Theoretical Study of Granular Computing

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Abstract. We propose a higher order logic called as the granular logic. This logic is introduced as a tool for investigating properties of granular computing. In particular, constants of this logic are of the form $m(F)$, where F is a formula (e.g., Boolean combination of descriptors) in a given information system. Truth values of the granular formula are discussed. The truth value of a given formula in a given model is defined by a degree to which the meaning of this formula in the given model is close to the universe of objects. Our approach generalizes the rough truth concept introduced by Zdzisław Pawlak in 1987. We present an axiomatization of granular logic. The resolution reasoning in the axiomatic systems is illustrated by examples, and the resolution soundness is also proved.

Keywords: Granular logic, granular computing, closeness degree.

1 Introduction

Information granulations belong to a specific class of sets. Granulation is a collection of entities, arranged together due to their similarity, functional relativity, indiscernibility, coherency or alike. The properties of entities or relationships between entities can be described by meanings of logical formulas, hence information granulations may be considered as sets defined from formulas.

We propose a higher order logic, with two types of formulas: the individual and the set formulas. Constants may be of the form $m(F)$, where F is an individual formula. The meaning of constant $m(F)$ in an information system is the set of all objects satisfying F. Binary relational symbols with arguments of the set type are the inclusion to a degree $\subseteq_\lambda$ and the closeness to a degree CL_λ. In this paper we discuss mainly the set formula type in such granular logic. Granular logic may hopefully be a theoretical tool to study granular computing.

For computing the truth value of the set formulas in a model (e.g., defined by an information system), we use 1-*ary* functional symbol T with the following interpretation: The value of T on a given set of objects is equal to the degree of closeness of this set to the universe of objects. Pawlak introduced in 1987 the concept of rough truth [1], assuming that a formula is roughly true in a given information system if and only if the upper approximation of its meaning is equal to the whole universe. So, our approach extends Pawlak's approach in [1].

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The paper is organized as follows. In Section 2 we define the granular logic and its reasoning systems. In Section 3 we present some basic properties of granular logic. Section 4 presents a resolution reasoning in granular logic. Section 5 concludes the paper.

2 Granular Logic and Its Reasoning Systems

Zadeh proposed data granules in 1979 [2]. The data granule g is characterized by proposition of general form

$$g = (x \text{ is } G \text{ is } \lambda) \tag{1}$$

where x is a variable on U and the value of x belongs to the fuzzy subset $G \subseteq U$ to a degree at least λ, $0 \leq \lambda \leq 1$. Formally, g – as induced by x, G, and λ – is specified by

$$g = \{u \in U : v(x) = u, v \text{ is an assignment symbol on } U, u \in_{\lambda} G\} \tag{2}$$

From the viewpoint of fuzzy sets, we could also write $\in_G (e) \geq \lambda$ or $\mu_G(e) \geq \lambda$. From the viewpoint of fuzzy logic, λ approximates from below the truth value or probability of fuzzy proposition g.

Lin defined binary relational granulation from a viewpoint of neighborhood in 1998. Subsequently, he published many papers on granular computing [3 − 8]. Consider information system $IS = (U, A, V, f)$, where U is the universe of objects, A is a set of attributes, V is a set of attribute values, and f is the information function. Let $B : V \rightarrow U$ be a binary relation. The granulation defined by B is defined as follows:

$$g_p = \{u \in U : uBp\}, \text{ where } p \in V \tag{3}$$

Obviously, whether g_p is clear or vague depends on properties of B [7, 8].

In 2001, Skowron reported the information granules and granular computing. He called the meaning set of formula defined on information table an information granule corresponding to the formula, and introduced the concepts of syntax and semantics of the language L_{IS} defined on information systems IS [9 − 14].

In 2002, Yao studied granular computing using information tables [15 − 19]. In particular, Yao and Liu proposed a generalized decision logic based on interval-set-valued information tables in 1999 [19].

In $IS = (U, A, V, f)$, a_v, which can be denoted also as (a, v), is defined as a descriptor defined by $a(x) = v$, where v is the value of attribute a with respect to individual variable $x \in U$. Thus a_v is considered as a proposition in rough logic [21, 24]. The meaning set of a_v can be also formulated as

$$m(a_v) = \{x \in U : x \mid\approx_{IS} a_v\} \tag{4}$$

where $\mid\approx_{IS}$ is the symbol of satisfiability to a degree on IS. The granule is defined via propositional formula a_v in rough logic, so it is called elementary granular logical formula. If φ is the combination of descriptors a_v with regard to usual logical connectives $\neg$ (negative), $\vee$ (disjunctive), $\wedge$ (conjunctive), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence), then

$$m(\varphi) = \{x \in U : x \mid\approx_{IS} \varphi\} \tag{5}$$

is granular combination of $m(a_v)$ with regard to usual set operation symbols $\cup$ (union), $\cap$ (intersection), $-$ (complement). In this way we construct so called granular logic [1, 23, 24].

