SC/CSE 3213 Winter 2014

L12: Basic Baseband Digital Comms

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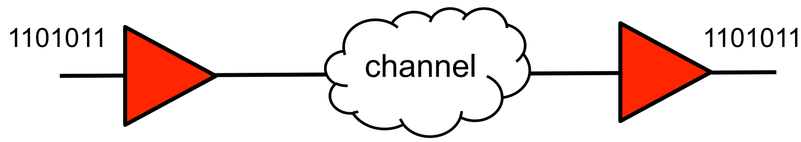
Outline

- Basic digital communicator blocks
- Nyquist pulses
- Channel capacity

Underlying Idea

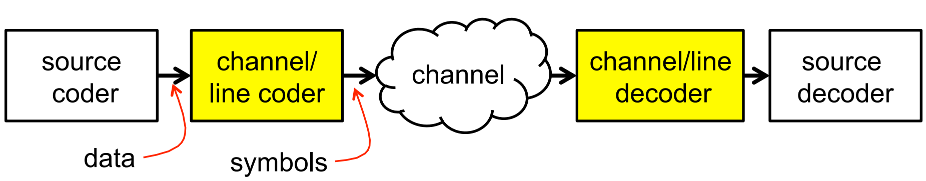
- Attempting to send a sequence of digits through a continuous channel



- Not easy…
- …modulerize the design

The Information Theory Perspective

- Exchanging sequences of discrete symbols
- source coding
  - increase redundancy
- channel coding
  - error correction (add redundancy)
- line coding
  - map bits to symbols

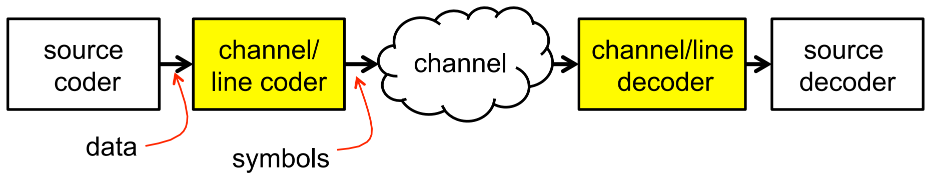


0 —— –1
1 —— +1

00 —— –3
01 —— –1
10 —— +1
11 —— +3

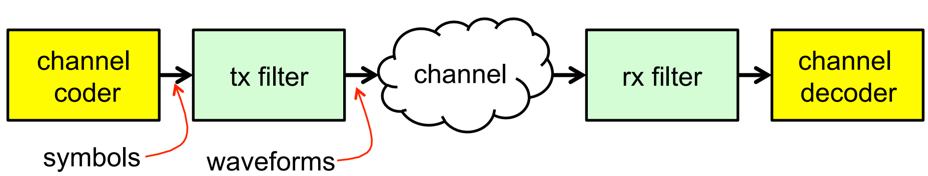
00 —— –1
01 —— –j
10 —— +1
11 —— +j

Line Coding Examples



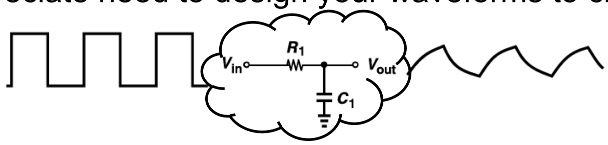
Continuous-Time Communication

- Transmit filter (TX filter)
- Map symbols into analog waveforms that match channel response
  - bandwidth
  - location (baseband/passband)

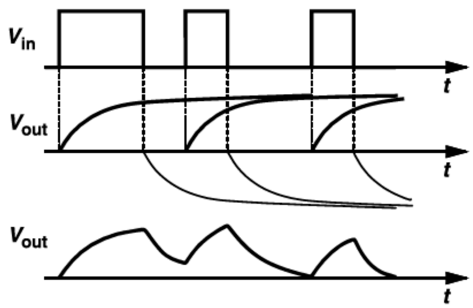


Baseband Channel Problems

- Appreciate need to design your waveforms to channel



- Prolonged rise and fall times corrupt energies of adjacent symbols: intersymbol interference (ISI)



