Chapter Objective

- Understanding phasor concept and be able to perform phasor transform and inverse phasor transform.
- Be able to transform a circuit with sinusoidal source into the frequency domain using phasor transform
- Know how to use circuits analysis techniques to solve circuits in the frequency domain.
- Be able to use phasor in analyzing circuits with ideal transformers.

Figure 9.1 A sinusoidal voltage.

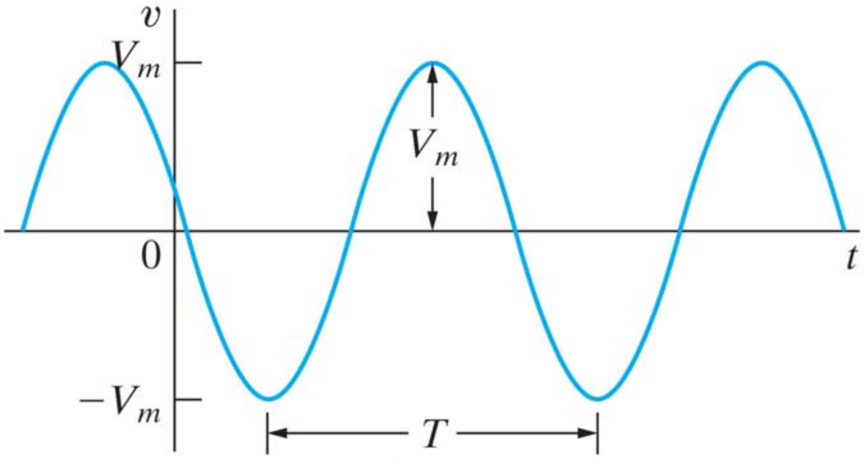


Figure 9.2 The sinusoidal voltage from Fig. 9.1 shifted to the right when $\phi = 0$.

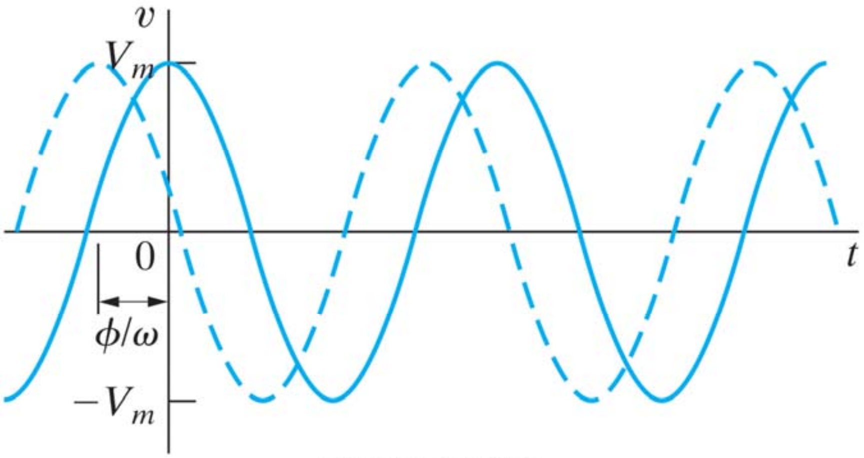


Figure 9.3 Periodic triangular current.

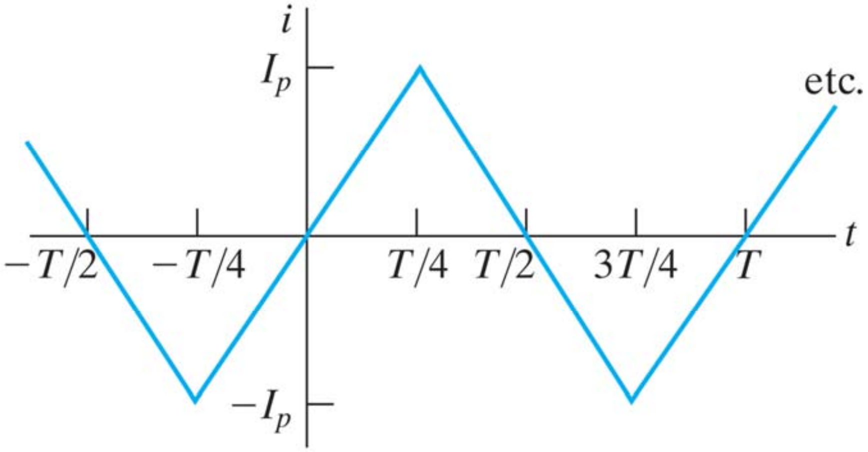
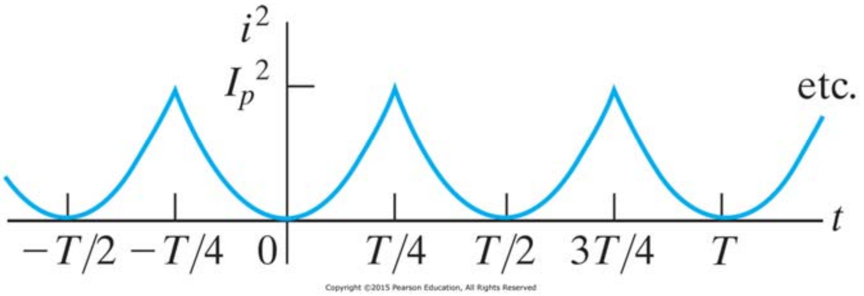
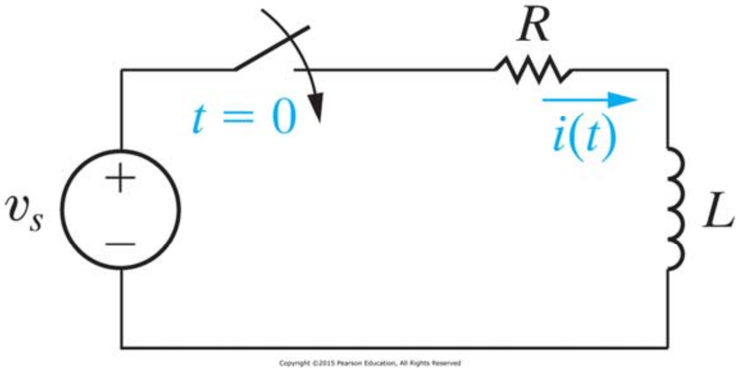


