1. Problem 3.44

- As discussed in class the attenuation curves are related to the transfer functions of the physical media
- The more that a signal is attenuated at a particular frequency the less ~~that~~ of that signal passes through the physical medium (obviously)
- In Fig. 3.47 & 3.50 the attenuation per unit distance is greater for higher frequencies than for lower frequencies

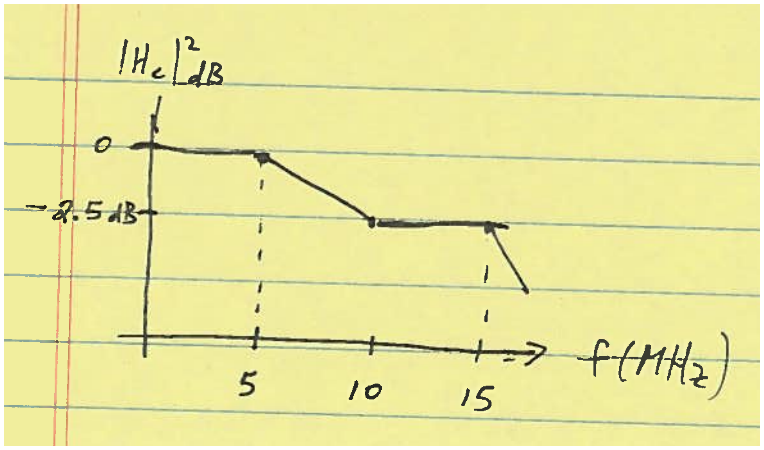
- For example for 1.2/4.4 mm coax the attenuation is about 2 dB/km @ 0.1 MHz & 15 dB/km @ 10 MHz
- a drop of 13 dB for a 1 km cable
- if the cable were 2 km long the drop would be

  $15 \times 2 - 2 \times 2 = 30 - 4 = 26\text{ dB}$ even bigger
- in fact to get back my 13 dB drop I would need a loss of...

$$x \times 2 - 2 \times 2 = 13$$
$$x = 8.5\text{ dB/km}$$

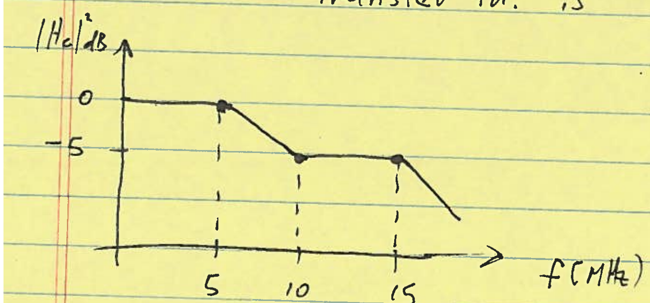
- looking @ Fig. 3.50 this occurs at roughly 2 MHz and is a reflection of the fact that my channel bandwidth has dropped when considering communication (through the same physical medium) over a longer distance

2. Sketch the transfer function over 500-m.

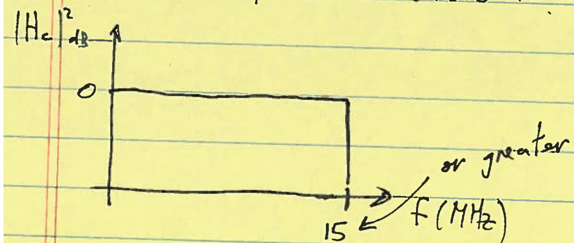


3. Equalizer response

for 1 km transfer fn. is

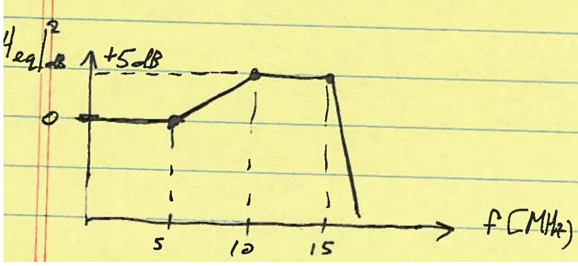


for 30 Mbps reasonable to expect spectrum of



30 Mbps in the form of sinc(t/T) pulse representation of bits requires at least 15 MHz bandwidth from channel to pass through without distortion

∴ make equalizer with profile



4. Problem 3.45

for 1200nm to 1400nm

velocity of light in fiber

$f = \frac{v}{\lambda}$ $f_1 = \frac{2\times10^8 \text{ m/s}}{1.2\times10^{-6} \text{ m}}$ $f_2 = \frac{2\times10^8}{1.4\times10^{-6}}$

