Granular Computing for Data Mining

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ABSTRACT

Granular computing, as an emerging research field, provides a conceptual framework for studying many issues in data mining. This paper examines some of those issues, including data and knowledge representation and processing. It is demonstrated that one of the fundamental tasks of data mining is searching for the right level of granularity in data and knowledge representation.

1. INTRODUCTION

As an emerging field of study, Granular Computing (GrC) is both new and old. On the one hand, the term "granular computing" was first suggested in 1997.[32, 93] On the other hand, the ideas and principles of granular computing have been studied under various names in many different fields, such as diakoptics, divide and conquer, structured programming, interval analysis, quantization, rough set theory, Dempster-Shafer theory of belief functions, chunking, cluster analysis, machine learning, data analysis and data mining, databases, and many others.[4, 19, 38, 76, 81, 82, 84, 92] In the past few years, we have witnessed a rapid development of, and a fast growing interest in the topic.[4, 22, 34, 38–40, 50, 51, 53, 54, 61, 63, 70, 73, 74, 86, 90, 99] Many models and methods of granular computing have been proposed and studied. The results enhance our understanding of granular computing.

The concept of granular computing has been defined and studied by many authors from different points of views, using different notions, based on different conceptual models, and in different contexts.[4, 19, 34, 38, 50, 63, 92] Although a concise and precise definition of granular computing is desirable, any such a definition may unnecessarily limit its scope, generality and potential. For the time being, it is suffice to rely on our intuitive interpretation. Broadly speaking, granular computing may be considered as a label of a new field of multi-disciplinary study, dealing with theories, methodologies, techniques, and tools that make use of granules in the process of problem solving.[76]

While concrete granular computing models have been proposed, there is still a lack of a well-accepted framework. It is evident that we must describe and study granular computing from many perspectives, in a wider context, and independent of any particular problem domain.[81, 82, 84] The results from recent studies seem to converge to a view that granular computing provides a common unifying, conceptual framework for modeling human thinking and problem-solving, as shown by the following quote:[84]

> Granular computing, in our view, attempts to extract the commonalities from existing fields to establish a set of generally applicable principles, to synthesize their results into an integrated whole, and to connect fragmentary studies in a unified framework. Granular computing at philosophical level concerns structured thinking, and at the application level concerns structured problem solving. While structured thinking provides guidelines and leads naturally to structured problem solving, structured problem solving implements the philosophy of structured thinking.

More specifically, granular computing is a multi-disciplinary study with the objectives to investigate and model a way of thinking, a family of granule-oriented problem solving methods, and a paradigm of information processing.[4, 76, 82, 84] It is a study of a general theory of problem solving based on different levels of granularity and detail.[20, 76, 82, 92, 94, 95]

If granular computing is to be accepted as a general theory of problem-solving, one must demonstrate its potentials in understanding, modeling, and solving many concrete real world problems. Many authors have indeed applied ideas of granular computing to reexamine many classic problems, in order to obtain new understandings and more insights.[4, 19, 21, 22, 51, 70] Data mining is one of such problems explored by some researchers. This paper is another attempt to establish a basis for data mining based on granular computing. It serves dual purposes: demonstrating the potential of granular computing on one hand and exploring a new perspective of data mining on the other.

Mjolsness and DeCoste suggested that machine learning can support scientists at every stage of the scientific process.[45] Yao and Zhao observed that machine learning, data mining and scientific research are much in common in terms of their goals, tasks, processes and methodologies.[87] The support is therefore in two ways, one can benefit from the other. The modeling of data mining in terms of granular computing may also be easily applied to a wider context of scientific research as a human problem-solving activity.

The rest of this paper is organized as follows. Section 2 presents an overview of granular computing. Section 3 first briefly reviews a few typical examples of granular computing based studies on data mining, and then makes an attempt to establish a basis of data mining from the view point of granular computing. Finally, some additional remarks are given in Section 4.