Figure 9.4 i^2 versus t.





$$L\frac{di}{dt} + iR = V_m \cos(\omega t + \phi)$$

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

RMS Values

- Why RMS

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

The Phasor

- The phasor is a complex number that carries the amplitude and phase angle information of a sinusoidal function.
- Euler's identity $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

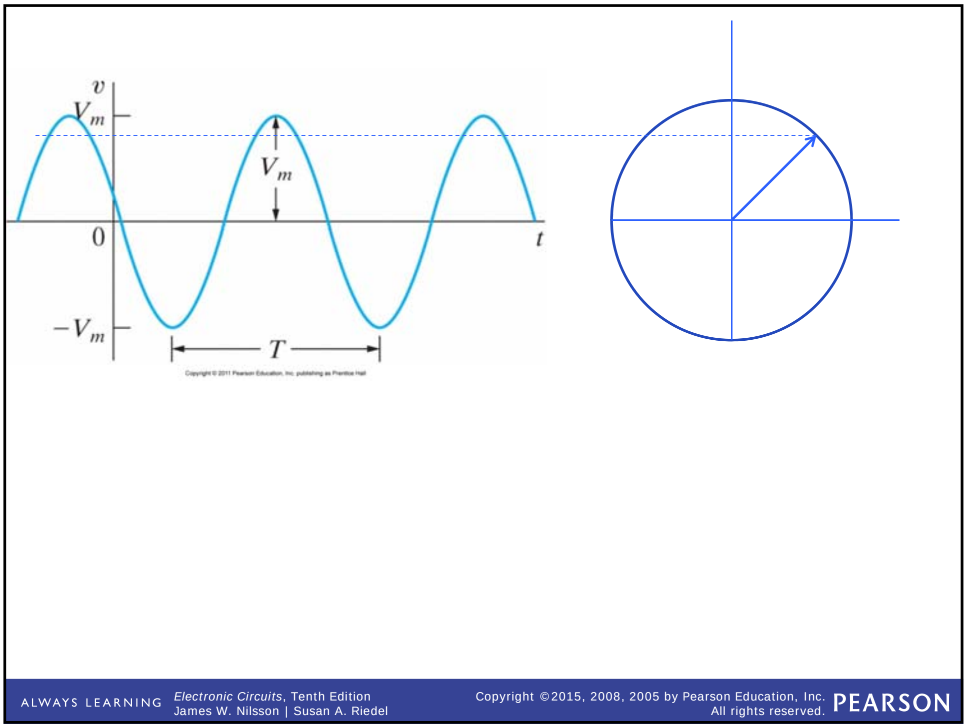
$$\cos\theta = \Re\left\{e^{j\theta}\right\}$$

$$\sin\theta = \Im\left\{e^{j\theta}\right\}$$

$$v = V_m \cos(\omega t + \phi)$$

$$v = \Re\left\{V_m e^{j\phi} e^{j\omega t}\right\}$$

$$Ae^{j\phi} = A\angle\phi^\circ$$


v
V_m
V_m
0
t
$-V_m$
T
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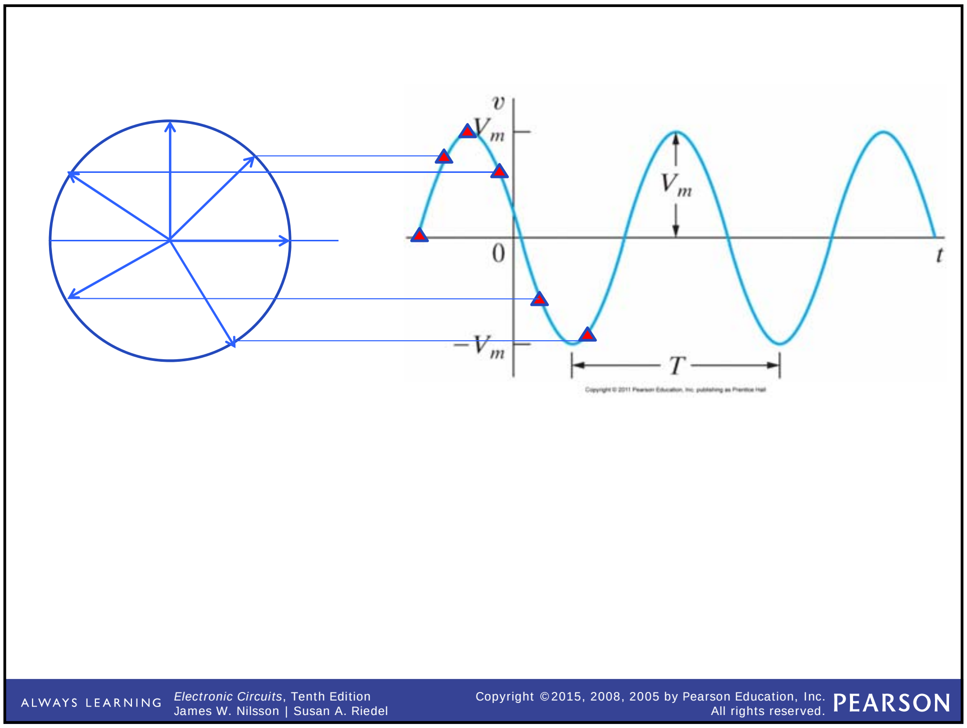
v
V_m
V_m
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t
$-V_m$
T
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The inductor