$BW = f_1 - f_2 = 23.8\ THz$

$\frac{23.8\times10^{12}}{6\times10^9} = 5.95\times10^3\ Hz$ ~6 kHz per person

5. Problem 3.50

for $\Delta\lambda = 0.2nm$

$B = \frac{v\cdot\Delta\lambda}{\lambda^2} = \frac{2\times10^8\times0.2\times10^{-9}}{(1550\times10^{-9})} = 16.6\ GHz$

∴ assuming normal bandpass comms you can achieve bit rates up to 16.6 Gbps per wavelength used

standards at 10 and 2.4 Gbps exist, either of which could be used

6. Problem 3.55

- signal gets attenuated by a factor of d^2
- referenced relative to the loss experienced over a meter the loss in dB is

$$10 \times \log_{10}\left(\frac{36\times10^7 m}{1 m}\right)^2 = 20 \log_{10}(36\times10^7) = 151 dB$$

7. Problem 3.1

a) 1 MB file consists of...

$$8 \times 2^{20} = 8\,388\,608 \text{ bits}$$

@ 32kbps this takes

$$\frac{8\,388\,608}{32\times10^3} = 262.144 \text{ seconds to send}$$

c) at 1:6 compression obviously the size of the data is 6x smaller so it should only take...

$$\frac{262.144}{6} = 43.69 \text{ seconds}$$

8. Problem 3.3

computer monitor has 1200×800 pixels
$= 960{,}000$ pixels per screen

65,536 colors requires $\log_2 65{,}536 = \dfrac{\log_{10} 65{,}536}{\log_{10} 2}$
$= 16$ bits

$\therefore$ need $9.6 \times 10^5 \times 16 = 15.36$ Mbits per screen (Mb)

9. Problem 3.5

PCM is 8 bits every $\frac{1}{8000} = 125\mu s$

$= \dfrac{8}{125 \times 10^{-6}} = 64$ kbps

$\dfrac{45 \times 10^6}{64 \times 10^3} = 703.125 \Rightarrow 704$ channels needed

10. Problem 3.10

10^6 people $1\% = 10^{-2}$ on the phone

$\Rightarrow 10^6 \times 10^{-2} = 10^4$ on the phone @ any given time

w/ each using PCM's 64 kbps rate thats

$10^4 \times 64 \times 10^3 = 640$ Mbps

11. Problem 3.11

after n repeater stages...

... signal power is: σ_x^2

... noise power is: $n\sigma_n^2$

SNR @ i/p to repeater chain is $\dfrac{\sigma_x^2}{\sigma_n^2}$

SNR @ o/p of repeater chain is $\dfrac{\sigma_x^2}{n\sigma_n^2}$

$$SNR[dB] = 10 \log_{10} \frac{\sigma_x^2}{n\sigma_n^2}$$

using our relations

$$= 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} + 10 \log_{10}\left(\frac{1}{n}\right)$$

$$= 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} - 10 \log_{10}(n) + 10 \log_{10}(1)$$

$$= 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} - 10 \log_{10}(n)$$

12. Problem 3.12

- 50 ~~pr~~ repeaters in a link

chance of failure

- 1% chance of repeater failure over 1 year $p = 0.01$
- for link not to fail none of the 50 repeaters can fail over the course of a year
- chance of repeater not failing over 1 year is

$$(1-p)$$

- chance of 50 not failing is

$$(1-p)^{50} = (1-0.01)^{50} = 0.605$$

- only a 60% change of the link not failing!

13. Problem 3.13

if the signal is 10^k the noise power then your SNR is 10^k

so $\text{SNR [dB]} = 10 \log_{10} 10^k = 10 \cdot k \cdot \log_{10}(10)$

$$= 10 \cdot k$$

14. Problem 3.18

part a) ⇒ the problem comes down to the fact that our modem's data rate limits the number of bits we can send per second

BUT to adequately capture & send the voice signal in real-time we need to gather samples at least at the Nyquist rate

~~TUS~~ THUS to accommodate the needed sample rate our modem's limited bit rate bounds the number of bits we can use per sample & hence limits the SNR (via quantization noise)

$W = 8\,kHz$ ∴ by Nyquist 16 k samples/sec. are needed

with $R = 28.8$ kbps

→ only $\frac{28.8}{16} = 1.8$ bits per sample can be sent

→ since our following calcs. are approximate assume ≈ 2 bits / sample

→ assuming our ADC is set to $V_{max}/\sigma_x = 4$

$SNR\,[dB] = 6m - 7.2 = 12 - 7.2 = 5\,dB$

15. Problem 3.19

4 MHz signal requires $2W = 8 \times 10^6$ samples/sec
$(= 4 \times 10^6 \text{ Hz})$

assuming $\frac{V_{max}}{\sigma_x} = 4$

desired SNR

$$\text{SNR [dB]} = 6m - 7.2 = 60$$

$$\therefore 6m = 67$$

$$m \approx 11 \text{ bits}$$

$$\therefore R = \underbrace{2W}_{\text{sample rate}} \times \underbrace{11}_{\text{bits per sample}} = 88 \text{ Mbps}$$

