York University
Department of Electrical Engineering and Computer Science
EECS 4215

Lab #1(v1.0) Antennas

S. Magierowski

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1 Objectives

Our aim is to learn a little bit more about antennas and how they effect the signals we process.

2 Discussion

2.1 Some Basics

The transmitted power density (in W/m^2) at a distance r from a transmitting antenna in empty (homogenous) space is

$$S_T(\theta,\phi,r) = \frac{U(\theta,\phi)}{r^2} \tag{1}$$

where $U(\theta,\phi)$ is the power-per-steradian (W/Ω_A) radiated in the (θ,ϕ) direction. θ is take to be the elevation angle and ϕ the azimuth angle (see Fig. 1 for a sketch of these directions relative to the rectangular coordinate system). In terms of the radiated power (out of the transmit antenna and into the air), P_{rad} we can write

$$S_T(\theta,\phi,r) = \frac{1}{r^2} \cdot P_{rad} \cdot \Omega_T(\theta,\phi) \tag{2}$$

where $\Omega_T(\theta,\phi)$ is the effective solid-angle of the transmit antenna (in steradians) in the (θ,ϕ) direction; this variable indicates the density of the power in a particular direction at a unit distance from the antenna terminals. It is a sort of reference radiation pattern.

In terms of the injected power (into the transmit antenna from the electronic source), P_{inj}, we can also write the transmitted power density as

$$S_T(\theta,\phi,r) = \frac{\eta}{r^2} \cdot P_{inj} \cdot \Omega_T(\theta,\phi) \tag{3}$$

The efficiency, at which injected power is converted to transmitted radiation power, P_{rad} (and vice-versa for a receiving antenna) is represented by η which is a number between 0 and 1 (the more efficient the antenna the closer that η is to 1).

The distance at which an antenna "sees" (in receive-mode) or emits (in transmit-mode) far-field radiation is proportional to the wavelength, λ, of that radiation. For this reason, the above solid angle is normalized to an area removed by a distance equal to λ from the antenna terminals and the above equation is almost always written as

$$S_T(\theta,\phi,r) = \frac{1}{r^2} \cdot P_{inj} \cdot \frac{A_T(\theta,\phi)}{\lambda^2} \tag{4}$$

where $A_T(\theta,\phi)$ is the effective area of the transmit antenna (in m^2) λ meters away from the "center of the antenna" (admittedly a somewhat nebulous concept) in the (θ,ϕ) direction. So, A_T is a way for us to keep track of what effective surface area that the antenna can "suck in" or "blow out" far field radiation (assuming that it can only "interact" with far-field radiation one λ away from its "center" — no farther and no closer).

Shifting gears a bit, another way to express the transmitted power density is with

$$S_T(\theta,\phi,r) = \frac{1}{r^2} \cdot \frac{P_{inj}}{4\pi} \cdot G_T(\theta,\phi). \tag{5}$$

In this case we consider the injected power, P_{inj}, distributed over a unit sphere (the second term, $P_{inj}/4\pi$ in units W/Ω_A) multiplied by the transmit gain $G_T(\theta,\phi)$ which gives the relative amount of power propagating in the (θ,ϕ) direction relative to the injected power distributed over a unit sphere. The maximum value of gain is typically denoted by

$$G_T = \max\{G_T(\theta,\phi)\}. \tag{6}$$

The gain of any particular antenna is the same wether in transmit or receive mode and is typically denoted by G, the subscripts are used only to identify any potential differences between transmit and receive antennas in communication system (where at any one time the transmit and receive antennas are physically different structures).

Now, the total power absorbed by a receiving antenna and available to a receive circuit (assuming conjugate match) is given by

$$P_R = S_T(\theta,\phi,r) \cdot A_R(\theta,\phi) \tag{7}$$

where $A_R(\theta,\phi)$ is the effective area of the receive antenna (roughly speaking λ away from the antenna terminals) looking in the (θ,ϕ) direction (now assumed relative to the receive antenna). This particular equation assumes only one path from transmitter to receiver (note that the arguments in S_T and A_R may be different), if more paths need to be accounted for they can be added to the above expression (including any potential phase offsets in the electric and magnetic fields registered by the receive antenna since the sum of the signals arriving from different paths is not necessarily constructive).