2. OVERVIEW OF GRANULAR COMPUTING

The basic ideas and principles of granular computing are not entirely new and have indeed been investigated in many disciplines of social and natural sciences. It is unfortunate that they are examined in relatively isolated and independent ways, expressed in much domain dependent concepts and notions, buried in details and scattered in many places. The study of granular computing therefore aims at arriving at a new powerful philosophical view and a general problem-solving theory. They are referred to as structured thinking and structured problem-solving.[84] Through granular computing, we hope to obtain a more holistic view of science as an integrated whole, in contrast to fragmented views.[5, 10]

Broadly, granular computing can be studied based on the notions of representation and process, which were also used by Marr in the study of vision.[41] The representation concerns granules and their organizations in terms of levels, networks, and hierarchies. One focuses on common features and universally applicable principles for the understanding, description, organization, and formulation of various problems across many different disciplines. The process deals with (computational) methods that manipulate granules and granular structures. One focuses on practical and systemic methods of problem solving. Based on this simplified view, we list some fields and specific research areas where the ideas of granular computing have been investigated.

- Computational intelligence: The explicit study of granular computing starts within the computational intelligence community.[4, 34, 37, 38, 50, 53, 54, 61, 63, 77, 84, 90, 97] In 1979, Zadeh first introduced the notion of information granulation and suggested that fuzzy set theory may find potential applications in this respect.[91] Unfortunately, this notion did not receive much attention in more than 10 years. In 1982, Pawlak proposed the theory of rough sets,[48, 49] which in fact provides a concrete example of granular computing. To some extent, rough set theory makes more people realize the importance of the notion of granulation. For example, Lin,[33, 34] Pawlak,[50] Peters, Pawlak, Skowron,[53] Polkowski and Skowron,[54] Skowron and Stepaniuk,[63] and Yao[77] examined information granulation based on rough sets. In 1997, Zadeh revisited information granulation,[92] which led to a renewed interest. In the same year, Lin suggested the term "granular computing" to label this new and growing research field.[32, 93] Lin proposed a method for granular computing based on neighborhood systems.[34, 37, 38] Yao,[75] and Yao and Zhong[90] also examined a few granular computing methods with neighborhood systems.

  The above studies may be broadly characterized as a set-theoretic study of granular computing. Each granule is defined and represented as a (fuzzy) set, and the granular structure is a family of (fuzzy) sets.

  Additional studies of granular computing, within the context of computational intelligence, can be found in recently edited books,[22, 39, 51] a book by Bargiela and Pedrycz,[4] and conferences proceedings of *International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing* and *IEEE International Conference on Granular Computing*.

- Artificial intelligence: The ideas of granular computing have been investigated in artificial intelligence through the notions of granularity and abstraction. In fact, the notion of granules plays an important role in knowledge representation, searching, and reasoning. A few examples are given to illustrate the main ideas.

  Hobbs proposed a theory of granularity,[20] which is similar to the theory of rough sets in terms of formulation. The theory indeed captures some of the essential features of granular computing. That is, we perceive and represent the world under various grain sizes, and abstract only those things that serve our present interests. The ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility. This enables us to map the complexities of real world into computationally tractable simpler theories.

  Giunchigalia and Walsh proposed a theory of abstraction.[16] Like the conceptualization in levels of granularity, abstraction is a process for us to consider what is relevant and to ignore irrelevant details. Knoblock proposed a theory of hierarchical planning,[25] in which plans of different granularities are considered.

  Zhang and Zhang developed a quotient space theory of problem solving based on hierarchical description and representation of a problem.[94,95] The quotient space theory motivates us to view granular computing as a way of structured problem solving. The theory has been successfully used to study efficient state-space search as a general problem-solving method.

- The theory of hierarchy: The hierarchy theory focuses on the understanding and representation of complex systems using multiple level structures.[1,2,46,47,57–59,72] Hierarchical structure can be observed in many natural, artificial, and abstract systems. It reflects the orderness, control, and stability of such systems. One can conceptualize a complex system by discriminating entities, relations, processes and levels as the basic ingredients of a hierarchical structure. A hierarchy links the parts or components into a whole, and hence provides a multi-level and multi-resolution description of a system.