$$v = L\frac{di}{dt}$$

$$v = V_m\cos(\omega t)$$

$$di = \frac{1}{L}V_m\cos(\omega t)dt$$

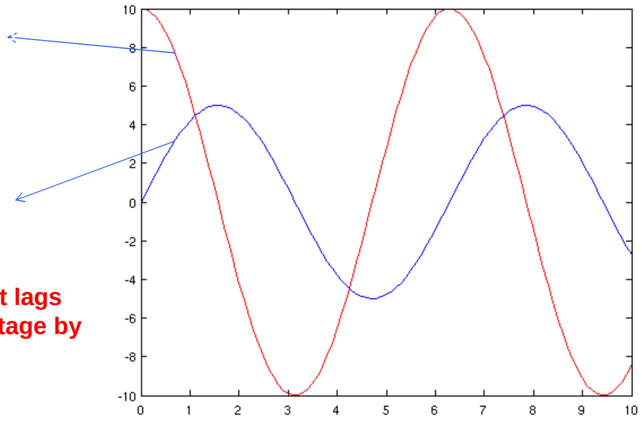
$$i = \frac{1}{L}V_m\int\cos(\omega t)dt$$

$$i = \frac{V_m}{\omega L}\sin(\omega t) = \frac{V_m}{\omega L}\cos(\omega t - \frac{\pi}{2})$$

$$I = \frac{V_m}{\omega L}e^{-j\pi/2} = \frac{V_m}{\omega L}\angle -\pi/2$$

$$Z = \frac{v}{i} = \omega L\angle\pi/2 = jwL$$

A plot showing the phase relationship between the current and voltage at the terminals of an inductor



Current lags the voltage by 90°

The Capacitor

$$i = C\frac{dv}{dt}$$

$$v = V_m\cos(\omega t)$$

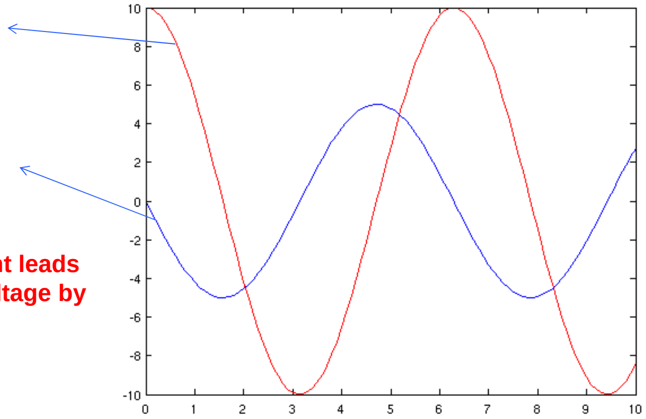
$$i = CV_m\frac{d}{dt}\cos(\omega t)$$

$$i = -C\omega V_m\sin(\omega t) = \omega CV_m\cos(\omega t + \frac{\pi}{2})$$

$$I = \omega CV_m e^{j\pi/2} = \omega CV_m\angle\pi/2$$

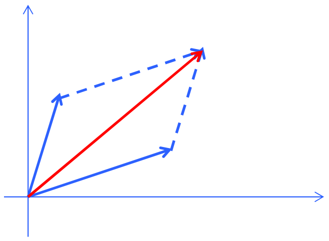
$$Z = \frac{v}{i} = \frac{1}{\omega C}\angle -\pi/2 = \frac{-j}{\omega C} = \boxed{\frac{1}{j\omega C}}$$

A plot showing the phase relationship between the current and voltage at the terminals of a capacitor



Current leads the voltage by 90°

Adding Complex Numbers



Multiplication

$$\begin{array}{lr} & x_1+jy_1 \\ \times & x_2+jy_2 \\ & \text{-------------} \\ \multicolumn{2}{l}{(x_1\,x_2-y_1y_2)+j(x_1y_2+x_2+y_1)} \end{array}$$

$A_1\angle\theta_1 \times A_2\angle\theta_2 = A_1\,A_2\,\angle\theta_1{+}\theta_1$

TABLE 9.1 Impedance and Reactance Values

Circuit Element	Impedance	Reactance
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Figure 9.14 Impedances in series.

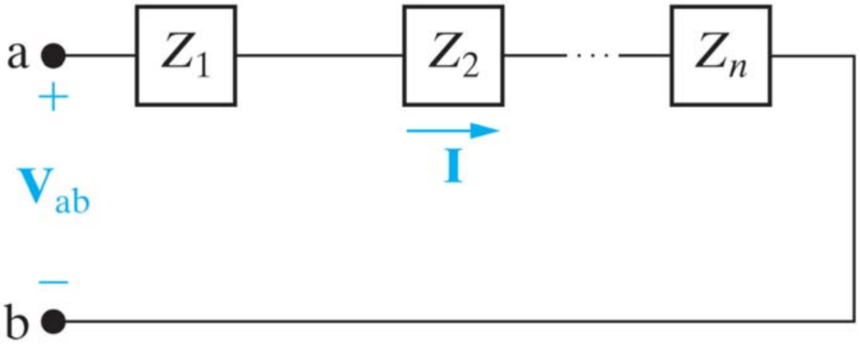


Figure 9.15 The circuit for Example 9.6.

