SC/CSE 3213 Winter 2014

L11: Digitization

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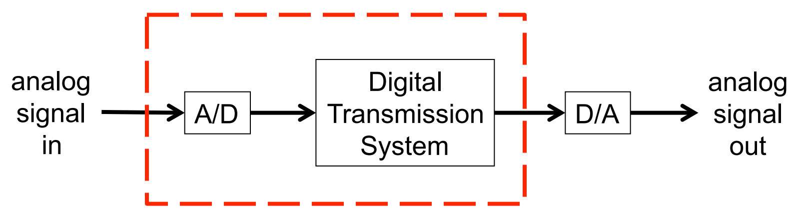
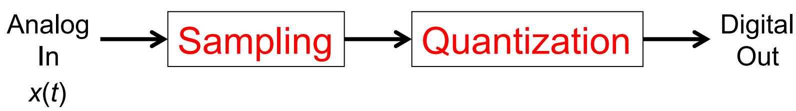
Outline



- How are analog signals digitized?
- How are digital signals transmitted/communicated?
  - Subject of following lecture
- Describe these mostly from the conceptual level

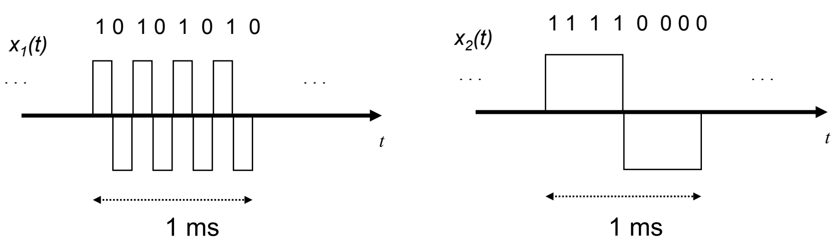
Digitization of Analog Signals

1. Sampling: obtain samples of $x(t)$ at discrete time intervals
2. Quantization: map each sample into an approximation value of finite precision
   – Pulse Code Modulation: telephone speech
   – CD audio



Sampling Rate and Bandwidth

- A signal that varies faster needs to be sampled more frequently
- Bandwidth measures how fast a signal varies



- What is the bandwidth of a signal?
- How is needed sampling rate related to bandwidth?

Periodic Signals

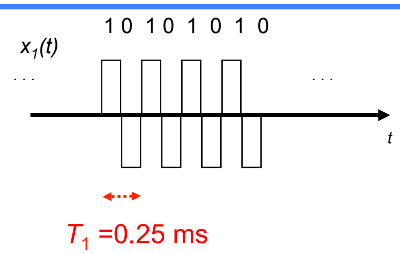
- A periodic signal with period T can be represented as a sum of sinusoids using Fourier Series:

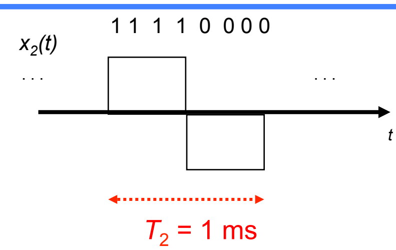
$$x(t) = a_0 + a_1\cos(2\pi f_0 t + \phi_1) + a_2\cos(2\pi 2 f_0 t + \phi_2) + \ldots + a_k\cos(2\pi k f_0 t + \phi_k) + \ldots$$

- a_0: "DC" long-term average
- $a_1\cos(2\pi f_0 t + \phi_1)$: fundamental frequency $f_0=1/T$ first harmonic
- $a_k\cos(2\pi k f_0 t + \phi_k)$: kth harmonic

- $|a_k|$ determines amount of power in kth harmonic
- Amplitude spectrum $|a_0|, |a_1|, |a_2|, \ldots$

Example Fourier Series





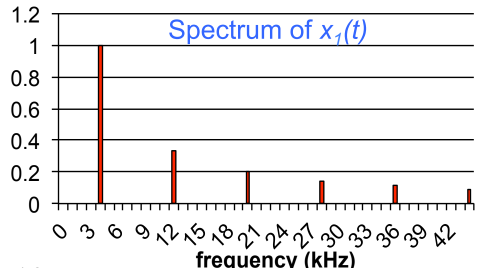
$$x_1(t) = 0 + \frac{4}{\pi}\cos(2\pi 4000 t) + \frac{4}{3\pi}\cos(2\pi 3(4000)t) + \frac{4}{5\pi}\cos(2\pi 5(4000)t) + \ldots$$

