Lab 4: RLC Circuits

OBJECTIVE:

- Understand the response of an RLC circuit.
- Understand the difference between over damped, under damped and critically damped circuit.
- Implement an RLC circuit and display the step response of the circuit on the scope

INTRODUCTION:

Consider the following Circuit

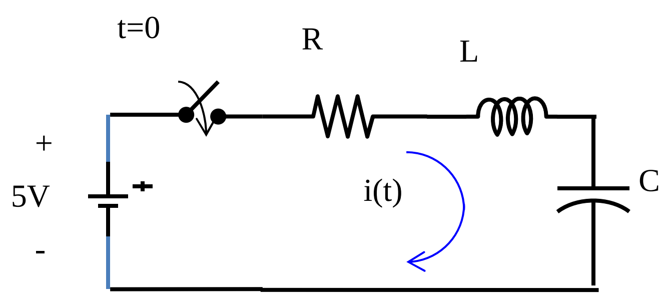

Figure L4.2 The natural response of an RLC circuit.

By applying KVL, we get

$$v_R + v_L + v_c = 0$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0$$

Differentiating with respect to t, we get

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

The characteristic equation for this differential equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Another way to put this

$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$ Where ω_0 is the resonant radian frequency, and ζ is the damping ratio, where

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The solution to the above equation depends on the value of ζ (relation of ω to α)

CASE I (OVER DAMPED) $\zeta>1$ or ($\alpha>\omega$)

The two roots are real numbers (s1,s2), the solution is

$$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

CASE II (CRITICALLY DAMPED) $\zeta=1$ or ($\alpha=\omega$)

Two identical real roots (σ)

$$i(t) = D_1e^{\sigma t} + D_2te^{\sigma t}$$

CASE I (Under DAMPED) $\zeta<1$ or ($\alpha<\omega$)

Two complex conjugate roots, $s_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2} = -\sigma \pm j\omega_d$

$$i(t) = e^{-\sigma t}\left(B_1\cos(\omega_d t) + B_2\sin(\omega_d t)\right)$$

Once we know i(t), the voltages across the different elements are easy to calculate,

$v_R = i(t)R$, $v_L = L\frac{di}{dt}$, $v_C = -v_R - v_L$ or $v_C = -\frac{1}{C}\int_0^t i(\tau)d\tau + V_0$

We still need to determine the constants in the above equations (A1,A2,B1,B2, D1,D2). Determining the constants can be done from the initial conditions.

Two important rules, voltage across a capacitor and current in an inductor cannot change suddenly, that is to say

$$v_C(0^-) = v_C(0^+) \quad , \quad i_L(0^-) = i_L(0^+)$$

From these initial conditions we can solve to find the value of the 2 constants in the current equation.

For the natural response of an RLC circuit, the voltage across the capacitor can be found similarly.

$$v_c(t) = \begin{cases} A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{Overdamped} \\ e^{-\sigma t}\left(B_1 \cos(\omega_d t) + B_1 \sin(\omega_d t)\right) & \text{Underdamped} \\ D_1 e^{-\alpha t} + D_2 e^{-\alpha t} & \text{Critically damped} \end{cases}$$

For the step response, the solution is the same as the natural response with an added constant VF to reflect the forced response

$$v_c(t) = \begin{cases} V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{Overdamped} \\ V_f + e^{-\sigma t}\left(B_1 \cos(\omega_d t) + B_1 \sin(\omega_d t)\right) & \text{Underdamped} \\ V_f + D_1 e^{-\alpha t} + D_2 e^{-\alpha t} & \text{Critically damped} \end{cases}$$

PRELAB

Consider the circuit shown in Fig L4.1.

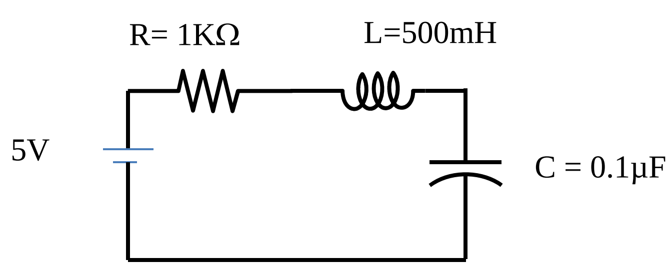

Figure L4.1 RLC Circuit

1. Solve the circuit to determine the voltage across the capacitor assuming no voltage or current at t=0
2. Using matlab plot the voltage as a function of time
3. What type of damping is that?

4. Using SPICE simulate the above circuit
5. Plot the voltage across the capacitor as a function of time
6. What damping is that? is there an overshot? Undershot? How much?
7. How long will it takes until the voltage reaches the steady state
8. What values for R, L, and C you suggest for the response to be overdamping? Underdamping? Critically damped? Simulate each case and show the plot in your report.

LAB:

In the lab, construct the circuit as the one in Figure L4.1

9. The input to the circuit is a square wave
10. The values of the R,L, C and the frequency will be given to you in the lab.
11. Use the scope to show the voltage across the capacitor
12. What kind of damping is that?
13. Sketch the waveform in your notebook