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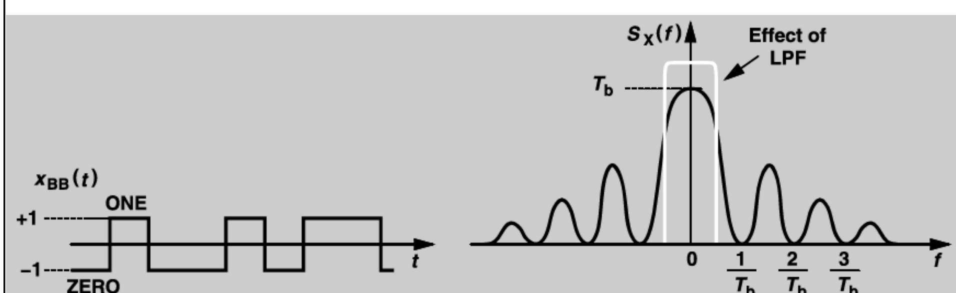
A Spectral Perspective

- Random bit patterns sinc^2 spectrum



- Need extremely wide LPF to pass this without distortion
- Otherwise deal with ISI (intersymbol interference)

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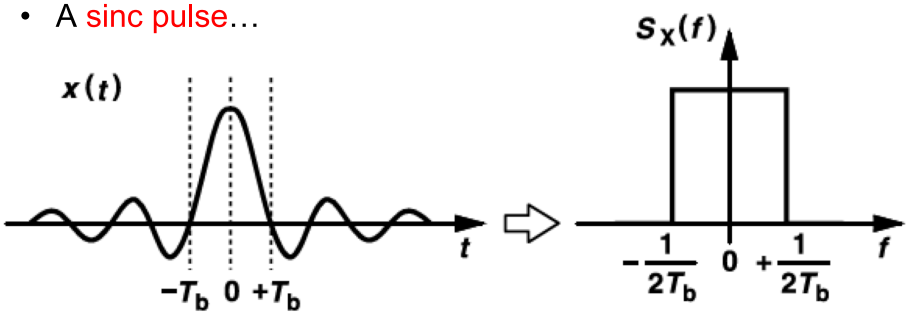
Pulse Shaping, Sinc Pulses

- What is the ideal bandlimited pulse shape for the tx filter to produce?
- A sinc pulse…



- …has spectrum that is neatly confined (to $\pm 1/2T_b$)
  - And there's another important advantage of this…

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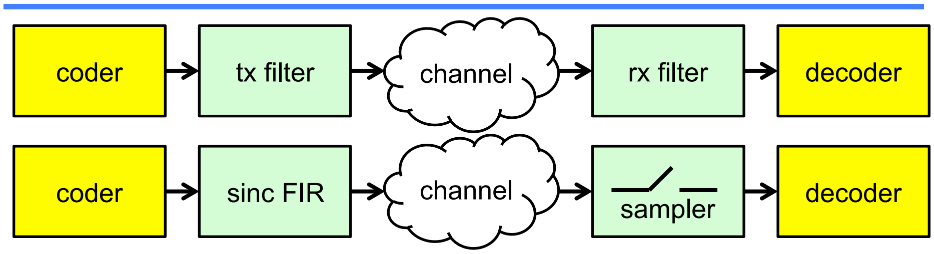
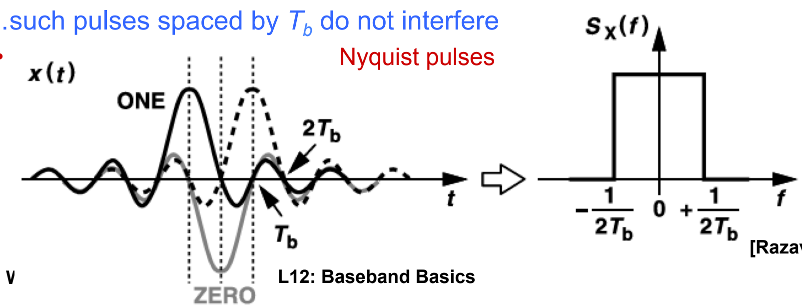
Simple Baseband Modulator (Pulse Shaper)