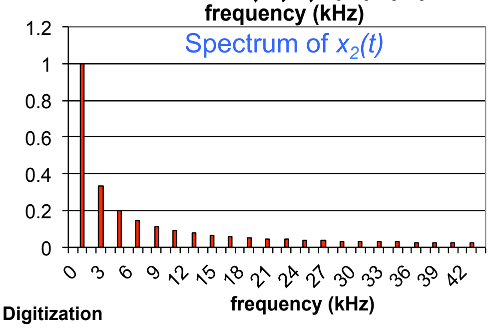
$$x_2(t) = 0 + \frac{4}{\pi}\cos(2\pi 1000 t) + \frac{4}{3\pi}\cos(2\pi 3(1000)t) + \frac{4}{5\pi}\cos(2\pi 5(1000)t) + \ldots$$

Only odd harmonics have power

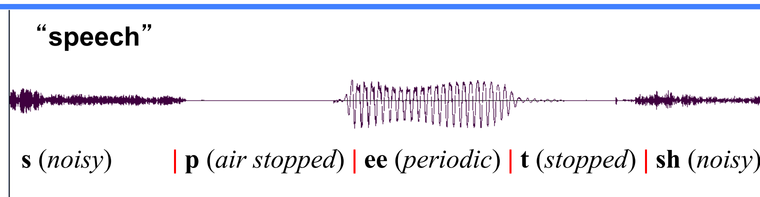
Spectra & Bandwidth

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- $x_1(t)$ varies faster in time & has bigger HF content than $x_2(t)$
- Bandwidth W_s is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power

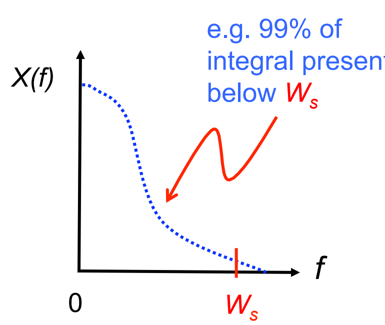




Bandwidth of General Signals

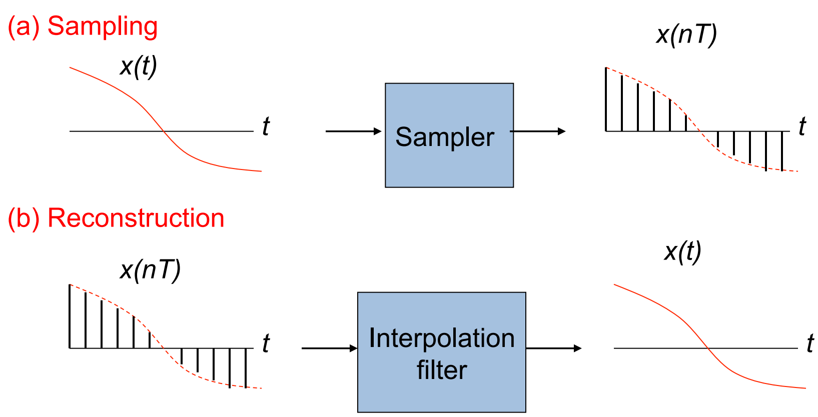


- Not all signals are periodic
  - E.g. voice signals varies according to sound
  - Vowels are periodic, "s" is noiselike
- Spectrum of long-term signal
  - Averages over many sounds, many speakers
  - Involves Fourier transform
- Telephone speech: 4 kHz
- CD Audio: 22 kHz