Example 1. Let $IS = (U, A, V, f)$ be an information system, $\varphi = a_3 \wedge c_0$ be a rough logical formula on IS. By the definition above, the granulation may be computed using the following information table.

$$m(\varphi) = m(a_3 \wedge c_0) = m(a_3) \cap m(c_0) = \{2, 3, 5\} \cap \{1, 2, 3, 4, 6\} = \{2, 3\} \tag{6}$$

Table 1. Information Table

U	a	b	c	d	e
1	5	4	0	1	0
2	3	4	0	2	1
3	3	4	0	2	2
4	0	2	0	1	2
5	3	2	1	2	2
6	5	2	1	1	0

2.1 Syntax and Semantics for Granular Logic

Definition 1. *(Syntax) The granular logic consists of granular formulas of the set formula type derived via atoms or their combination in rough logic on IS:*

1. *The descriptor of the form a_v is an atom in rough logic, thus $m(a_v)$ is defined as the elementary granular formula in granular logic;*
2. *Let $B \subseteq A$ be a subset of attributes. Any logical combination φ of atoms a_v, where $a \in B$, is the formula in rough logic, thus $m(\varphi)$ is the granular formula in granular logic;*
3. *If $m(\varphi)$ and $m(\psi)$ are granular formulas, then $m(\neg\varphi)$, $m(\varphi \vee \psi)$, $m(\varphi \wedge \psi)$ are also granular formulas;*
4. *The formulas defined via finite quotation $(1\!-\!3)$ are considered in the granular logic.*

Definition 2. *(Inclusion) Let φ and ψ be rough logical formulas on IS. The granular formula $m(\varphi)$ is included in granular formula $m(\psi)$ to degree at least λ. Formally:*

$$\subseteq_\lambda (m(\varphi), m(\psi)) = \begin{cases} Card(m(\varphi) \cap m(\psi))/Card(m(\varphi)) & m(\varphi) \neq \emptyset \\ 1 & m(\varphi) = \emptyset \end{cases} \tag{7}$$

Definition 3. *(Closeness) Let φ and ψ be rough logical formulas. The granulation $m(\varphi)$ is close to granulation $m(\psi)$ to degree at least λ. Formally, it is defined as follows:*

$$| T_{I_{IS}u_{IS}}(m(\varphi)) - T_{I_{IS}u_{IS}}(m(\psi)) | < 1 - \lambda \wedge m(\varphi) \subseteq_\lambda m(\psi) \wedge m(\psi) \subseteq_\lambda m(\varphi) \tag{8}$$

for short denoted by $CL_\lambda(m(\varphi), m(\psi))$*, where:*

1. CL_λ *is called* λ-closeness *relation, abbreviated by* $\sim_\lambda$*, to have* $\sim_\lambda (m(\varphi), m(\psi))$*,*
2. $T_{I_{IS}u_{IS}}$ *is the united assignment symbol defined by*

$$T_{I_{IS}u_{IS}}(m(\varphi)) = Card(m(\varphi))/Card(U) \tag{9}$$

where I_{IS} *is an interpretation symbol of set formula* $m(\varphi)$ *in a given information system* IS*, and* u_{IS} *is an evaluation symbol to individual variable in set formula in a given information system* IS *(to see* $[22-32]$*).*

Truth value of a formula in GL_{IS} is defined by the means of assignment model $T_{I_{IS}u_{IS}}(m(\varphi))$. So, satisfiability of granular logical formula means the formula is true or roughly true in the model.

Definition 4. *(Truth) For* $\varphi \in RL_{IS}$*, truth value of* $m(\varphi)$ *is the ratio of the number of elements in* U *satisfying* φ *to the total of objects in* U*.Truth value of granular formula in granular logic is defined as follows:*

1. *If* $\sim (m(\varphi), U) = 0$*, then truth value of* $m(\varphi)$ *is thought of as false in* IS*;*
2. *If* $\sim (m(\varphi), U) = 1$*, then truth value of* $m(\varphi)$ *is thought of as true in* IS*;*
3. *If* $\sim (m(\varphi), U) = \lambda$*, then truth value of* $m(\varphi)$ *is thought of as being true to degree at least* λ*, where* $0 \leq \lambda \leq 1$*.*