- Pulse goes through zero at equally spaced intervals of T_b
  - …such pulses spaced by T_b do not interfere
    - Nyquist pulses



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Example of Composite Waveform

Three Nyquist pulses shown separately

- + $s(t)$
- + $s(t-T)$
- - $s(t-2T)$

Composite waveform

$r(t) = s(t)+s(t-T)-s(t-2T)$

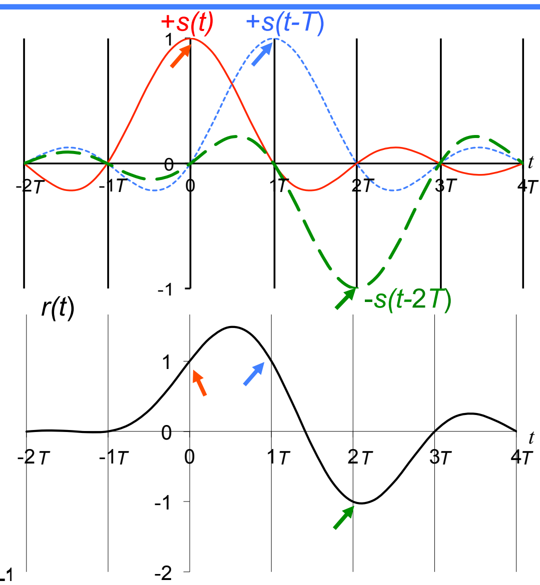
Samples at kT

$r(0)=s(0)+s(-T)-s(-2T)=+1$

$r(T)=s(T)+s(0)-s(-T)=+1$

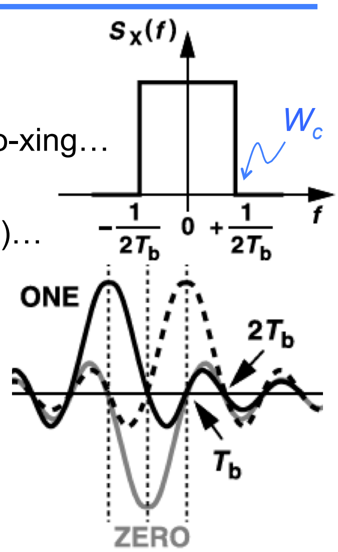
$r(2T)=s(2T)+s(T)-s(0)=-1$

Zero ISI at sampling times kT



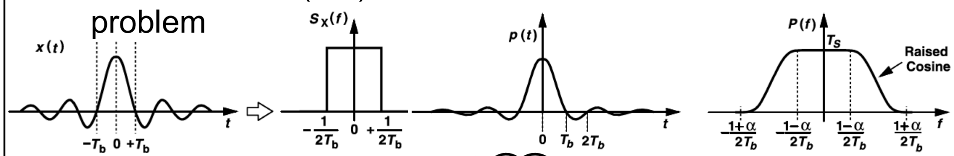
Achievable Symbol Rate

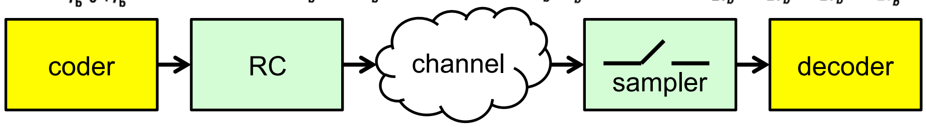
- For brickwall channel…
  - W_c: stop frequency
- For sinc pulses with $2T_b$ between zero-xing…
  - spectrum limited to $1/2T_b$
- Therefore to prevent distortion (no ISI)…
  - make: $1/2T_b = W_c$
- Therefore for W_c channel…
  - can send: symbols per second = $2 \cdot W_c$
- E.g.: 1 MHz channel allows 2 MSps



