Lecture 12

Bayesian Networks

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Graphs

- A graph is a pair $G = <V, E>$
  - $V = \{v_i \mid 0 \leq i < n, n > 0\}$ denotes a set of nodes (vertices)
  - $E = \{(u, v) \mid u, v \in V, u \neq v\}$ denotes a set of edges
  - The edge is said to be directed if (u, v) is ordered with u directing to v, which can be written as $<u, v>$
- A graph is *directed* if all edges are directed
- A graph is *undirected* if every edge is undirected
- A graph is *hybrid* if some edges are directed some not

Graphs

- Two nodes u and v are *adjacent* if $(u, v) \in E$
- A *path* is a sequence of nodes such that each pair of consecutive nodes are adjacent
- In a *directed path,* other than the first and last nodes, each node is a head of one edge in the path and a tail of another
- If there is a directed path from u to v, then u is called an ancestor of v and v a descendant of u
- A path is a *cycle* if it contains two or more distinct nodes and the first node is identical to the last one

Graphs

- In a *directed cycle*, each node in the cycle is a head of one edge in the cycle and tail of another
- A graph is a *tree* if it contains no cycles
- A directed graph is a *directed acyclic graph* (DAG) if it contains no directed cycles

Conditional independent

- Let X, Y, and Z be disjoint sets of variables. X and Y are *conditionally independent* give Z, denoted $I(X, Z, Y)$, iff for every $x \in D_X$, $y \in D_Y$, $z \in D_Z$ such that $P(y, z) > 0$, the following holds:

$$P(x|y, z) = P(x|z)$$

When Z is empty, X and Y are marginally independent, denoted by $I(X, \varnothing, Y)$

Bayesian networks

- A Bayesian network is a triplet $(V, G, \mathcal{P})$. V is a set of variables, G is a connected DAG whose nodes correspond one-to-one to members of V such that each variable is conditionally independent of its non-descendants given its parents.
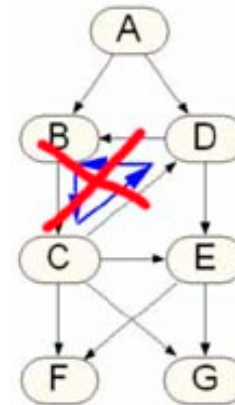
  Denote the parents of $v \in V$ in G by $\pi(v)$. $\mathcal{P}$ is a set of probability distributions:

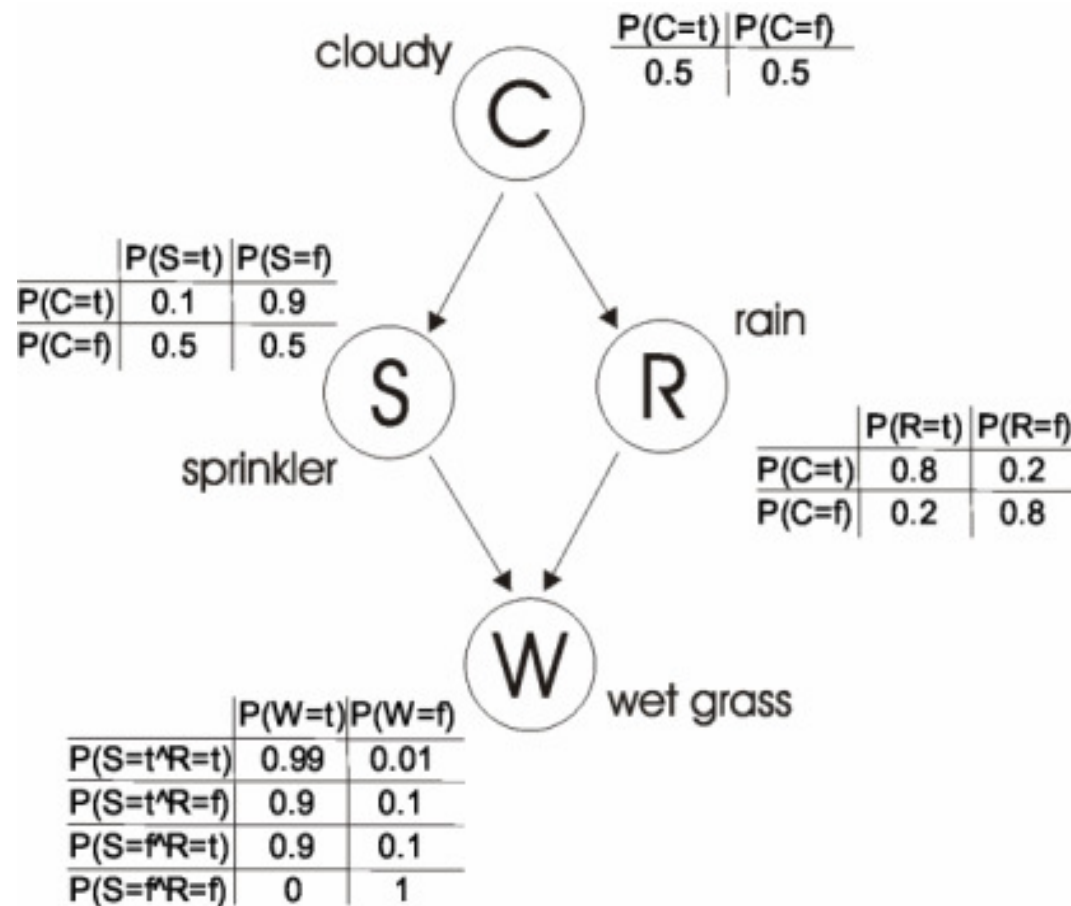$$\mathcal{P} = \{ P(v \mid \pi(v)) \mid v \in V \}.$$