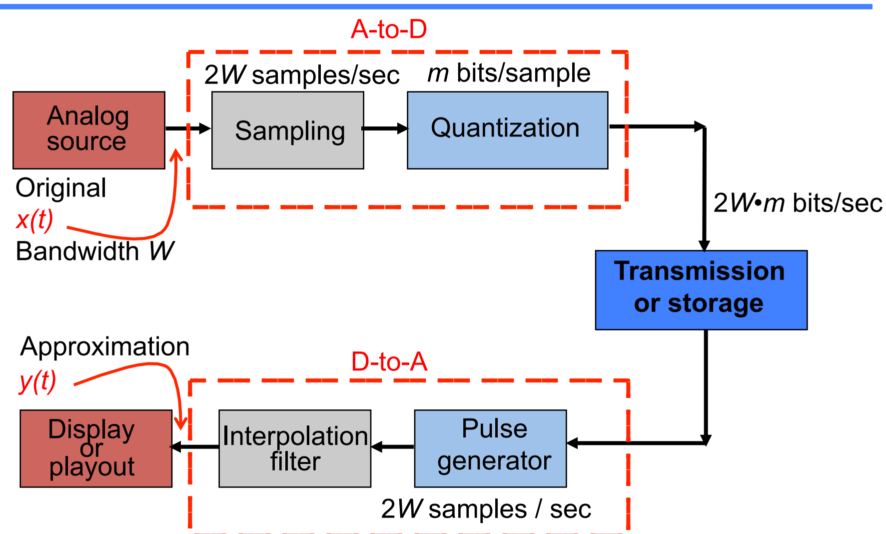


Sampling Theorem

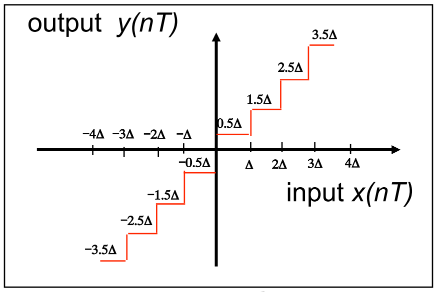
Nyquist: Perfect reconstruction if sampling rate $1/T \geq 2W_s$



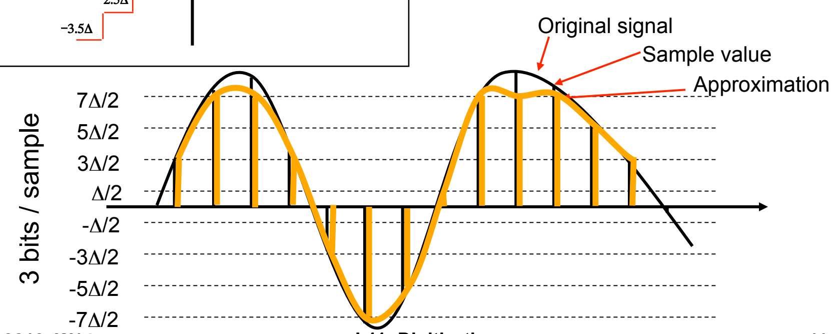
Digital Transmission of Analog Information



Quantization of Analog Samples

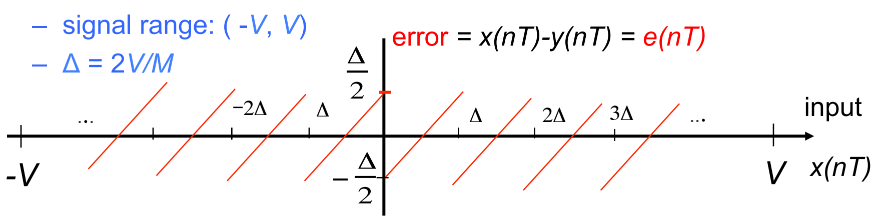


- Quantizer maps input
  - closest of 2^m representation values
- Quantization error
  - "noise" = $x(nT) - y(nT)$



Quantizer Performance

- $M = 2^m$ levels
  - signal range: $(-V, V)$
  - $\Delta = 2V/M$



- If M is large, then error is roughly uniformly distributed between $(-\Delta/2, \Delta/2)$
- Average Noise Power = Mean Square Error:

$$\sigma_e^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e(x)^2 dx = \frac{\Delta^2}{12}$$

Quantizer Performance

- Figure of Merit:
  - Signal-to-Noise Ratio = Avg signal power / Avg noise power
  - Let σ_x^2 be the signal power, then

$$SNR = \frac{\sigma_x^2}{\Delta^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3\left(\frac{\sigma_x}{V}\right)^2 M^2 = 3\left(\frac{\sigma_x}{V}\right)^2 2^{2m}$$

- The ratio $V/\sigma_x \approx 4$
- The SNR is usually stated in decibels:
- SNR [dB] = 10 $\log_{10} \sigma_x^2/\sigma_e^2$ = 6 + 10 $\log_{10} 3\sigma_x^2/V^2$
- SNR [dB] = 6m - 7.27 dB for $V/\sigma_x = 4$.

Example: Telephone Speech

W = 4 kHz, so Nyquist sampling theorem

⇒ 2W = 8000 samples/second

Suppose error requirement = 1% error

SNR = 10 log(1/.01)^2 = 40 dB

Assume V/σ_x =4, then

40 dB = 6m – 7

⇒ m = 8 bits/sample

PCM ("Pulse Code Modulation") Telephone Speech:

Bit rate= 8000 x 8 bits/sec= 64 kbps