$v_s = 700\cos(5000t + 30^\circ)$

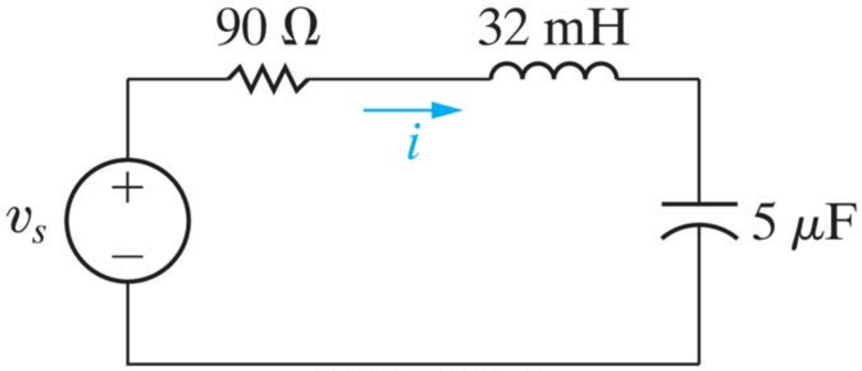


Figure 9.16 The frequency-domain equivalent circuit of the circuit shown in Fig. 9.15.

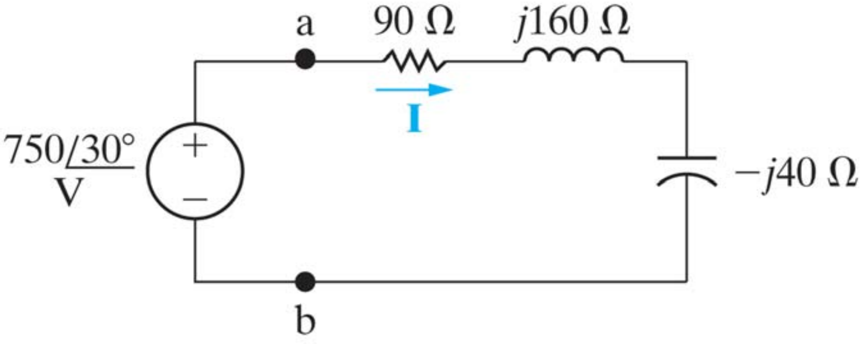
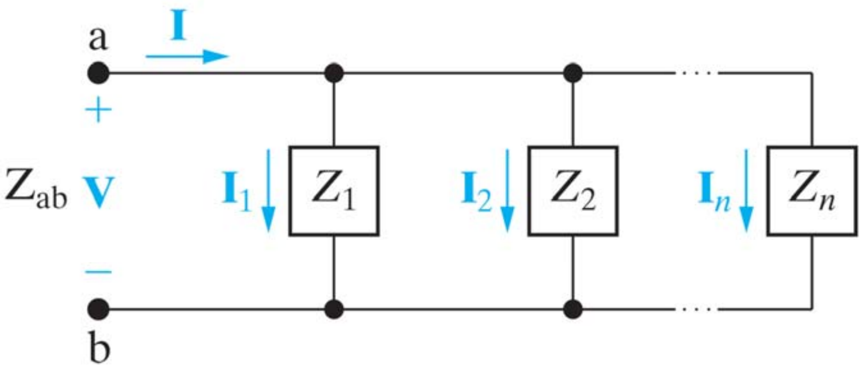


TABLE 9.2 **Admittance and Susceptance Values**

Circuit Element	Admittance (Y)	Susceptance
Resistor	G (conductance)	—
Inductor	$j(-1/\omega L)$	$-1/\omega L$
Capacitor	$j\omega C$	ωC

Figure 9.17 Impedances in parallel.



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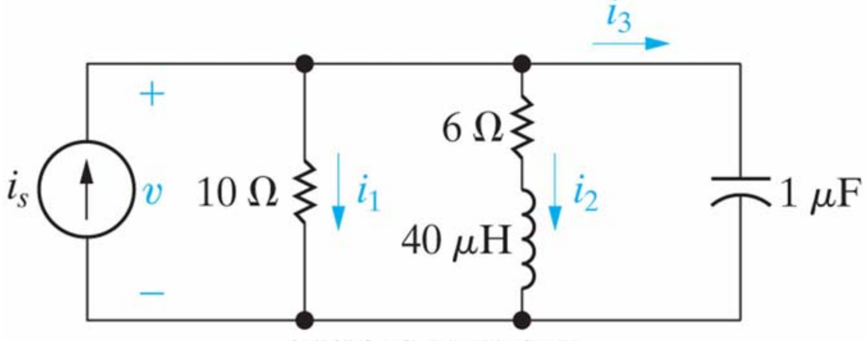
Example