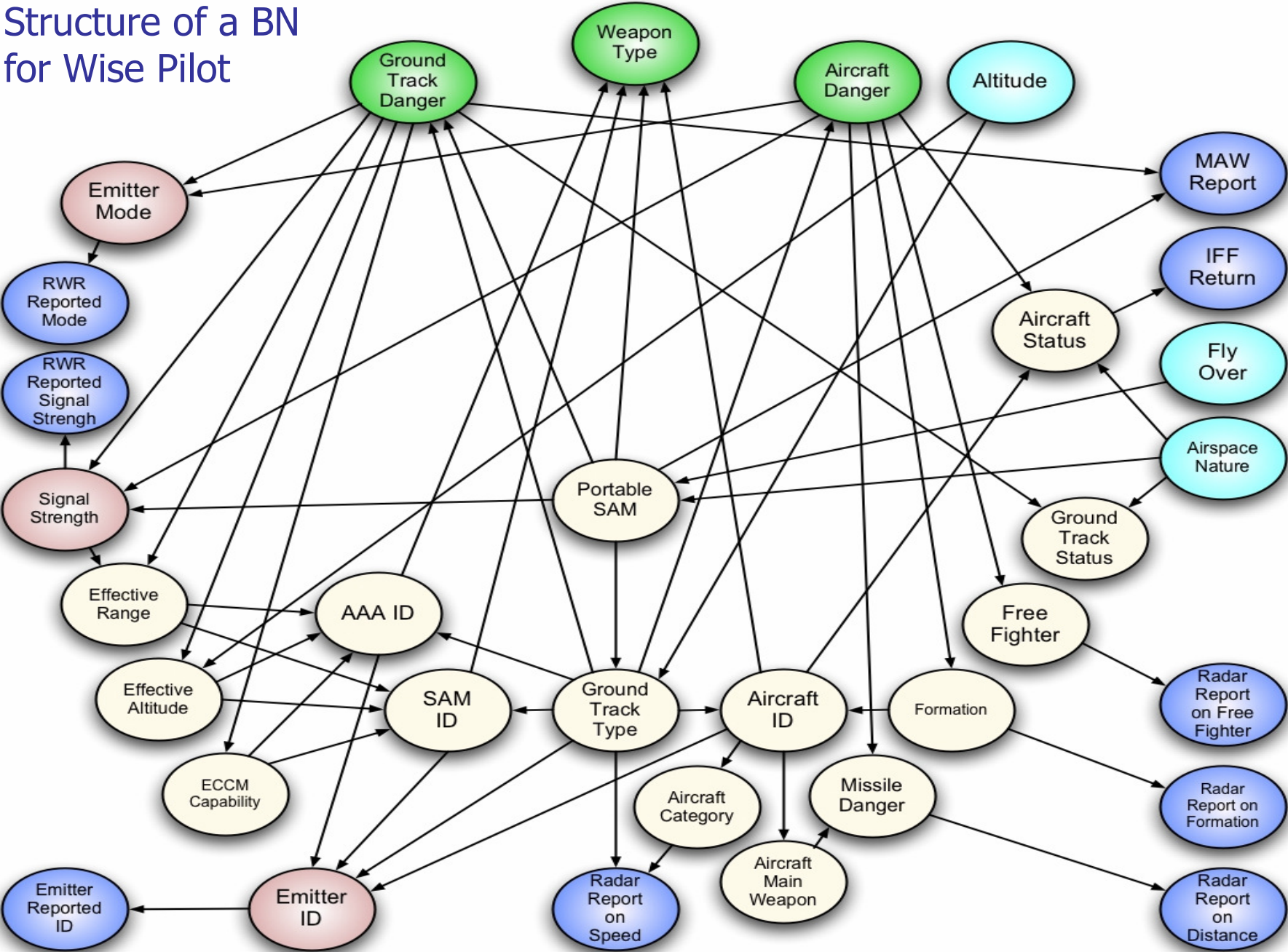# BN structures are DAGs
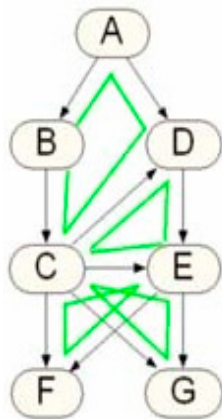