Definition 5. *(Semantics) Semantics of individual logical formula* φ *in a given information system is similar to usual logical formulas. The following discusses the meaning of the set formulas in a given information system, namely the value assignments to the constants, variables, functions and predicates occurring in the set formula* $m(\varphi)$*:*

1. *Each constant symbol* c *is interpreted as the set of an entity* $e \in U$*. That is* $m(\varphi) = I_{IS}(c) = \{e\}$*;*
2. *Each individual variable* x *is assigned the set of an entity* $e \in U$*. That is* $m(\varphi) = u_{IS}(x) = \{e\}$*;*
3. *Each n-tuple function symbol* π *is interpreted as a mapping from* U^n *to* U*, such that* $m(\varphi) = \{\overline{x} \in U^n : \pi(\overline{x}) = e\}$*;*
4. *Each n-tuple predicate symbol* P *is interpreted as an attribute – relation on* U *such that* $m(\varphi) = \{x \in U : x \mid\approx_{IS} P\}$ *.*

Let satisfiability model of granular formula $m(\varphi)$ in GL_{IS} be a five-tuple

$$M = (U, A, IR, VAL, m) \tag{10}$$

where:

- U is a set of entities. A is a set of attributes. Every attribute subset $B \subseteq A$ induces the indiscernibility relation on U.
- $IR = \{I_{IS}^1, \cdots, I_{IS}^h\}$ is the set of all interpretations on IS.

- $VAL = \{u_{IS}^1, \cdots, u_{IS}^t\}$ is the set of all evaluation symbols on IS.
- $u_{IS} \in VAL$ is to assign an entity to individual variable on U.
- m is to assign a granule/granulation to rough logical formula on IS.

Furthermore, for each $\varphi \in RL_{IS}$, the lower satisfiability, the upper satisfiability and satisfiability of granular logical formula $m(\varphi)$ with respect to interpretation $I_{IS} \in IR$ and evaluation $u_{IS} \in VAL$, are denoted, respectively, by

$$\begin{array}{c} M, u_{IS} \mid\approx_{L\varphi} \sim_\lambda (m(\varphi), U) \\ M, u_{IS} \mid\approx_{H\varphi} \sim_\lambda (m(\varphi), U) \\ M, u_{IS} \mid\approx_{m(\varphi)} \sim_\lambda (m(\varphi), U) \end{array} \tag{11}$$

Here, $L\varphi$ and $H\varphi$ are the lower and upper approximations of $m(\varphi)$, respectively [22, 32]. The meaning of the above types of satisfiability is $L\varphi \sim_\lambda U$, $H\varphi \sim_\lambda U$, and $m(\varphi) \sim_\lambda U$, respectively.

Definition 6. *(Operations) Let $m(\varphi)$ and $m(\psi)$ be two granular logical formulas, the operations of them with respect to usual logical connectives $\neg$, $\vee$, $\wedge$, $\rightarrow$ and $\leftrightarrow$ in the rough logical formula are defined as follows* [1, 21]*:*

1. $m(\neg\varphi) = U - m(\varphi)$;
2. $m(\varphi \vee \psi) = m(\varphi) \cup m(\psi)$;
3. $m(\varphi \wedge \psi) = m(\varphi) \cap m(\psi)$;
4. $m(\varphi \rightarrow \psi) = m(\neg\varphi) \cup m(\psi)$;
5. $m(\varphi \leftrightarrow \psi) = (m(\neg\varphi) \cup m(\psi)) \wedge (m(\neg\psi) \cup m(\varphi))$.

2.2 Axiomatics of Granular Logic

GA_1: Each axiom in the granular logical is derived from the corresponding axiom schema in classical logic.

GA_2: $m(a_v) \cap m(a_u) = \emptyset$, where $a \in A$, $v, u \in V_a$, and $v \neq u$.

GA_3: $\bigcup_{v \in V_a} m(a_v) = U$, for each $a \in A$.

GA_4: $\neg m(a_u) = \bigcup_{v \in V_a : v \neq u} m(a_v)$, for each $a \in A$.

GA_2–GA_4 are special axioms in the granular logic based on information systems.

2.3 Inference Rules

$G - MP$: If $|\sim m(\varphi) \subseteq_\lambda m(\psi)$ and $|\sim \sim_\lambda (m(\varphi), U)$, then $|\sim \sim_\lambda (m(\psi), U)$.

$G - UG$: If $|\sim \sim_\lambda (m(\varphi), U)$, then $|\sim \sim_\lambda ((\forall x) m(\varphi), U)$.

Where $|\sim$ is a reasoning symbol, to denote truth under degree at least $\lambda \in [0, 1]$.