$i_s = 8\cos 200{,}000t$



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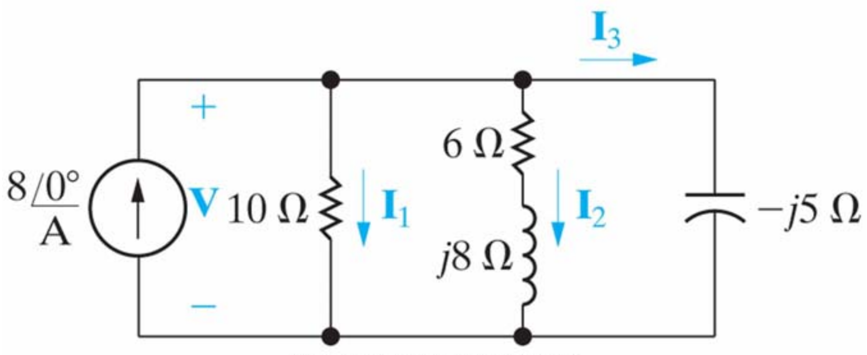
Example



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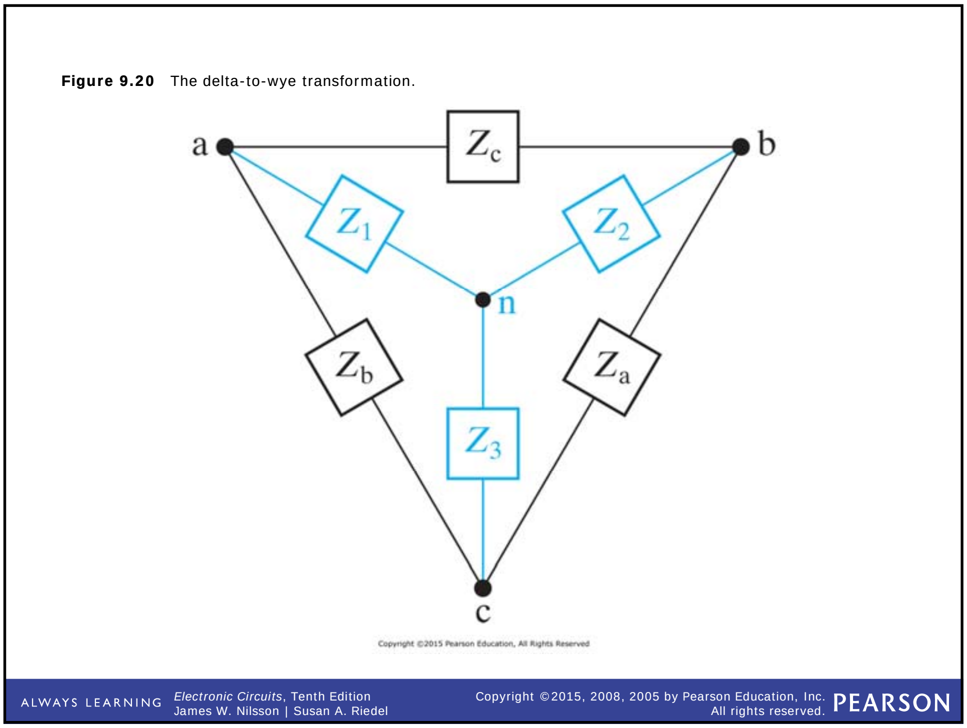

Figure 9.20 The delta-to-wye transformation.

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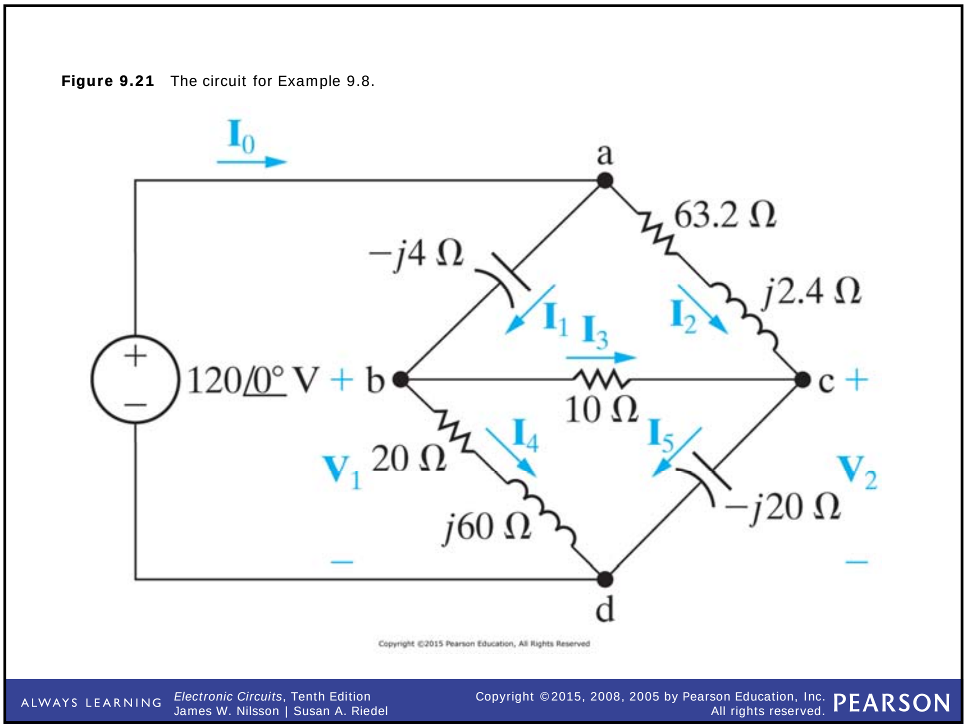

Figure 9.21 The circuit for Example 9.8.