Without directed
cycles, is a DAG
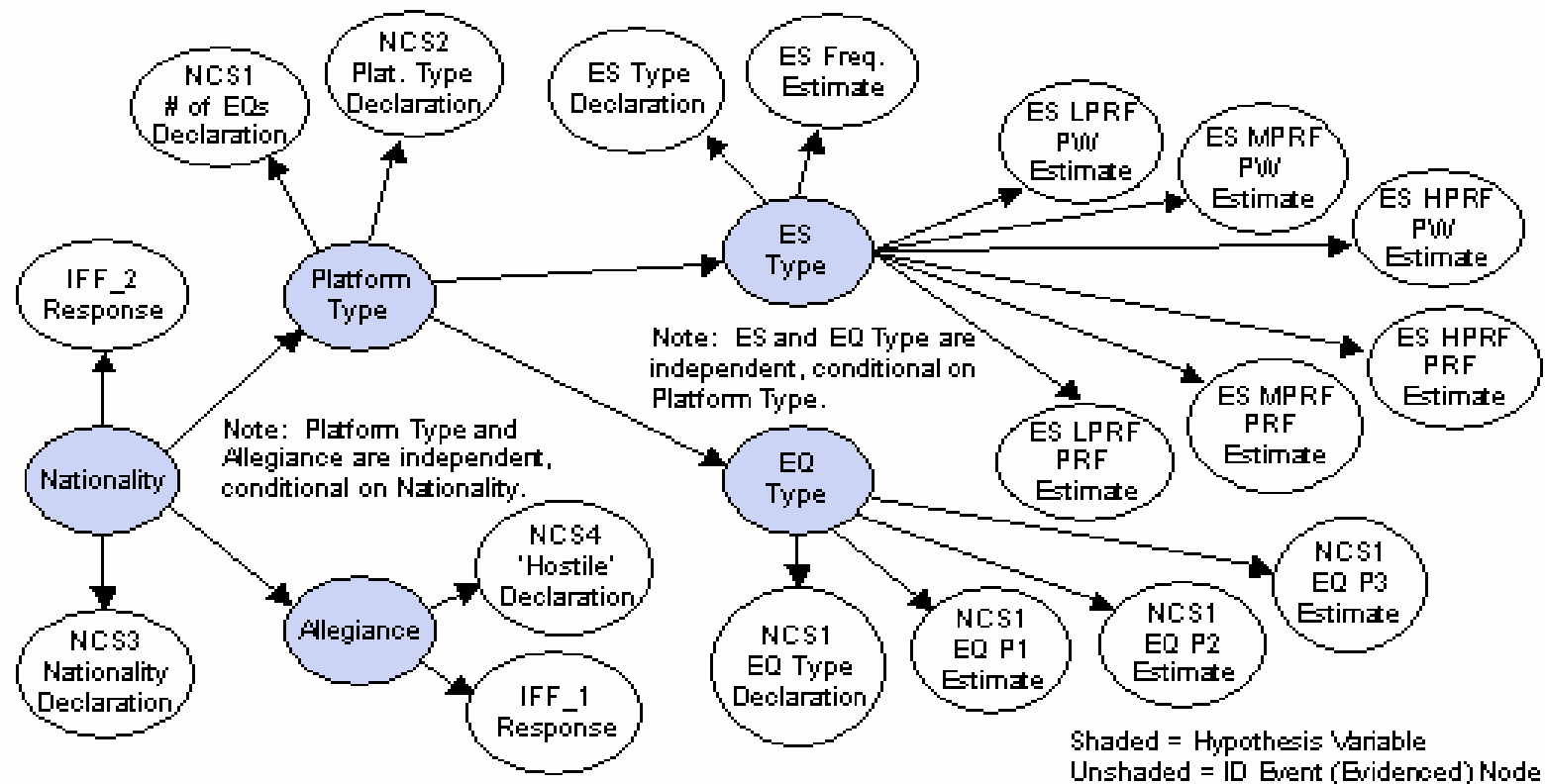
With a directed
cycle, not a DAG

