EECS 3213 Fall 2014

L12: Modulation

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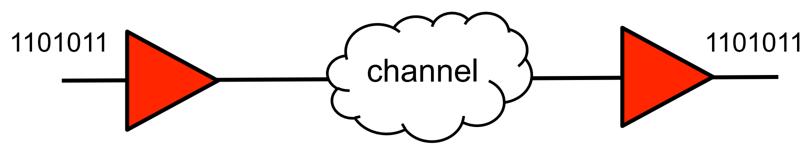
Outline

- Review
- Passband Modulation
  - ASK, FSK, PSK
- Constellations

Underlying Idea

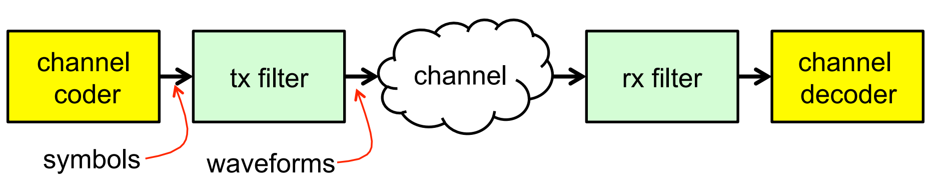
- Attempting to send a sequence of digits through a continuous channel



- Not easy…
- …modulerize the design

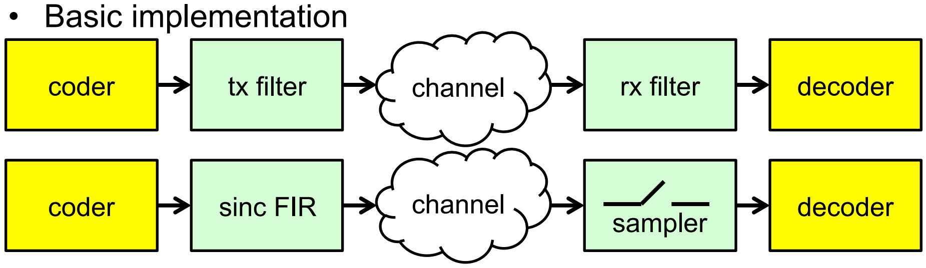
Continuous-Time Communication

- Map symbols into analog waveforms that match characteristics of channel
  - spectral shape
  - location (carrier)

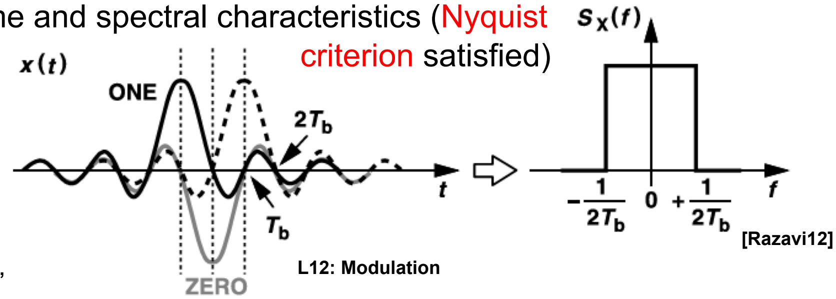


Simple Baseband Modulator (Pulse Shaper)

- Basic implementation



- Time and spectral characteristics (Nyquist criterion satisfied)



Spectral Efficiency

- What is the bit-rate relative to the spectrum used:

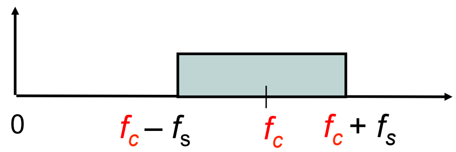
$$v = \frac{\text{bit-rate}}{\text{bandwidth}}$$

$$v = \frac{2f_s \cdot \log_2(M)}{f_s} \left[\frac{\text{bits}}{\text{s} \cdot \text{Hz}}\right]$$

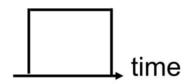
- Simple 2-level baseband modulation 2 bits/s•Hz
- or 2 symbols/s•Hz

Passband Modulation

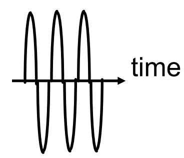
- What if the channel is not baseband?



