \supset ((p) \lor (q))) \supset ((q) \lor (r))$

6. Determine whether each of the following is (a) a term, (b) an at5omic formula, (c) a non-atomic wff, (d) none of the above

a)	$(x)\, A(x, y) \supset (D(y) \lor f(x, a)))$	*none of these*
b)	$f(x, a)$	*term*
c)	$(x)\,(y)\, D(z, a)$	*nonatomic wff*
d)	$A(a, b) \lor B(x, a)$	*nonatomic wff*
e)	$B(a, f(x, a))$	*atomic*
f)	$(x)\, C(x, y, a)$	*nonatomic wff*
g)	a	*term*
h)	$h(a)$	*term*
i)	$(x)\, A(f(x, y), b) \land B(x, a)$	*nonatomic wff*
j)	$A(x, a, y)$	*atomic*
k)	$G(x, f(x, y), a, g(a, b, x, y))$	*term*
l)	$x \equiv y$	*none of these*
m)	$D(a, b, c)$	*atomic*
n)	$F(x, A(x, y))$	*none of these*
o)	$(x)\, x$	*none of these*

p) *~(∃x) ~f(x)* *none of these*

q) *A(x) ⊃ ~ ~ (∃x) ~ B(x, y)* *nonatomic wff*

Fuzzy Logic & Sets

7. (a) Let **a** be a crisp number. We might define a truth value for the fuzzy statement "x <= a" as follows:

   value = 1 if x <= a (using the normal definition of <=)

   value = (1 + a - x) if a <= x <= a+1

   value = 0 if x > a+1

   Draw the truth function for the fuzzy statement "x <= 10"

   (b) Let **a** be a crisp number. We might define a truth value for the fuzzy statement "x >= a" as follows:

   value = 0 if x < a-1

   value = (1 + x - a) if a-1 <= x <= a

   value = 1 if x >= a

   Draw the truth function for the fuzzy statement "x >= 5"

   (c) Using the definitions just given, sketch the truth functions for each of the following fuzzy statements

   "x is <= 5 and x is >= 6"

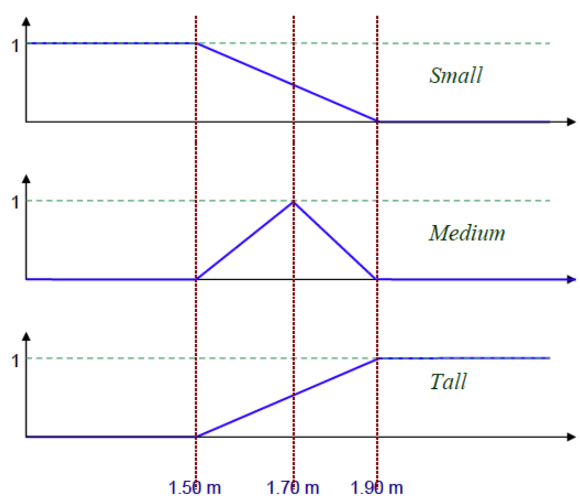
   "x is <= 5 or x is >= 6"

   "x is >= 2 and x is <= 4"

   "x is >= 2 or x is <= 4"

   "x is >= 4 and x is <= 4"

(d) Consider these (very subjective) membership functions for the length of a person:



Compute the graphical representation of the membership function of:

(1) $Small \cap Tall$ (2) $(Small \cup Medium) \cap Tall$

Rough Sets

8. Given a set of objects, OBJ, a set of object attributes, AT, a set of values, VAL, and a function f:OBJ x AT -> VAL, so that each object is described by the values of its attributes, we define an equivalence relation R(A), where A is a subset of AT:

   given two objects, o1 and o2,

   $o1\ R(A)\ o2 \Leftrightarrow f(o1,a) = f(o2, a), \forall a \text{ in } A$

   We say o1 and o2 are indiscernible (with respect to attributes in A). Now, we use this relation to partition the universe into equivalence classes, {e_0, e_1, e_2, ... , e_n} = R(A)*.

   The pair (OBJ, R) form an "approximation space" with which we approximate arbitrary subsets of OBJ referred to as "concepts". Given O, an arbitrary subset of OBJ, we can approximate O by a union of equivalence classes:

   the LOWER approximation of O (also known as the POSITIVE region):

   $LOWER(O) = POS(O) \Leftrightarrow \forall [o_R] \subseteq O\}$

   the UPPER approximation of O:

   $UPPER(O) \Leftrightarrow \forall [o_R] \cap O \neq \Theta$

   NEG(O) = OBJ - POS(O)

   BND(O) = UPPER(O) - LOWER(O) (boundary)

   There are several versions of the exact definition of a rough set (unfortunately), the most common is that a roughly definable set is a set, O, such that BND(O) is non-empty. So a rough set is a set defined only by its lower and upper approximation. A set, O, whose boundary is empty is exactly definable.

   If a subset of attributes, A, is sufficient to create a partition R(A)* which exactly defines O, then we say that A is a "reduct". The intersection of all reducts is known as the "core".

This is the simplest model. There are several probabilistic versions. Many researchers have used rough set theory for inductive learning systems, generating rules of the form:

description(POS(O)) →positive decision class
description(NEG(O)) →negative decision class
description(BND(O)) ~~> (probabilistically) positive decision class

Given the sample table below

Name	Education	Decision (Good Job Prospects)
Joe	High School	No
Mary	High School	Yes
Peter	Elementary	No
Paul	University	Yes
Cathy	Doctorate	Yes

So, the set of positive examples of people with good job prospects:

O = {Mary, Paul, Cathy}

The set of attributes:

A = AT = {Education}

The equivalence classes:

R(A)* = {{Joe, Mary}, {Peter}, {Paul}, {Cathy}}

The lower approximation and positive region:

POS(O) = LOWER(O) = {Paul, Cathy}

The negative region:

NEG(O) = {Peter}

The boundary region:

BND(O) = {Joe, Mary}

The upper approximation:

UPPER(O) = POS(O) + BND(O) = {Paul, Cathy, Joe, Mary}

As an aside, decision rules we can derive:

des(POS(O)) --> Yes
des(NEG(O)) --> No
des(BND(O)) ~~> Yes (equivalently ~~> No)

That is:

(Education, University) or (Education, Doctorate) →Good prospects
(Education, Elementary) →No good prospects
(Education, High School) ~~> Good prospects (i.e., possibly)