A BN with parameters

cloudy

	P(C=t)	P(C=f)
	0.5	0.5

	P(S=t)	P(S=f)
P(C=t)	0.1	0.9
P(C=f)	0.5	0.5

sprinkler

rain

	P(R=t)	P(R=f)
P(C=t)	0.8	0.2
P(C=f)	0.2	0.8

wet grass

	P(W=t)	P(W=f)
P(S=t^R=t)	0.99	0.01
P(S=t^R=f)	0.9	0.1
P(S=f^R=t)	0.9	0.1
P(S=f^R=f)	0	1

Structure of a BN for Wise Pilot

BN structure for Combat Air Identification Fusion

A diagnosing BN, called asia
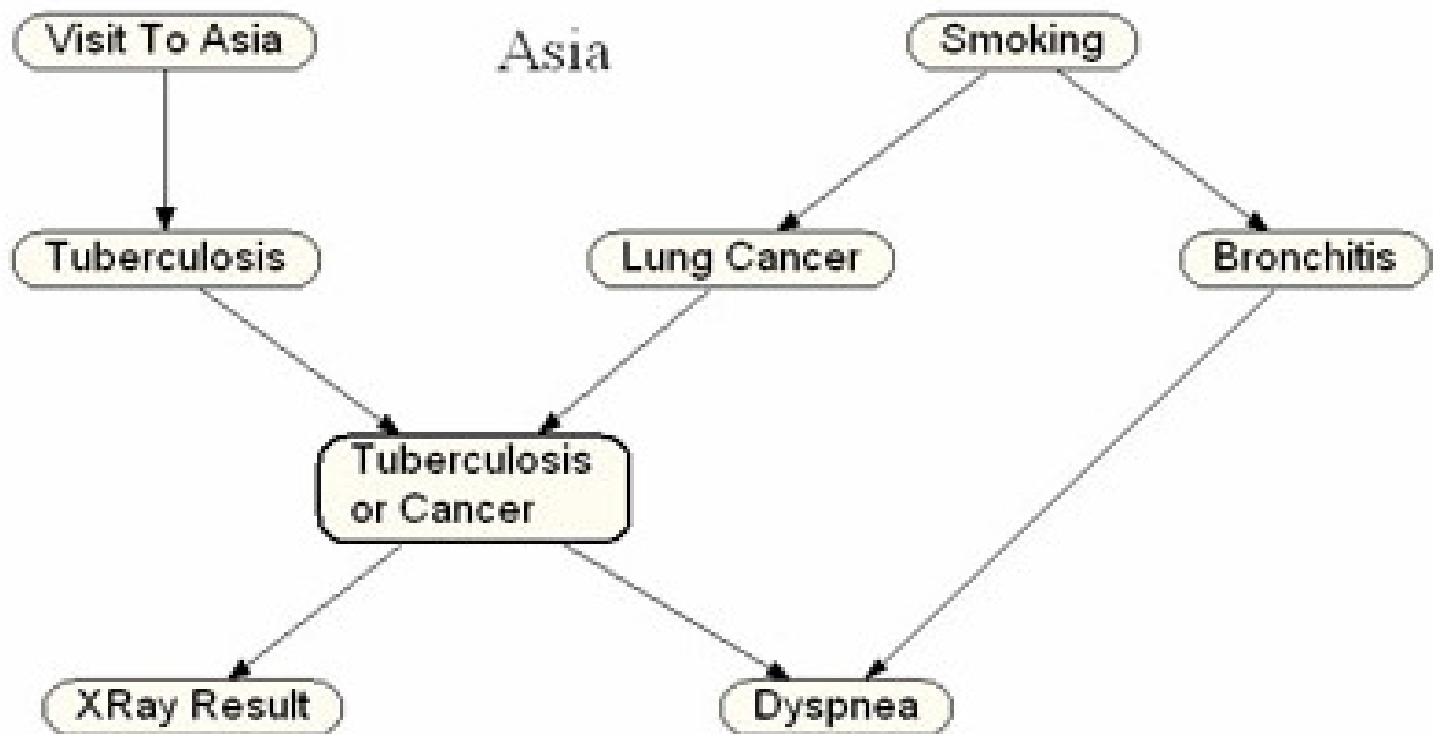
Inference with general population data
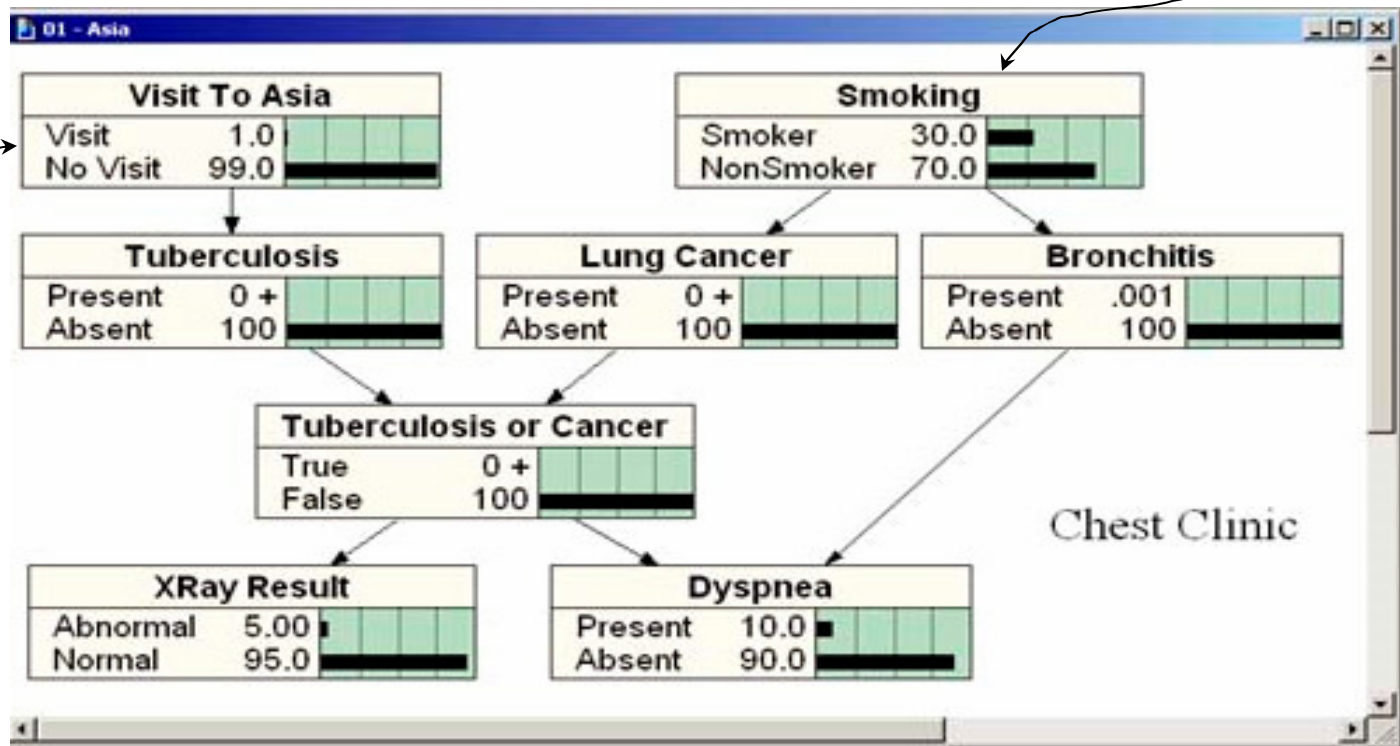
Marginal distributions