Another way to express this is with

$$P_R = [4\pi\lambda^2 S_T(\theta,\phi,r)] \cdot \frac{1}{4\pi} \cdot G_R(\theta,\phi) \cdot \frac{1}{4\pi}. \tag{8}$$

This expression is purposely expanded into four terms to help highlight some of the physics at work. With a nod to the fact that and antenna can "sense" electromagnetic radiation λ away from its terminals (again, no farther and no closer) the first takes the power, $S_T(\theta,\phi,r)$ flowing in from direction (θ,ϕ) and pretends as if all this power were flowing through the surface of a sphere with radius λ. This term expresses the total EM power λ away from the antenna.

However, the power is not actually coming in all around the antenna, as mentioned, it is only coming into one steradian in direction (θ,ϕ) (from the perspective of the receiving antenna), so the second term accounts for this by dividing by the solid angle of a whole sphere.

Next we multiply by the gain of the receiving antenna, $G_R(\theta,\phi)$ which describes how much of the power coming into the antenna from direction (θ,ϕ) is converted to available power at the antenna terminals (connected to the receiving circuit).

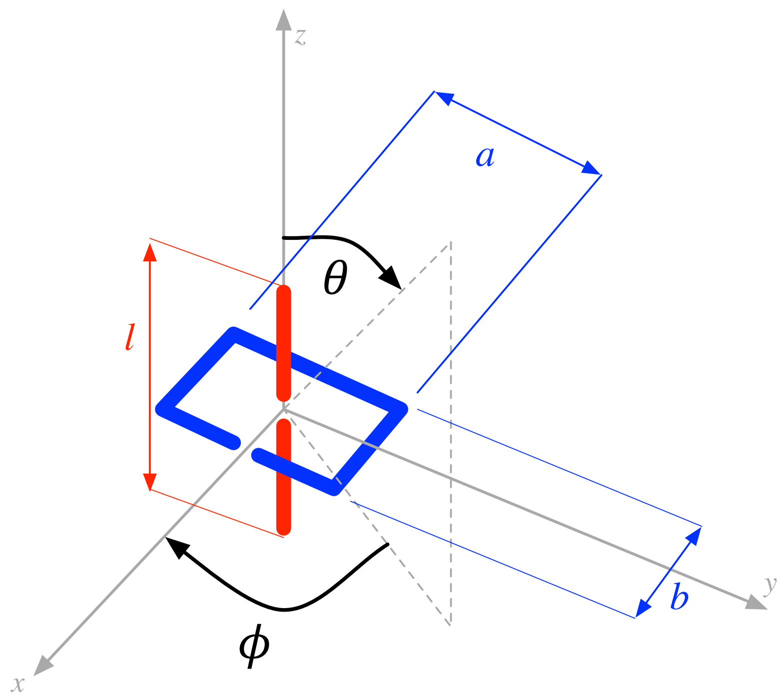

Figure 1: Simplified sketch of a dipole (red) and loop (blue) antenna with dimensions and coordinate references.

However it must be noted that the gain term, $G_R(\theta,\phi)$, relates the available power to an incoming power spread uniformly along the surface of the sphere, hence in the fourth term we divide by 4π again. Physically this means that we take the power coming in at (θ,ϕ) (i.e. $\lambda^2 S_T(\theta,\phi,r)$ left after multiplying the first term in Eq. (8) by the second term) and spread it over the surface of a unit sphere (again, to satisfy the meaning of the antenna gain coefficient).

Similar to the terminology used above

$$G_R = \max\{G_R(\theta,\phi)\}. \tag{9}$$

Summing up the above, assuming that the radiation patterns of the transmit and receive antennas are optimally aligned then the maximum (because they are optimally aligned) available power at the receive antenna is given by

$$P_R = \frac{\lambda^2 G_T G_R P_{inj}}{(4\pi r)^2} \tag{10}$$

for a single unobstructed EM path. This relation is better known as Friis' formula and highlights the importance of the antenna gain parameter.

2.2 A Couple of Antennas

Now consider a couple of common antenna arrangements, a dipole and a loop antenna. Simplistic pictures of the two antennas with spherical reference coordinates defined are shown in Fig. 1. The dipole, a couple of wires with a centre feed (i.e. where the electronics driving the antenna is attached) is shown in red with a total length of l meters. The rectangular antenna, an $a \times b$ m^2 structure with a feed on one of the a sides is shown in blue.