  The hierarchy theory reflects, to some degree, the philosophy of reductionism, where the understanding of a whole is decomposed into the understanding of its smaller parts. In spite of some criticisms, hierarchical analysis is one of the successful methods used in the investigation and understanding of complex systems. For example, social hierarchy is a well studied concept in many branches of social sciences.[23]

  Simon convincingly argued that hierarchical organization leads to efficient solutions.[59] The hierarchical organization explores the notion of loose coupling of parts and provides a practical model of a nearly-decomposable system. In the context of granular computing, this implies that we may search for a nearly-decomposable system from a web of granules.

  A main criticism for the reductionism-based approaches is that they do not consider complex relations and interactions between parts. In order to overcome such limitations, many researchers promote systems thinking, representing a shift from the parts to the whole.[6] That is, complex systems, such as living systems, are integrated wholes whose properties cannot be the properties of their smaller parts. Instead of using a simply hierarchy, one needs to adopt the notions of networks and a web.

  If we use a broader meaning for hierarchies, instead of the restricted mathematical notion defined by a partial ordering, it is possible to combine the theory of hierarchy and the systems thinking, as well as taking advantages of both. For example, although a complex system may be modeled as a web of entities, one can still investigate in different levels of details. It may also be useful to examine a web of sub-webs, where each sub-web can be viewed as a granule.

- Divide and conquer: The strategy of divide and conquer can be used to effectively solve many types of problems. It is also related to the philosophy of reductionism in the sense that a large problem is decomposed into a family of smaller problems, and the solution of the large problem is obtained by combining the solutions of smaller problems. Two example applications of the divide and conquer strategy are structured programming and diakoptics.

  The top-down structured programming is an effective technique to deal with the complex problem of programming. The principles and characteristics of the top-down design and stepwise refinement, as discussed by Ledgard, Gueras and Nagin,[29] provide a good example demonstrating the ideas of granular computing. More specifically, the following issues are considered: (a) design in levels; (b) initial language independence; (c) Postponement of details to lower levels; (d) formalization of each level; (e) verification of each level; and (f) successive refinements. In a wider context, Foster studied algorithms, abstraction, and implementation in terms of levels of detail..[13] Since programming is a typical problem-solving problem, one can easily apply the same principles elsewhere. For example, it has also been suggested that the top-down approach is effective for developing, communicating and writing mathematical proofs.[14, 16, 28, 30]

Diakoptics may be viewed as system-tearing.[26, 27] By applying the strategy of divide and conquer, large systems with a large number of variables, such as electrical circuits, are torn into subdivisions; each subdivision is solved independent of others; and the partial solutions of subdivisions are integrated into a solution of the entire system.[27]

- The theory of small groups: Small group research is a field in psychology.[3] Its basic issues and methods are very relevant to granular computing, if we view a small group as a granule. Arrow, MaGrath and Berdahl developed a general theory of small groups as complex systems.[3] Groups are studied as adaptive, dynamic systems determined by three factors: (a) interaction among group members; (b) interaction between different groups; and (c) the embedding contexts of groups. Obviously, we need to study similar types of factors in granular computing.

  Many ideas from the small group research, as well as its research methodologies, can be readily applied to the study of granular computing. In the development of the general theory of small groups, Arrow, MaGrath and Berdahl established five propositions addressing the following fundamental issues:[3]

  – the nature of groups;
  – causal dynamics in groups;
  – group purposes or functions;
  – group composition and structure;
  – modes of group life.

  They are in fact basic issues we face in granular computing. The methodologies used by Arrow, MaGrath and Berdahl also have significant implications to the study of granular computing. They drew on a broad interdisciplinary foundation that seamlessly incorporates ideas and perspectives from general systems theory, social network theory, dynamical systems theory, and complexity theory. A general framework for granular computing may be similarly developed based on those theories and related ones mentioned in this paper.

- The memory-predication framework of intelligence: In the book *On Intelligence*, Hawkins used the notion of cortical hierarchies for deriving a memory-predication framework for explaining intelligence.[18] In his top-down approach for understanding the brain, a model of cortex is given by highlighting its hierarchical connectivity and information flow up and down the hierarchy. Their framework may have a significant impact on the study of granular computing.