Inference with clinic data
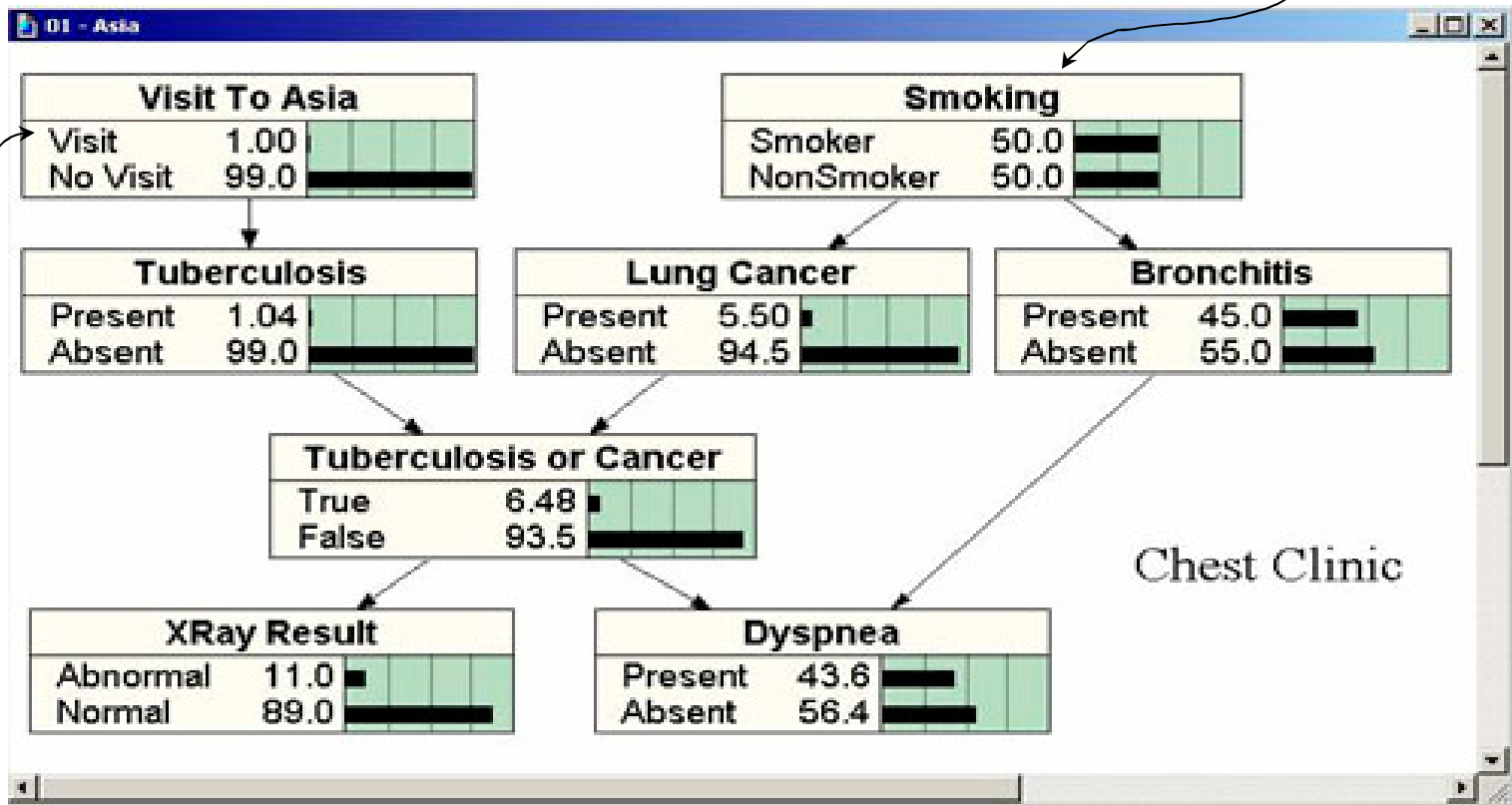
Marginal distributions