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Figure 9.22 The circuit shown in Fig. 9.21, with the lower delta replaced by its equivalent wye.

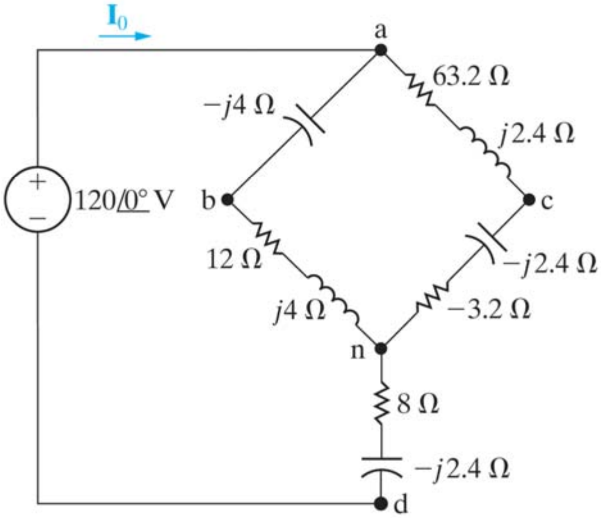

Figure 9.23 A simplified version of the circuit shown in Fig. 9.22.

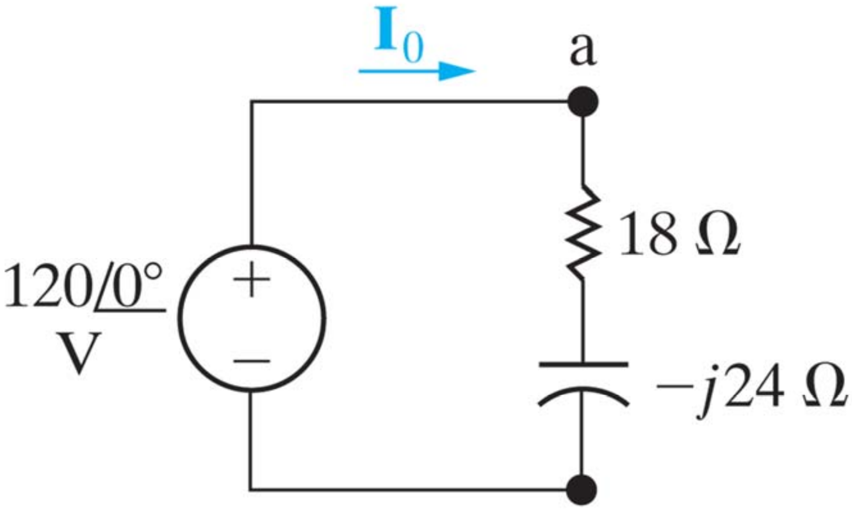

Figure 9.24 A source transformation in the frequency domain.

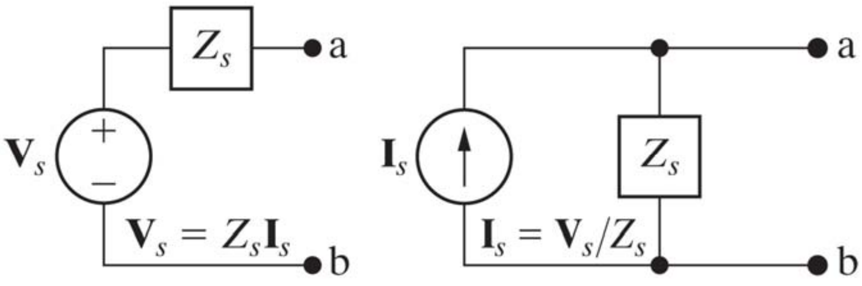

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Figure 9.25 The frequency-domain version of a Thévenin equivalent circuit.

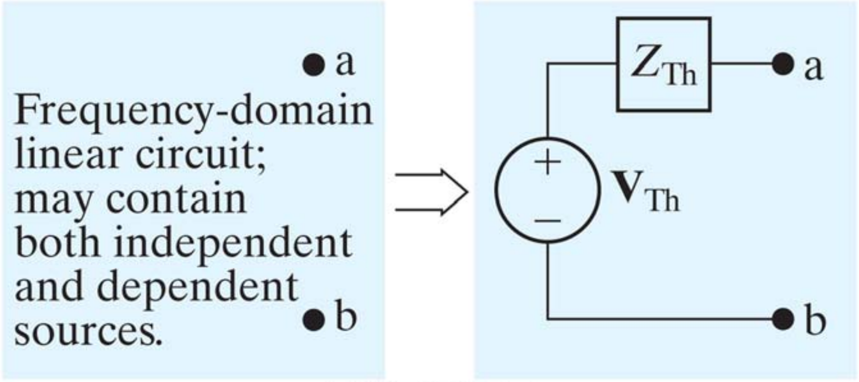

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Figure 9.26 The frequency-domain version of a Norton equivalent circuit.