The power density (per steradian) for the dipole is

$$U_D(\theta,\phi) = \frac{I_0^2}{8\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left[\frac{\sin\left(\frac{\pi l}{\lambda}\right)\cos\left(\frac{\pi l}{\lambda}\cos\theta\right) - \cos\theta\cos\left(\frac{\pi l}{\lambda}\right)\sin\left(\frac{\pi l}{\lambda}\cos\theta\right)}{\sin\theta}\right]^2. \tag{11}$$

The power density for the loop is

$$U_L(\theta,\phi) = \frac{2I_0^2}{\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)\sin\left(\frac{\pi b}{\lambda}\sin\theta\sin\phi\right)}{\sin\theta\sin 2\phi}\right]^2. \tag{12}$$

In both cases I_0 is the amplitude of the sinusoidal current flowing in the loop (in Amperes), λ is the wavelength of the radiation signal (in meters), θ is the elevation angle (with respect to the plane of the loop), ϕ is the azimuth angle (in the plane of the loop), μ_0 is the permeability of vacuum, and ε_0 is the permittivity of vacuum. The ratio $\sqrt{\mu_0/\varepsilon_0}$ works out to 376.7 Ω. Thus, if you use values of I_0 in terms of Amperes in equations (11) and (12) you will get radiated power density values in Watts per steradian.

The total amount of power radiated (or absorbed) by an antenna can be expressed as

$$P_{rad} = \int_\Omega U(\theta,\phi)\mathrm{d}\Omega \tag{13}$$

$$= \int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}U(\theta,\phi)\sin\theta\mathrm{d}\theta. \tag{14}$$

People commonly express this in the form

$$P_{rad} = \frac{I_0^2 R_r}{2} \tag{15}$$

where R_r is the radiation resistance. That is, the antenna is abstracted to a resistor, R_r. Instead of just radiating heat however R_r gives us an idea of how the antenna is radiating the electronic signal we wish to send.

This also makes it easy to calculate the efficiency of the antenna which is the ratio of "good" resistance, R_r, (i.e. the radiation resistance that tells us how good we are at converting the input electronic current to desired electromagnetic radiation) to the total resistance of the antenna which includes the "bad" resistance, R_l (or "loss" resistance) which is your typical resistance that converts the input electronic current into heat. Generally we write

$$\eta = \frac{R_r}{R_r + R_l}. \tag{16}$$

For a wire of length l and radius r made out of a metal with resistivity ρ (in units of Ω/m operating at a frequency f (in Hz) the loss resistance in Ω is

$$R_l = \frac{\rho l}{2\pi r\delta} \tag{17}$$

where, δ, is the so-called "skin-depth" and is expressed with

$$\delta = \sqrt{\frac{\rho}{\pi f}} \tag{18}$$

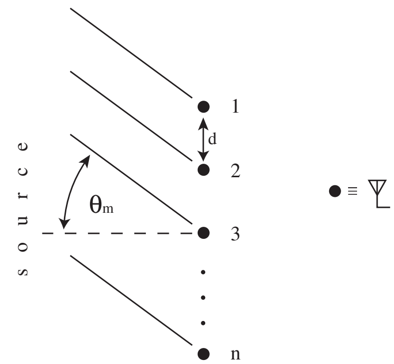

Figure 2: A linear antenna array (top-view).

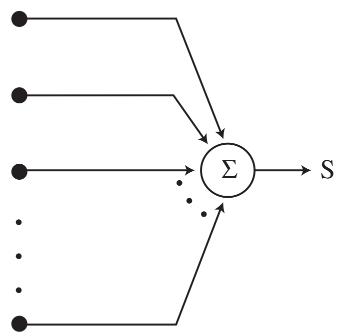

Figure 3: Summing the signals from an antenna array.

2.3 Antenna Arrays

A common way of improving the gain of a radiator (i.e. an antenna), is to use more than one copy. That is to use an array of antennas. The simplest scenario is one that uses a bunch of antennas arranged in an evenly distributed row (a linear array) as shown in Fig. 2.3. This picture is intended to show the top view of a bunch of dipole antennas (looking at dipoles from the top makes them look like dots) separated from their neighbours by a distance, d.