Entering evidence (finding)

14

Another finding

Visit to asia confirmed
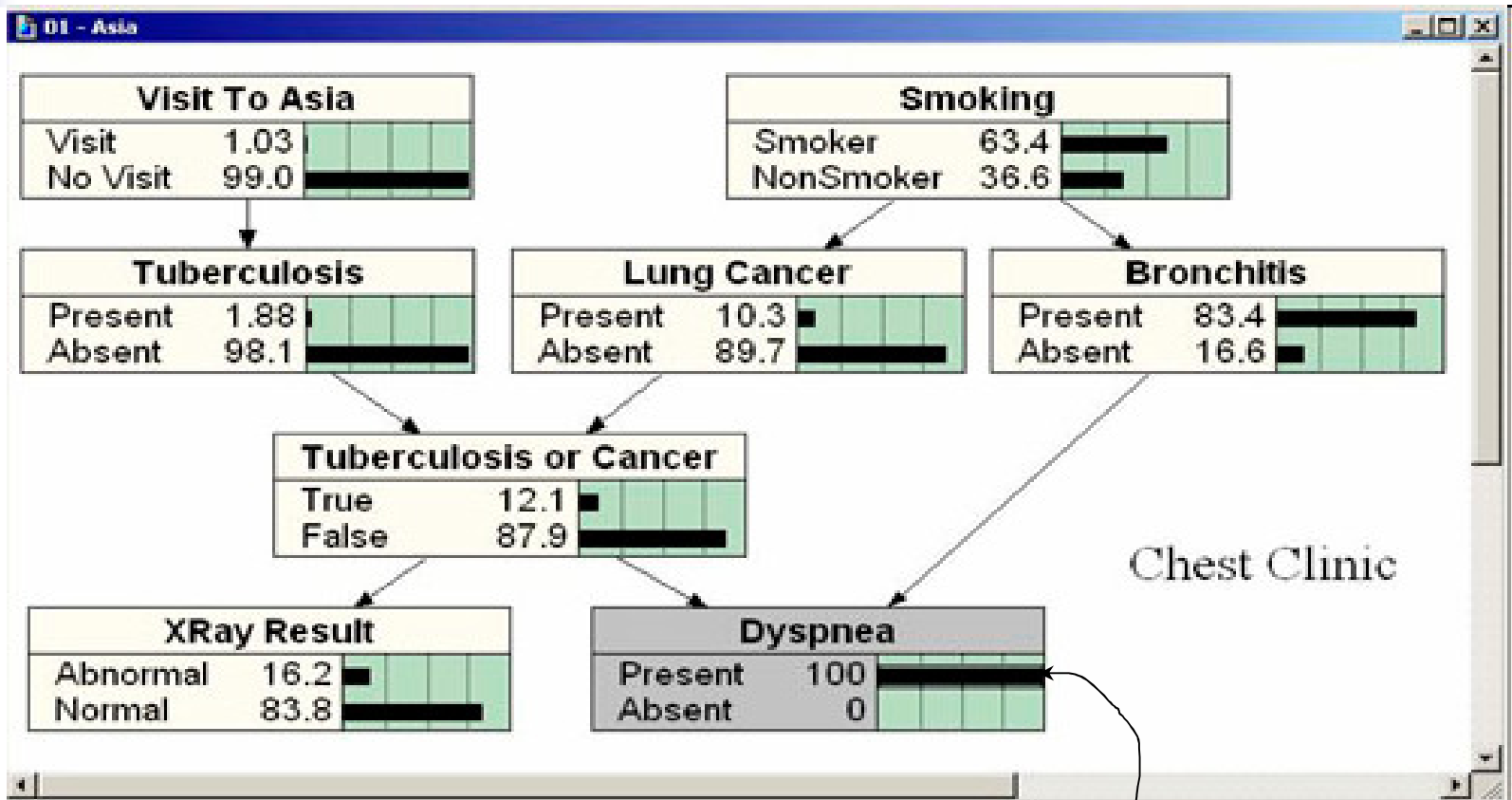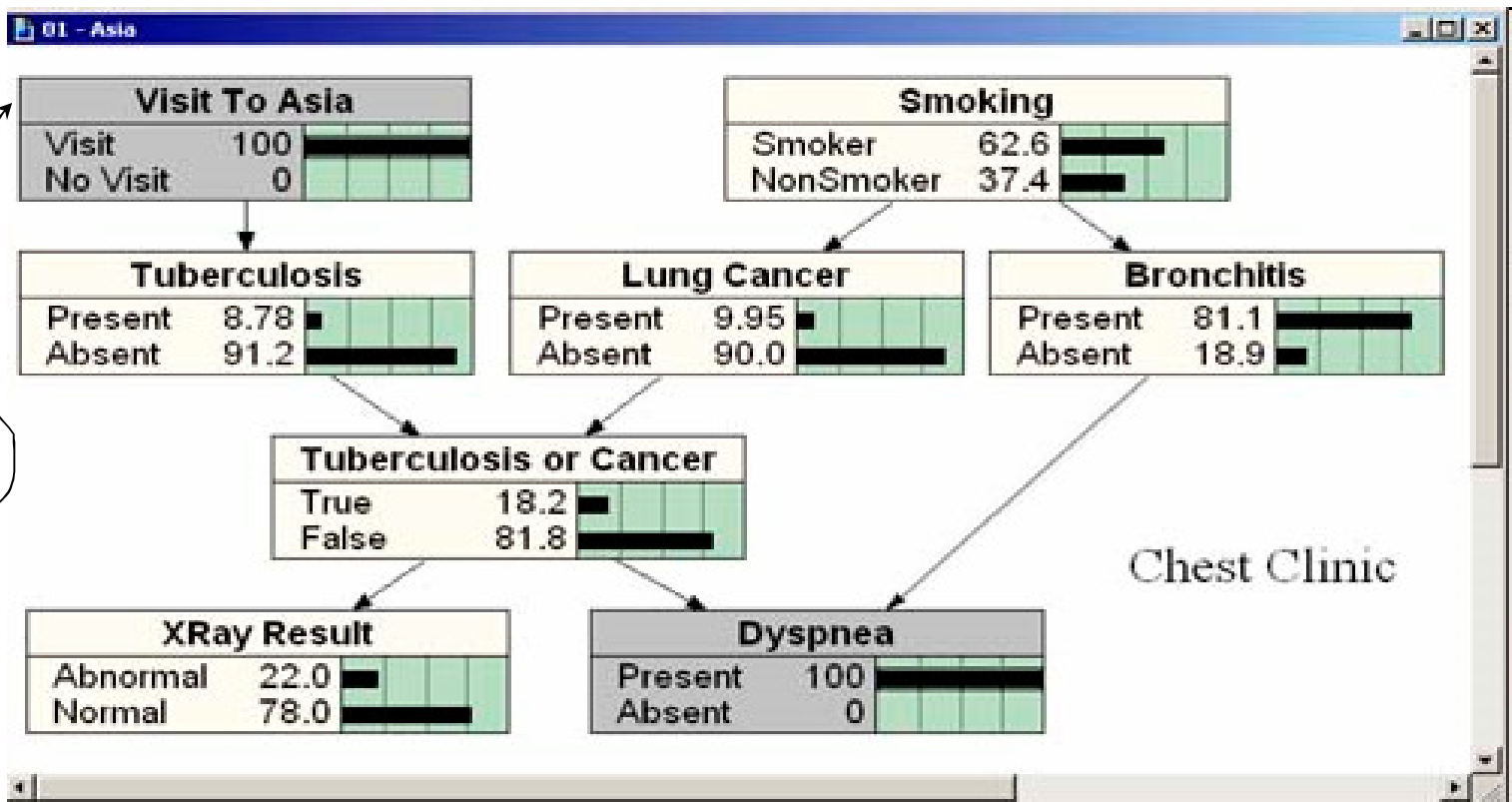