  Granular computing based on hierarchies shares some basic elements with Hawkins' model: cortex regions corresponding to granules and hierarchical structures to granular structures, as well as different types of information processed at different levels of a hierarchy. A concrete model of granular computing may be established based on the memory-predication model. Hawkins' model provides further support for modeling granular computing based on the notion of hierarchies.

  Some additional topics related to memory, information processing, and granular computing, are hierarchical organization in memory and chunking.[24, 43] Through hierarchical organizing and chunking, experts can efficiently retrieve knowledge from memory.[11]

The above discussions are not intended to provide an exhausted list of theories and topics from which granular computing can be benefited. Instead, it only attempts to demonstrate by examples the universality of the ideas and principles of granular computing. Although the list is far away from being complete, the scopes, significance, the potential, and the vision of granular computing emerge immediately from a detailed study of those examples.

An underlying assumption of granular computing is that the basic principles and methodologies are common in most types of problem solving, independent of disciplines and problem domains. Granular computing, therefore, focuses on everyday and commonly used concepts and notions, such as granule, granulated view, granularity, and hierarchy. The notions of granular computing may be interpreted in terms of abstraction, generalization, clustering, levels of abstraction, levels of detail, and so on in various domains.

The basic ingredients and issues of granular computing are summarized below,[82, 84] with reference to the previously discussed theories and topics:

- Granule: A granule may be interpreted as one of the numerous small particles forming a larger unit. By considering a small group as a granule, we can draw results from the theory of small groups. We need to consider at least three basic properties of granules:
  - internal properties reflecting the interaction of elements inside a granule;
  - external properties revealing its interaction with other granules;
  - contextual properties showing the relative existence of a granule in a particular environment.

  A granule is treated both as a collection of individual elements characterized by its internal properties and as a whole characterized by its external properties. The existence of a granule is only meaningful in a certain context. Elements of a granule can be granules, and a granule can also be an element of another granule.

- Granular structures: Granular structures provide structured descriptions of a system or a problem under consideration. By combining ideas from systems thinking, complex systems theory, and theories and techniques of hierarchies, we can identify at least three levels of structure on a web of granules:
  - internal structure of a granule;
  - collective structure of a family of granules;
  - hierarchical structure of a web of granules.

  A collective structure of family of granules may be interpreted as a level or a granulated view in an overall hierarchical structure. Itself may be an inter-connected network of granules. For the same system or the same problem, many interpretations and descriptions may co-exist. Granular structures need to be modeled as multiple hierarchies and multiple levels in each hierarchy.[84]

- Granulation: Granulation involves the construction of the basic components of granular computing, namely, granules, granulated views, web of granules, and hierarchies. Issues involved are:[82]
  - granulation criteria;
  - granulation algorithms/methods;
  - representation/description of granules and granular structures;
  - qualitative/quantitative characterization of granule and granular structures.

- Computing with granules: Computationally, granular computing can solve a problem by systematically exploring the granular structures. This involves two-way communications upwards and downwards in a hierarchy and moving within a hierarchy. Some of the issues are:
  - mappings connecting granules and levels;
  - granularity conversion;
  - operators of computing;
  - property preservation or invariant properties.

Additional discussions and descriptions of a general framework of granular computing can be found in some recent papers.[76, 81, 82, 84]

In summary, granular computing is a multi-disciplinary study that draws ideas, principles, and perspectives from many fields. Its objective is to investigate and model a way of thinking, a family of granule-oriented problem solving methods, and a paradigm of information processing.[4, 76, 82, 84] It is a study of a general theory of problem solving based on different levels of granularity and detail.[20, 76, 82, 92, 94, 95] With a unified study under the umbrella of granular computing, there are many advantages. It is possible to see the connections between different disciplines with respect to their underlying principles, independent of subject matters. It is also possible to save the efforts of rediscovering old ideas in a new domain. Once the abstract ideas of structured thinking and structured problem-solving of granular computing are mastered, one can easily carry them over to any domain. A set of fundamental concrete ideas and principles of granular computing will also be valuable for solving various problems across many disciplines.

The potential of granular computing can perhaps be derived from the previously described new understanding and perception. This probably will bring more changes than any concrete model.