3 Properties of Granular Logic

In this paper a granular logic based on rough logic in information systems is proposed and this granular logic is used as the tool for granular computing. The granulations derived by rough logical formulas are also called granular logical formulas. The operation rules of granular logic depend on usual logical connectives. Thus in the following we will discuss relative properties of granular logic.

Property 1. Identity:

$$|\!\sim (\forall x)(\sim_\lambda (m(x), m(x))); \tag{12}$$

Property 2. Symmetry:

$$|\!\sim (\forall x)(\forall y)(\sim_\lambda (m(x), m(y)) \rightarrow \sim_\lambda (m(y), m(x)); \tag{13}$$

Property 3. Transitive:

$$|\!\sim (\forall x)(\forall y)(\forall z)(\sim_\lambda (m(x), m(y)) \wedge \sim_\lambda (m(y), m(z)) \rightarrow \sim_\lambda (m(x), m(z))); \tag{14}$$

Property 4. Substitute:

$$|\!\sim (\forall x)(\forall y)(\sim_\lambda (m(x), m(y)) \rightarrow \sim_\lambda (m(P(x)), m(P(y)))); \tag{15}$$

Property 5. Forever True: For $\varphi \in RL$, where RL is the abbreviation of rough logic,

$$|\!\sim \sim_\lambda (m(\neg\varphi \vee \varphi), U); \tag{16}$$

It means that for arbitrary rough logical formula $\varphi \in RL$,$\neg\varphi \vee \varphi$ is forever true, so the granulation $(m(\neg\varphi \vee \varphi)$ is close to universe U ;

Property 6. Extension:

$$|\!\sim (\forall x)(\forall y)((\forall z)(\sim_\lambda (m(z \in x), m(z \in y)) \rightarrow \sim_\lambda (m(x), m(y))); \tag{17}$$

It means that a granule/granulation is defined by their elements.

Property 7. Right:

$$|\!\sim (\forall x)(\sim_\lambda (m((\exists y) y \in x), U) \rightarrow \sim_\lambda (m((\exists y) y \in x \wedge (\forall z)(z \in y \rightarrow \neg z \in x)), U); \tag{18}$$

For any granulation x, if $\exists y \in x$, then y is an object or a granule/granulation of object elements. If $z \in y$ for all z, then y is only granule/granulation. So x is the granule/granulation of granule/granulation y used as element, thus the elements in y cannot be used as any object element in x.

Property 8. Power set:

$$|\!\sim \sim_\lambda (m((\forall x)(\forall y)(\forall z)(z \in y \rightarrow z \subseteq x)), U); \tag{19}$$

For any granule/granulation x, $y = \rho(x)$ is the power set of x. For all z, if $z \in y$, then $z \subseteq x$.

Property 9. Choice axiom:

$$|\!\sim \sim_\lambda (m((\forall x)(x \neq \emptyset \rightarrow (\exists f)(\forall y)(y \in x \wedge y \neq \emptyset \rightarrow f(y) \in y))), U). \tag{20}$$

It means that for any granule/granulation $x \neq \emptyset$, there exists a function f, such that $\forall y \neq \emptyset$ and $y \in x$, then the functional value $f(y)$ on y is in y, that is, $f(y) \in y$.

4 Resolution Reasoning for Granular Logic

We discuss the reasoning technique called granular resolution. It is similar to the resolution of clauses in classical logic. This is because the resolution of complement ground literals in classical logic is false, which equals exactly to the intersection of two elementary granules corresponding to them is empty set.

Definition 7. *Let $\varphi \in RL_{IS}$, where RL_{IS} denotes rough logic defined for information system $IS = (U, A, V, f)$. If there is no free individual variable in φ, then the $m(\varphi)$ is called a ground granular formula in granular logic.*

Theorem 1. *For $\varphi \in RL_{IS}$, $m(\varphi)$ can be transformed equivalently into granular clause form $m(C_1) \cap \cdots \cap m(C_n)$, where each $m(C_i)$ is an elementary granule/granulation, which is the set of the form $m(a)$ or negation of $m(a)$, where $a \in A$ is an attribute on A.*

Definition 8. *Consider ground granular clauses $m(C_1)$ and $m(C_2)$ specified by $m(C_1) : m(C_1') \cup m(a)$ and $m(C_2) : m(C_2') \cup m(b)$. The resolvent of $m(C_1)$ and $m(C_2)$, $GR(m(C_1), m(C_2))$, is defined as follows: If the ground granular atoms $m(a)$ in $m(C_1)$ and $m(b)$ in $m(C_2)$ are a complement literal pair* [23, 25, 28] *in granular logic, then resolution of $m(C_1)$ and $m(C_2)$ is*

$$\frac{\begin{array}{c} C_1 : m(C_1') \cup m(a) \\ C_2 : m(C_2') \cup m(b) \end{array}}{C : m(C_1') \cup m(C_2')} \tag{21}$$

Namely, we have $GR(m(C_1), m(C_2)) = m(C_1') \cup m(C_2')$.