16. Problem 3.21

- a signal is uniformly distributed between $-V$ and V

- that means it has equal probability to occur at any of the values between $-V$ and V inclusive

- its average value is obviously zero, but this isn't of much use when trying to gauge our quantizer it it, so we instead gauge its average fluctuation from zero which is the variance σ_x^2 (alternately referred to as power because of its relation to that quantity)

$$\sigma_x^2 = \text{avg. value of the (signal squared)}$$

$$= \underbrace{\frac{1}{2V}\int_{-V}^{V}}\, x^2\,dx$$

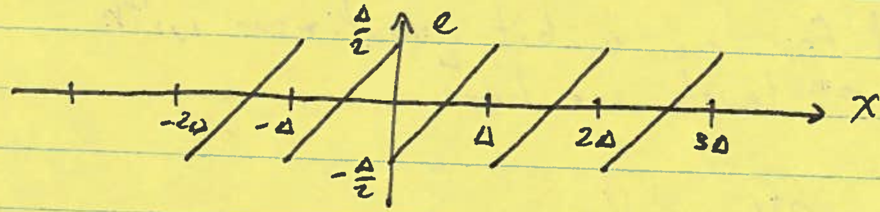
sum over $2V$ (from $-V$ to V) & then divide by the total span (which is $2V$)

$$= \frac{1}{2V}\,\frac{x^3}{3}\Bigg|_{-V}^{V} = \frac{1}{2V}\left(\frac{V^3}{3} - \frac{(-V)^3}{3}\right)$$

$$= \frac{1}{2V}\left(\frac{2V^3}{3}\right) = \frac{V^2}{3} = \sigma_x^2$$

$$\frac{V^2}{\sigma_x^2} = 3 \qquad \frac{V}{\sigma_x} = \sqrt{3}$$

- as outlined in the book the quantizer error for a simple quantizer takes on the form



- thus assuming the input signal, x, is uniformly distributed across the quantizing range the error signal is also uniformly distributed but only over the range $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$

- just as with the signal then we can calculate it's mean square error (i.e. noise power) with

$$\sigma_e^2 = \frac{1}{2\left(\frac{\Delta}{2}\right)} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^2 de = \frac{1}{\Delta} \left. \frac{e^3}{3} \right|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

$$= \frac{1}{\Delta} \left(\frac{(\Delta/2)^3}{3} - \frac{(-\Delta/2)^3}{3} \right)$$

$$= \frac{1}{\Delta} \left(\frac{\Delta^3}{24} + \frac{\Delta^3}{24} \right) = \frac{\Delta^2}{12}$$

- thus the SNR of the quantized signal becomes

$$SNR = \frac{\sigma_x^2}{\sigma_e^2} = \frac{\sigma_x^2}{\Delta^2/12}$$

- recall that in our case $\sigma_x^2 = \frac{V^2}{3}$
- and for an m-bit quantizer with $M = 2^m$ levels we have

$$\Delta = \frac{2V}{2^m} = \frac{V}{2^{m-1}}$$

$$\Delta^2 = \frac{V^2}{2^{2m-2}} = \frac{V^2}{2^{2m}2^{-2}} = \frac{4V^2}{2^{2m}}$$

$$\therefore \frac{\sigma_x^2}{\Delta^2/12} = \frac{V^2}{3} \times \frac{2^{2m}}{4V^2} \times 12 = 2^{2m} = SNR$$

$$\therefore SNR\ dB = 10 \cdot \log 2^{2m} = 20m \cdot \log 2$$

$$= 20m \cdot 0.3$$

$$SNR\ dB = 6m$$

17. Problem 3.27

10 kHz channel can accommodate

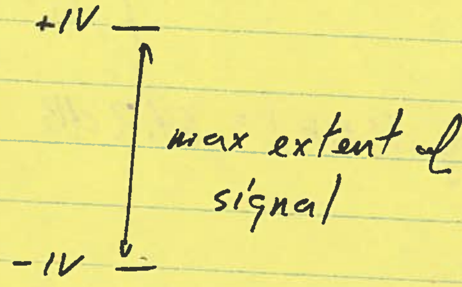
20×10^3 pulses per second (~~according to~~ for a pulse that satisfies the Nyquist criterion) (w/o ISI)

if each pulse takes ~~to~~ one of 16 possible levels than $\log_2 16 = 4$ bits are effectively being carried by each pulse

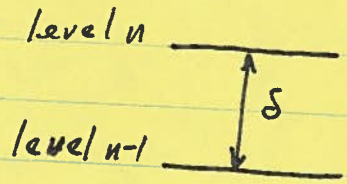
$\therefore$ the bit rate of the system is

$4 \times 20 \times 10^3 = 80$ kbps

18. Problem 3.28



2 levels within such a span ...