3. GRANULAR COMPUTING AS A BASIS FOR DATA MINING

A review of some existing studies points at the needs for a new framework of data mining based on granular computing.

3.1. Examples of granular computing based studies on data mining

Data mining aims at discovering knowledge embedded in data.[12] Rules are one of the most commonly used knowledge representation methods. Different types of rules can be studied based on their characteristics.[89] There are many studies on granular computing for data mining[19, 31, 35, 36, 44, 53, 54, 62, 66–68, 73, 74, 78, 80, 83, 88, 96–98, 100] We can examine some existing methods from several perspectives. For clarity, we restrict the discussion to rule mining.

- Rule representation/interpretation: A key notion of fuzzy set theory is linguistic variables. A fuzzy granule can be defined in terms of generalized constraints.[92] Fuzzy granules may be represented by words of a natural language. A rule summarizes a connection between two granules.[88] Consequently, we have a human friendly, natural interpretation of rules.[19, 44]

  As a concrete example of granular computing, rough set theory has been applied to data mining.[55] In this context, rules are expressed in terms of definable granules. Properties of rules can be interpreted and studied based on granules involved in the rules. For example, Skowron and Stepaniuk,[62] Peters, Pawlak and Skowron,[53] Polkowski and Skowron,[54] Tsumoto,[66, 67] Yao and Zhong,[88] Zhang *et al.*[96] and many authors interpreted rules based on properties of granules and inclusion relationships between granules.

- Rule mining: Granular computing techniques can be applied to rule mining. In order to mine more general or meaningful rules, one may group attribute values into granules, or a hierarchy of granules (i.e., a concept hierarchy[17]) by considering the semantic relationships between attribute values. For example, Zhong used granules of attribute values in rule mining.[98]

  Lin reformulated rule mining based on granular computing and proposed a machine-oriented modeling framework.[35, 36] A given attribute value is represented by the set of objects having the value, which in turn is coded as a bitstring. The mining process is then carried out through operations on bitstrings.

  By employing hierarchical structures, granular computing offers hierarchical interpretations of data. One can transform data into different levels of granularity. For example, Hirota and Pedrycz considered a pyramid architecture of data mining, in which different levels of granularity can be obtained by data transformation via linguistic granules.[19]

- Combination with other methods: Granular computing can be combined with other methods to produce new or more effective mining methods. In the context of computational intelligence, Hirota and Pedrycz pointed that neurocomputing, evolutionary computing, and granular computing (in particular, fuzzy and rough sets) can augment each other.[19] Many researchers have attempted to combine granular computing and other theories for data mining. For example, Zhang *et al.* considered the combination of granular computing and neurocomputing for data mining.[97]

The review of existing research suggests that although there exist a huge volume of and a great variety of studies, a conceptual framework is still missing. The rest of this section is therefore devoted to this problem.

3.2. A new framework

The new framework is developed from our previous study on modeling data mining with granular computing.[78, 80, 83] We will state explicitly the underlying assumptions of the framework and study their implications.

In building a granular computing based framework of data mining, we adopt the following assumptions:

- **Knowledge granule**: Each granule represents a piece of knowledge.
- **Structural knowledge**: Connections between a web of knowledge granules represents structural knowledge.
- **Mining task**: Searching for meaningful knowledge granules and structural knowledge is a basic task of data mining.

They are related to the basic components and task of granular computing: knowledge granules corresponding to granule, structural knowledge to granular structures, and mining task to granulation and computing with granules.

3.2.1. Knowledge granules

In order to treat a granule as a piece of knowledge, we need to represent or name a granule. For this purpose, we relate granules to the well studied notion of concepts.

Concepts are the basic units of thought that underlie human intelligence and communication. There are many theoretical views of concepts, concept formation and learning.[52, 64, 65, 69] The classical view treats concepts as entities with well-defined borderline and describable by sets of singly necessary and jointly sufficient conditions.[69] Other views include the prototype view, the exemplar view, the frame view, and the theory view.[69] Each view captures specific aspects of concepts, and has a different implication for concept formation and learning. The applications of different views for inductive data analysis have been addressed by many authors.[56, 65, 69]

We adopt the classical view of concepts, in which every concept is understood as a unit of thought that consists of two parts, the intension and the extension of the concept.[64, 65, 69] One can define a language so that the intension of a concept is expressed as a formula of the language and the extension is the set of objects satisfying the formula.[49, 83, 85] This enables us to study concepts in a logic setting in terms of intensions and also in a set-theoretic setting in terms of extensions.