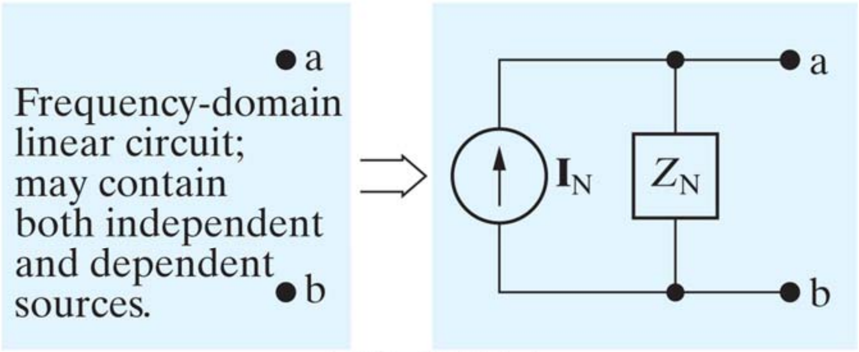


Figure 9.30 The circuit for Example 9.10.

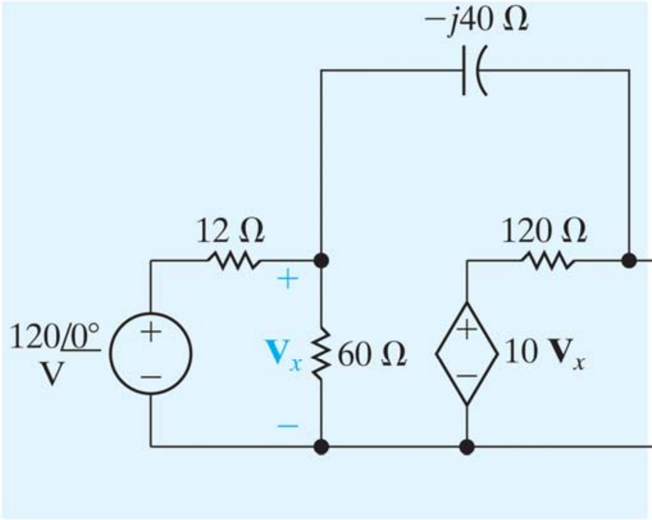


Figure 9.31 A simplified version of the circuit shown in Fig. 9.30.

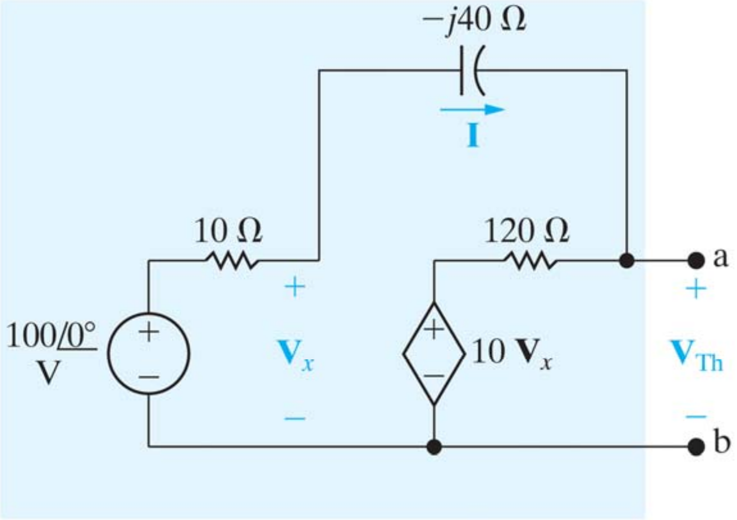
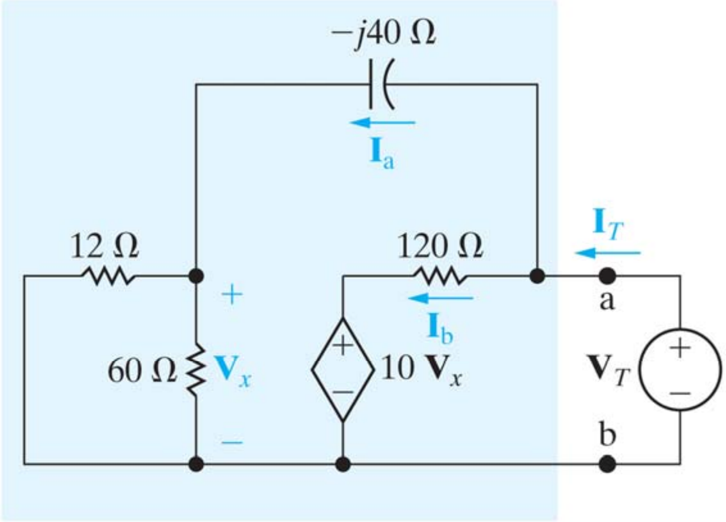


Figure 9.32 A circuit for calculating the Thévenin equivalent impedance.