Example 2. Let $IS = (U, A, V, f)$ be an information system, as given in Section 2. One can construct an axiomatic system of granular logic based on IS, as defined in [25 – 32]. We extract formula $\varphi \in RL_{IS}$ as follows:

$$\varphi(a_5, b_2, b_4, c_0, \neg e_0) = (a_5 \vee b_4) \wedge b_2 \wedge (c_0 \vee \sim e_0) \tag{22}$$

Formula (22) may be written as the following granular logical formula:

$$\varphi(a_5, b_2, b_4, c_0, \neg e_0) = (m(a_5) \cup m(b_4)) \cap m(b_2) \cap (m(c_0) \cup m(\neg e_0)) \tag{23}$$

By Theorem 1, this is the granular clause form, where each intersection item is a granular clause. By Definition 6, the ground granular clause form of the granular formula is defined as follows:

$$\varphi(a_5, b_2, b_4, c_0, \neg e_0) = (a_5^{\{1,6\}} \cup b_4^{\{1,2,3\}}) \cap b_2^{\{4,5,6\}} \cap (c_0^{\{1,2,3,4\}} \cup \neg e_0^{\{2,3,4,5\}}) \tag{24}$$

where each item is a ground granular clause. Obviously, $a_5^{\{1,6\}}$ and $\neg e_0^{\{2,3,4,5\}}$ is a complement ground granular literal pair. So, the resolvent $GR(m(C_1), m(C_2))$ of $a_5^{\{1,6\}} \cup b_4^{\{1,2,3\}}$ in $m(C_1)$ and $c_0^{\{1,2,3,4\}} \cup \neg e_0^{\{2,3,4,5\}}$ in $m(C_2)$ is defined as follows:

$$\frac{\begin{array}{c} a_5^{\{1,6\}} \cup b_4^{\{1,2,3\}} \\ c_0^{\{1,2,3,4\}} \cup \neg e_0^{\{2,3,4,5\}} \end{array}}{b_4^{\{1,2,3\}} \cup c_0^{\{1,2,3,4\}}} \tag{25}$$

Hence, the form (3) can be rewritten as

$$(b_4^{\{1,2,3\}} \cup c_0^{\{1,2,3,4\}}) \cap b_2^{\{4,5,6\}} \tag{26}$$

Theorem 2. *Let $\triangle$ be a set of granular clauses. If there is a deduction of granular resolution of granular clause C from $\triangle$, then $\triangle$ implies logically C.*

Proof. It is finished by simple induction on length of the resolution deduction. For the deduction, we need only to show that any given resolution step is sound. Suppose that $m(C_1)$ and $m(C_2)$ are arbitrary two granular clauses at the step i, $m(C_1) = m(C_1') \cup m(a)$ and $m(C_2) = m(C_2') \cup m(b)$ where $m(C_1')$ and $m(C_2')$ are still granular clauses. Assuming that $m(C_1)$ and $m(C_2)$ are two correct granular clauses, $m(a)$ and $m(b)$ are complement granular literal pair at the step i, then $m(a)$ and $m(b)$ are resolved to produce a resolvent $GR(m(C_1), m(C_2))$, which is a new granular clause $m(C) : m(C_1') \cup m(C_2')$.

Now let us prove that $m(C)$ is also a correct granular clause. By Definition 7, two granular clauses joined in resolution are $m(C_1)$ and $m(C_2)$. If there are the complement granular literals $m(a)^{\{\}}$ in $m(C_1)$ and $m(b)^U$ in $m(C_2)$ respectively, then $m(C_1')$ is a correct granular clause, so the new granular clause $m(C) : m(C_1') \cup m(C_2')$ is correct; If there are $m(b)^{\{\}}$ in $m(C_2)$ and $m(a)^U$ in $m(C_1)$ respectively, then $m(C_2')$ is correct, so $m(C) : m(C_1') \cup m(C_2')$ is correct new granular clause.

The extracting of resolution step i could be arbitrary, the proof of the soundness of granular resolution deduction is finished.

5 Conclusion

In this paper, we define a granular logic and study its properties. The logic is axiomatized, to get the deductive system. We may prove many relationships between granulations in the axiomatic system of granular logic, so the granular logic may be derived from the formulas in a given information system and used in granular computing. Hence, this logic could be hopefully a theoretical tool of studying granular computing.

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