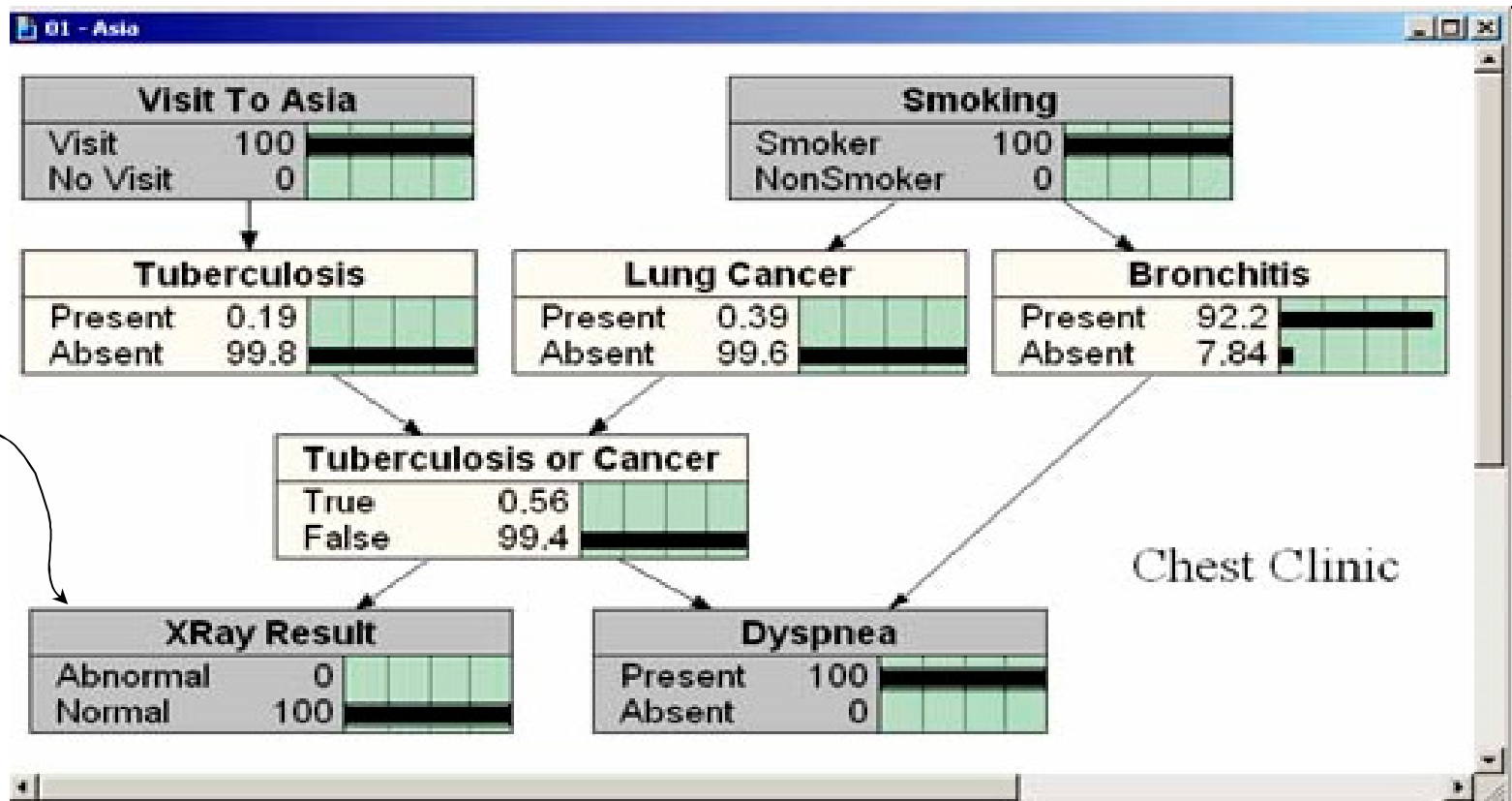# One more finding

Smoking is confirmed

One more finding

XRay result comes

Chest Clinic

17

What if xray is abnormal?