Raised Cosine (RC)

- Simple sinc has a lot of temporal energy away from main pulse
  - Sampling error can lead to lots of corruption
- Raised cosine (RC) waveform trades bandwidth for this problem





- excess-bandwidth/roll-off factor, α
  - 10-20%

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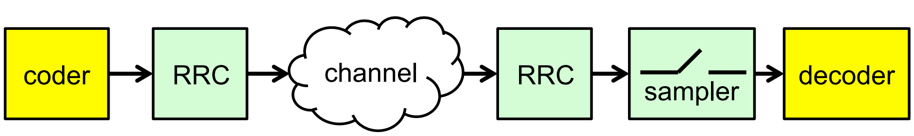
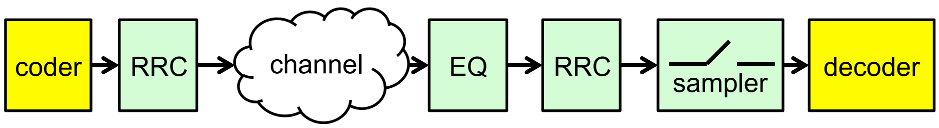
Root Raised Cosine (RRC)

- Very important for receiver to filter-out noise
  - minimize corruption to sampler
- Split up RC for simultaneous noise filtering and pulse shaping



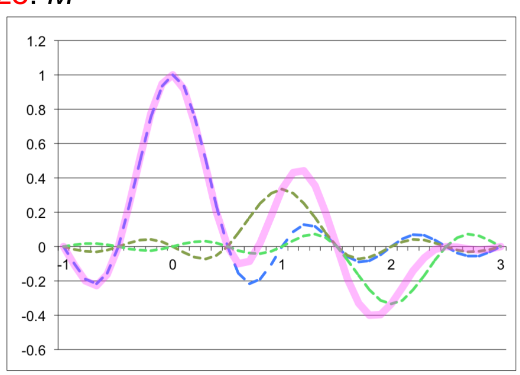
Equalization & Matched Filtering

- What if channel not a perfect brickwall or RC filter?
  - our previous assumptions would not match such a general channel and hence be distorted
- Compensate for channel with another filter: equalizer (EQ)



Achievable Bit Rate, R

- Account for alphabet size: M
  - binary, $M = 2$
  - multilevel $M > 2$
  - 4-level:
    - 00 —— −1
    - 01 —— −1/3
    - 10 —— +1/3
    - 11 —— +1



- $R = 2W_c \cdot \log_2(M)$
- For example:
  - W_c = 1 MHz
  - $M = 4$
  - R = 4 Mbps (2 MSps)

Spectral Efficiency

- What is the bit-rate relative to the spectrum used:

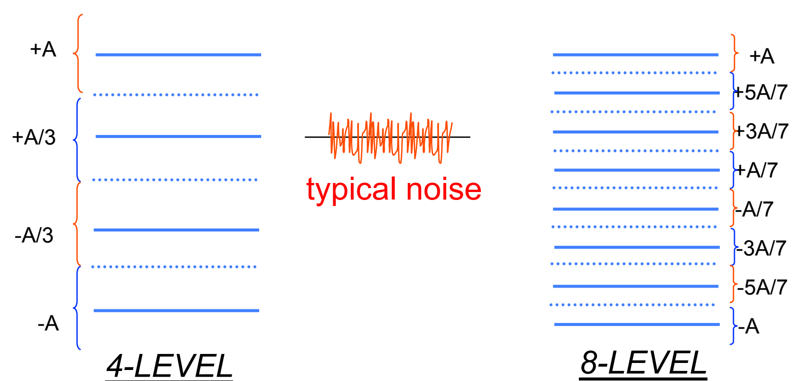
$$v = \frac{\text{bit-rate}}{\text{bandwidth}}$$

$$v = \frac{2W_c \bullet \log_2(M)}{W_c} \left[\frac{\text{bits}}{\text{s} \bullet \text{Hz}}\right]$$