... can be separated by some distance δ

to be able to tell n & n-1 apart the level representing them ∴ cannot stray by more than +/- $\frac{\delta}{2}$ since my noise is $\pm \frac{1}{16}$

∴ $\delta = \frac{1}{8}$ and the total # of levels that I can sustain w/o error is $= \frac{1-(-1)}{1/8} = 2 \cdot 8 = 16$

19. Problem 3.30

- this is handled by the Shannon channel capacity expression

want $C = 64$ kbps

have $W = 3$ kHz what SNR do I need?

$$64 \times 10^3 = \cancel{3\times10^3 (1 + SNR)} = 3\times10^3 \log_2(1+SNR)$$

$$\frac{64}{3} = \log_2(1+SNR)$$

$$2^{\frac{64}{3}} = 1 + SNR$$

$$SNR = 2.64 \times 10^6$$

$$SNR\,[dB] = 10 \log_{10}(SNR) = 64.2\text{ dB}$$

min. SNR needed to get the required bit rate through the given channel

20. Problem 3.36

(not baseband)

a) 6-MHz **bandpass** normally has

$R = 6 \times 10^6$ bps (not $2 \times 6 \times 10^6$ bps)

- however 4 point QAM effectively sends 2-bits per pulse

$\therefore R\big|_{4\text{-QAM}} = 12$ Mbps

- an 8-point constellation effectively sends $\log_2 8 = 3$ bits per pulse so

$R\big|_{8\text{-QAM}} = 18$ Mbps

b) for a 4 Mbps signal

$\frac{12}{4} = 3$ channels can be accommodated with 4-QAM

$\lfloor \frac{18}{4} \rfloor = 4$ channel can be accommodated with 8-QAM

21. Problem 3.37

- in radio telegraphy operators listened to the presence or absence of a tone

- this is essentially amplitude modulation (over varying periods of time) a "dot" was a carrier ~~on~~ for a "short time" (a "0")
  a dash was a carrier on for a "long time" (a "1")

- the absence of a tone (silence) or "space" ~~but~~ allowed the operator to "frame" the bits

22. Problem 3.38

a) each tone can be present or absent
$\therefore$ there are 2^8 tone combination which can be sent every T seconds

$\therefore$ $\cancel{\log_2(8)} = \log_2(2^8) = 8$ bits can be sent every T seconds

$\therefore$ bit rate is $R = \frac{8}{T}$

b) for a tone of ~~exact~~ unknown amplitude I need to observe the signal for at least the length of the period $\therefore$ T must be longer than the period of the lowest freq. tone

23. Problem 3.42

a) $40 \times 10^3 \ \frac{\text{samples}}{\text{sec}} \times 16 \ \frac{\text{bits}}{\text{sample}} = 640 \text{ kbps}$

∴ for both channels (of stereo) we have
$640 \times 2 = 1.28 \text{ Mbps}$

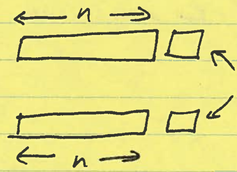
b) 200 kHz (bandpass) allows 200×10^3 pulses per s (w/o ISI) ∴ to send 1.28 Mbps need

$$\frac{1.28 \times 10^6}{200 \times 10^3} = 6.4 \text{ bits/pulse}$$

∴ need $2^7 = 128$ point constellation

24. Problem 3.59

a)



2 checkbits

effectively 2 codewords
each codeword is n+1 bits long

- error detectable if each codeword has odd # of errors

b) Can think of the ~~old~~ net 2n+2 bit codeword as 2 n+1 ~~codewords~~ bit (sub-codewords)

- let P_d be the probability that ~~either of the~~ a sub-codeword can detect an error

(this is just the probability that a single parity code word can detect an error)

- then the probability of both sub-codewords failing to detect an error is

$1 - P_d \cdot P_d$

c) no, it does not help to add a third parity check that just sums all the information bits

as what fools the parity bit for each sub-codeword (i.e. an even # of errors in each sub-codeword) would fool a parity bit summing over all sub-codewords as well (since ~~2~~ evens sum to evens)