For data mining, extensions of concepts are normally defined with respect to a particular training set of examples. In this case, we need to consider additional problems such as the definability of granules and the approximation of undefinable granules.[81] We also need to describe and study other properties of granules discussed in the last section.

3.2.2. Structural knowledge

Knowledge granules serves as building blocks to derive structural knowledge. Human knowledge is conceptual and forms an integrated whole. In characterizing human knowledge, one needs to consider two topics, namely, context and hierarchy.[52, 60] Structural knowledge can therefore be expressed in terms of connections between a web of knowledge granules.

The granular structures provide a plausible way to describe structural knowledge. Reasoning about intensions is based on logic.[65] For data mining, we need to derive relationships between the intensions of concepts based on the relations between the extensions of concepts. Through the connections between extensions of concepts, one may establish relationships between concepts.[78, 79]

The results of granular structures can be easily applied to the study of structural knowledge.

3.2.3. Mining task

Both knowledge granules and structural knowledge are useful. Conceptually, knowledge hidden in a dataset is embedded in the entire knowledge space defined by using only intensions of concepts. More often than not, it is only a small portion of the knowledge space. The mining task is therefore to search for the most suitable knowledge granules and structural knowledge.

The idea of learning as search suggested by Mitchell[42] is equally applicable here. In order to have practical algorithms, we need to have a search space with manageable size. For example, we can search in the space of conjunctively definable concepts.[83] We also need to study search heuristics.

For a given dataset, different types of knowledge may co-exist. Following our discussion of granular computing, we need to have a multiple view approach for data mining.[7] For example, two types of structural knowledge are formal concept lattices[15, 71] and hierarchical classes.[8, 9]

3.2.4. Discussions

By considering three basic issues, namely, knowledge granules, structural knowledge, and mining task, the new framework may provide a basis for data mining. Instead of dealing with irrelevant details, the framework offers a powerful conceptual view. It will not be a difficult job to interpret existing models of data mining in this framework.

As we have shown, once the basic concepts, ideas, and principles of granular computing are well understood, their applications to data mining seem to be very straightforward. Lin argued that data mining can be reduced to granular computing.[36] While his formulation is on the algorithm level, our discussion is on the conceptual level.

4. CONCLUSIONS AND REMARKS

A cross-disciplinary enquiry into human understanding and problem-solving leads to the emergence of granular computing. Although each field has its version of the problem-solving process, the basic way of thinking is shared across disciplines. Granular computing may be viewed as a study of such emergent properties from many disciplines. While each discipline may consider only certain aspects, the study of granular computing provides an integrated whole. At philosophical level, granular computing concerns structured thinking. Its implementation at the application level leads to structured problem solving. This powerful view enables us to establish a solid basis for data mining.

The application of granular computing for data mining illustrates two points. For one, granular computing is indeed a powerful view that can be used to model many problems. For the other, like many other fields, data mining follows the principles of granular computing.

The new framework of data mining focuses on high conceptual level issues by ignoring much irrelevant details. It brings more insights into data mining as a scientific field of study. The three fundamental assumptions, namely, (a) granules as knowledge granules, (b) granular structures as structural knowledge, and (c) mining as search, seem to be reasonable. Those assumptions are in fact implicitly used by many data mining models. In this paper, we in fact only presented an outline of the general framework. As future research, one needs to carefully examine the assumptions and claims with respect to a concrete model. Some preliminary work can be found in another paper.[83]

This paper draws results and perspectives from many fields. Although detailed discussions of each field are not provided in many cases, an extensive (yet non-exhaustive) list of references is given. In order to build a more coherent and complete framework of granular computing in general and data mining in specific, one needs to examine carefully results from other fields. This not only brings new insights, but also prevents rediscovering of old theories and techniques.

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