- Simple 2-level baseband modulation 2 bits/s•Hz
- or 2 symbols/s•Hz

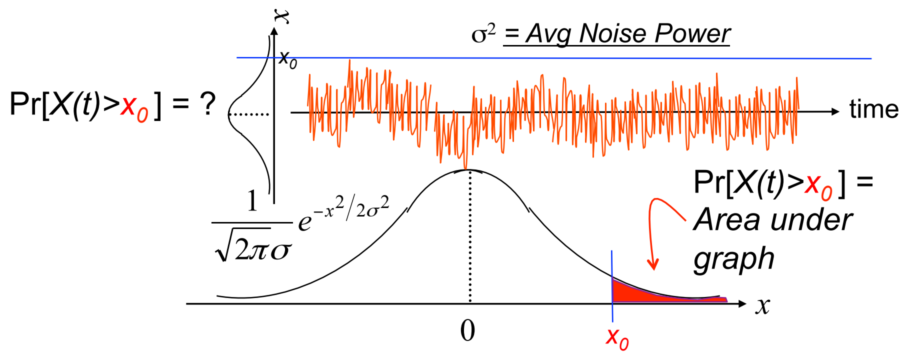
Noise Limits Accuracy

- Receiver makes decision based on transmitted pulse level + noise
- Error rate depends on relative value of noise and level spacing
- Large (positive or negative) noise values can cause wrong decision
- Noise level impacts 8-level signaling more than 4-level signaling



Noise Distribution

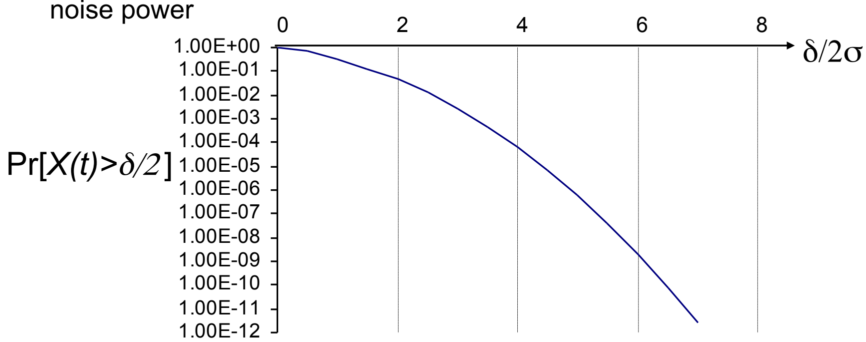
- Noise is characterized by probability density of amplitude samples
- Likelihood that certain amplitude occurs
- Thermal electronic noise is inevitable (due to vibrations of electrons)
- Noise distribution is Gaussian (bell-shaped) as below



$$\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$$

Probability of Error

- Error occurs if noise value exceeds certain magnitude
- Prob. of large values drops quickly with Gaussian noise
- Target probability of error achieved by designing system so separation between signal levels is appropriate relative to average noise power



Shannon Channel Capacity

Shannon Channel Capacity:

- The maximum reliable transmission rate over an ideal channel with bandwidth W_c Hz, with Gaussian distributed noise, and with SNR S/N is

  $$C = W_c \log_2(1 + S/N) \text{ bits per second}$$

- Reliable means error rate can be made arbitrarily small by proper coding

Example

- Consider a 3 kHz channel with 8-level signaling. Compare bit rate to channel capacity at 20 dB SNR

- 3 kHz telephone channel with 8 level signaling

  Bit rate = 2•3000 pulses/sec • 3 bits/pulse = 18 kbps

- 20 dB SNR means $10 \log_{10}(S/N) = 20$

  Implies $S/N = 100$

- Shannon Channel Capacity is then

  $C = 3000 \log(1 + 100) = 19{,}963$ bits/second