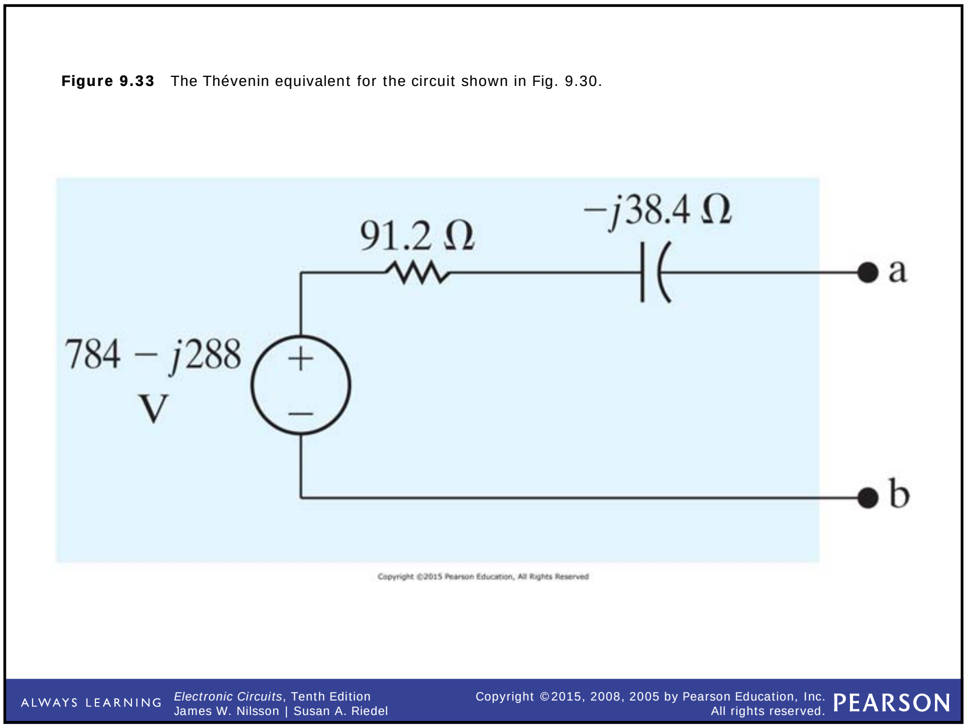


Figure 9.33 The Thévenin equivalent for the circuit shown in Fig. 9.30.

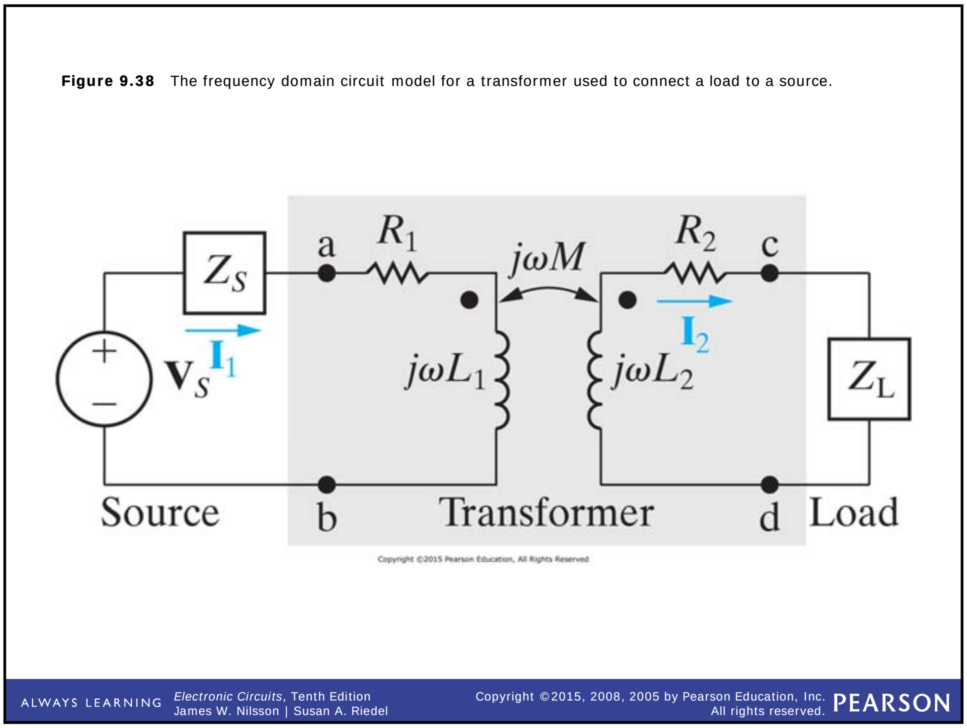
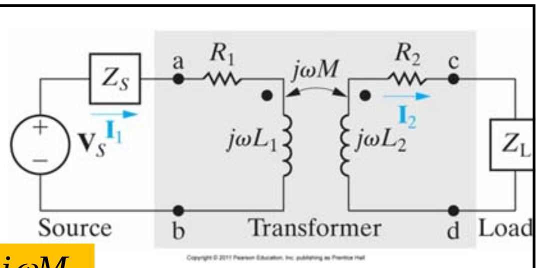


Figure 9.38 The frequency domain circuit model for a transformer used to connect a load to a source.



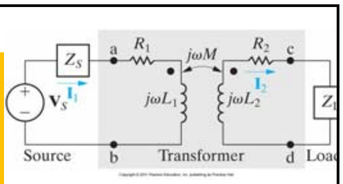
$$V_s = I_1(Z_s + R_1 + j\omega L_1) - I_2 j\omega M$$
$$0 = -j\omega M I_1 + I_2(R_2 + j\omega L_2 + Z_L)$$

$$j\omega M I_1 = I_2(R_2 + j\omega L_2 + Z_L)$$
$$I_2 = \frac{j\omega M I_1}{(R_2 + j\omega L_2 + Z_L)}$$
$$Z_{11} = Z_s + R_1 + j\omega L_1$$
$$Z_{22} = Z_L + R_2 + j\omega L_2$$

$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$
$$\frac{V_2}{I_1} = Z_{total} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

Impedance in
the primary loop

Reflected
impedance



$$Z_{reflected} = \frac{\omega^2 M^2}{Z_{22}}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + R_L + j\omega X_L}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{(R_2 + R_L) + j\omega(L_2 + X_L)}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{|Z_{22}|^2}\left[(R_2 + R_L) - j\omega(L_2 + X_L)\right]$$

Figure 9.39 The frequency-domain equivalent circuit for Example 9.13.