The focusing ability of such an arrangement lies in the fact that radiation from a distant source arrives at a different phase for each antenna depending on the angle of arrival. Radiation of wavelength λ coming in from a distant source at an angle θ_m to the normal arrives with a phase difference of

$$\phi = 2\pi \frac{d \sin \theta_m}{\lambda} \tag{19}$$

between adjacent antennas. Assuming ideal isotropic antennas, if we sum the signals of the antennas as shown in Fig. 2.3 we have

$$S = A\left[\cos \omega t + \cos(\omega t + \phi) + \cos(\omega t + 2\phi) + \ldots + \cos(\omega t + (n-1)\phi)\right]. \tag{20}$$

which works out to

$$S = A \frac{\sin(n\phi/2)}{\sin(\phi/2)}. \tag{21}$$

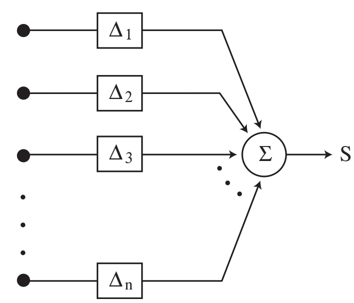

Figure 4: Applying phase shifts (weights) to pick out a direction.

When $\theta_m = \theta_1$ is zero ϕ is zero so $S = A$ irrespective of n. When $\theta_m = \theta_2 = \arcsin\left(\frac{\lambda}{nd}\right)$ you can show that $S = 0$. But rather then a straight sum we can add up the antenna signals as shown in Fig. 4. With the phase shifters providing shifts in phase of

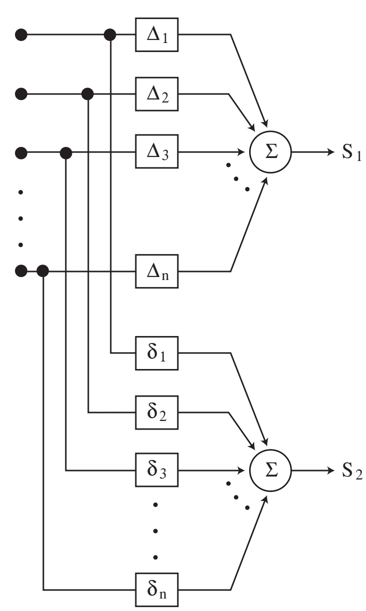

Figure 5: Choosing two directions simultaneously.

$$\Delta_k = 2\pi(n-k) \qquad\qquad k = 1, 2, \ldots, n \tag{22}$$

the signal from direction θ_2 can be recovered. So, by properly arranging the phase shifts we can electrically control the direction that the array "looks at" – this is called **beamsteering**. A number of applications arise from this simple property.

1) An antenna array can be focused in a particular direction, delivering all it's energy to a particular user. This leads to improved efficiency.

2) In a multipath environment, a focused beam can help reduced the scattering that a signal is subject to (a corollary to the first point). This helps reduce loss.

3) Conversely, in a multipath environment the antenna array can be used to gather up all signal of interest from all the different directions that it was scattered to.

4) An antenna array can ignore interferers coming from undesired directions. This helps to operate in a multiuser environment.

The third point highlights an important strength to the directionality of array receivers: that we can look at different directions simultaneously. Fig. 5 sketches the function of a system that can do this. Basically, a different set of phase shifters for each direction that we desire to look at. Of course we have to be intelligent about the implementation of such a system. It is quite difficult to make this approach practical and the procedure to do so remains an open research question.

3 Questions

1) On a polar coordinate system plot the gain of the dipole antenna in dBi for $l = 2.5$ cm at an operating frequency of 3 GHz as a function of θ.

2) Plot the maximum gain of the dipole in dBi as a function of l ranging from 1 cm to 30 cm intended for operation at 1.8 GHz.

3) Plot the maximum gain of the loop in dBi as function of $a = b$ ranging form 1 cm to 30 cm intended for operation at 1.8 GHz.

4) Plot the dipole's radiation resistance, R_r, in Ω as a function of l ranging from 1 cm to 30 cm intended for operation at 1.8 GHz.

5) Plot the loop antenna's radiation resistance, R_r, in Ω as function of $a = b$ ranging form 1 cm to 30 cm intended for operation at 1.8 GHz.

6) What is the skin-depth of copper operating at 1.8 GHz in microns (μm)?

7) Plot the η for a copper dipole antenna with l ranging from 1 cm to 30 cm while operating at 1.8 GHz.

8) Plot the gain pattern (superimposed them) for the summing signal S (as a function of θ_m) for a linear array of 2, 4, and 8 ideal isotropic antennas whose total length (from the first element to the last) is $\lambda/2$.