- Replace DC 1/0 representation…

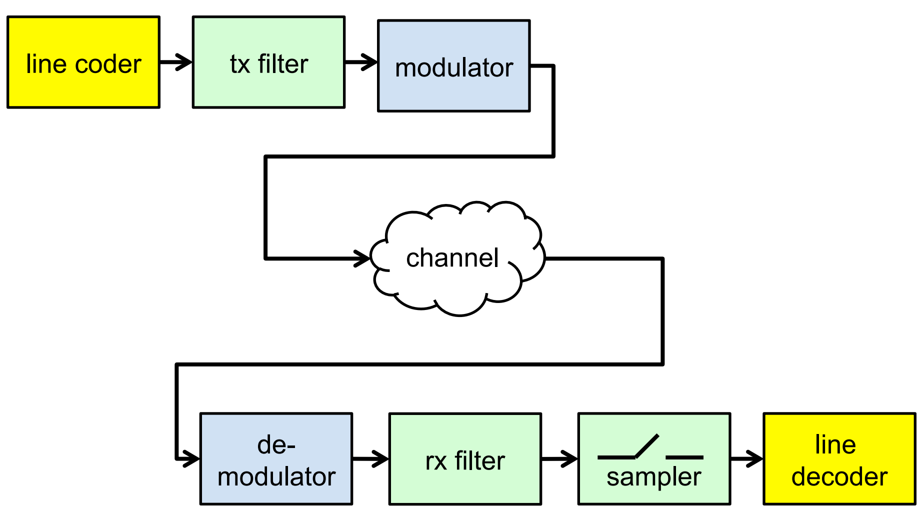


- …with AC representation…



- Have 3 ways to modulate a sinusoid: $A_c\cos(\omega_c t + \phi)$

Modulator in a Communication System

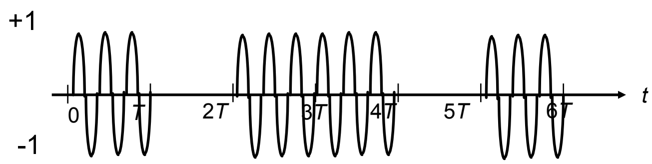


Amplitude Modulation

- $A_c \cos(\omega_c t + \phi)$
- Map bits into amplitude of sinusoid: "1" send sinusoid; "0" no sinusoid
- Demodulator looks for signal vs. no signal

Information 1 0 1 1 0 1

Amplitude Shift Keying

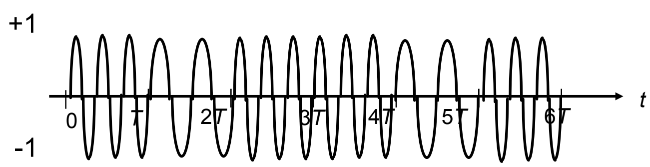


Frequency Modulation

- $A_c \cos(\omega_c t + \phi)$
- Map bits into frequency: "1" send frequency $f_c + \delta$; "0" send frequency $f_c - \delta$
- Demodulator looks for power around $f_c + \delta$ or $f_c - \delta$

Information 1 0 1 1 0 1

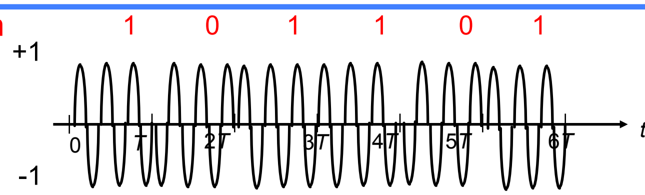
Frequency Shift Keying



(Binary) Phase Modulation

Information

Binary Phase Shift Keying (BPSK)



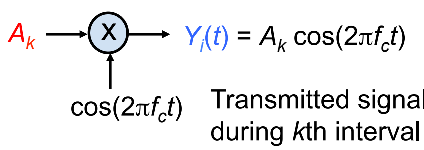
- Map bits into phase of sinusoid:
  - "1" send A $\cos(2\pi ft)$, i.e. phase is 0
  - "0" send A $\cos(2\pi ft+\pi)$, i.e. phase is π

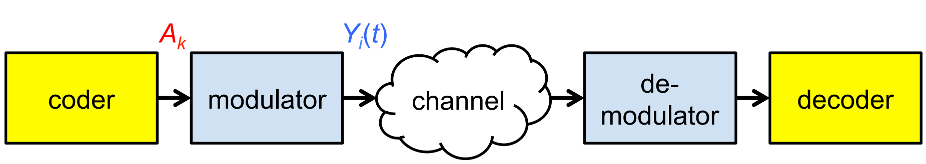
- Equivalent to multiplying $\cos(2\pi ft)$ by +A or -A
  - "1" send A $\cos(2\pi ft)$, i.e. multiply by 1
  - "0" send A $\cos(2\pi ft+\pi)$ = - A $\cos(2\pi ft)$, i.e. multiply by -1

- We will focus on phase modulation

AM Modulator

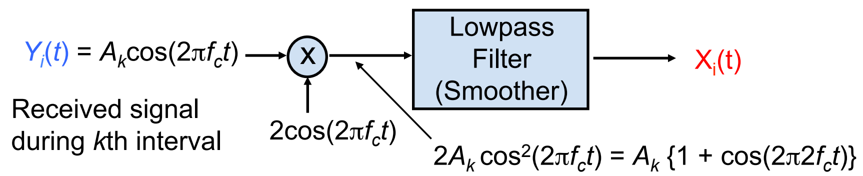
- Simplest and most dominant case, multiply symbol by a carrier
- Modulate $\cos(2\pi f_c t)$ by multiplying by A_k for T seconds:

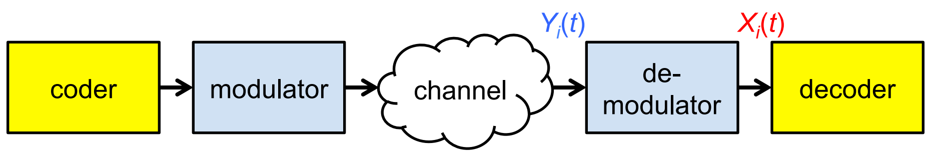




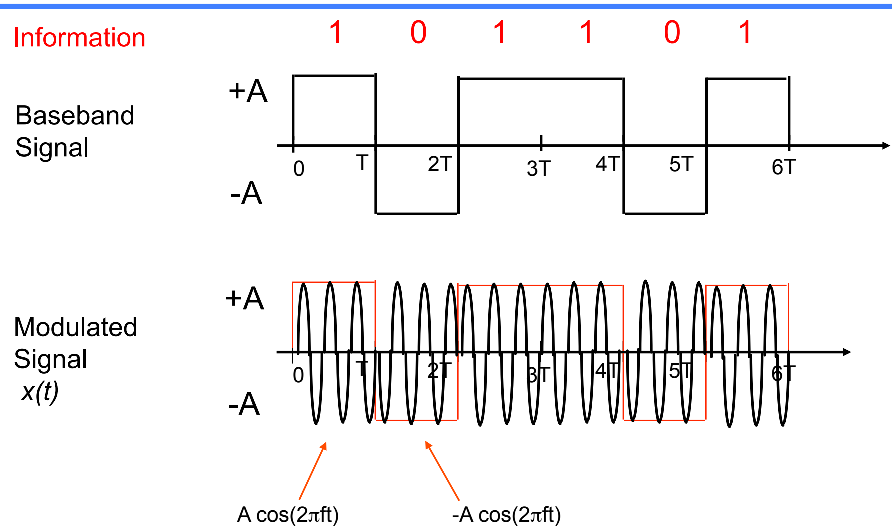
AM Demodulator

- Demodulate (recover A_k) by multiplying by $2\cos(2\pi f_c t)$ for T seconds and lowpass filtering (smoothing):

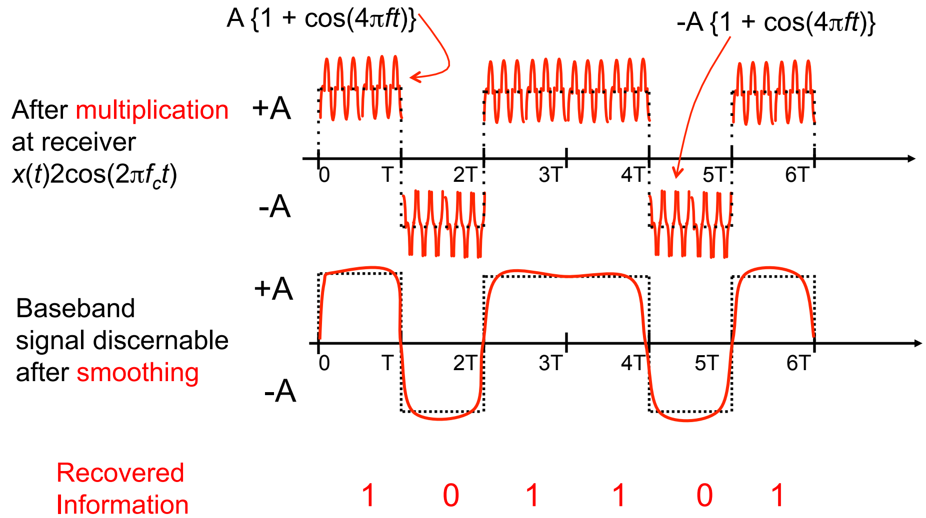




Example of Modulation



Example of Demodulation



Signaling Rate and Transmission Bandwidth

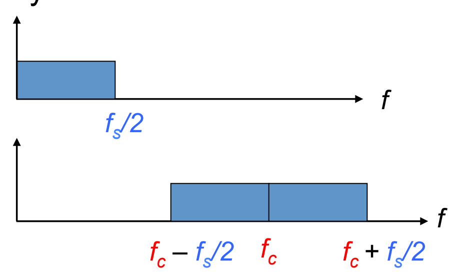
- Fact from modulation theory:

If

Baseband signal $x(t)$ with bandwidth $f_s/2$ Hz

then

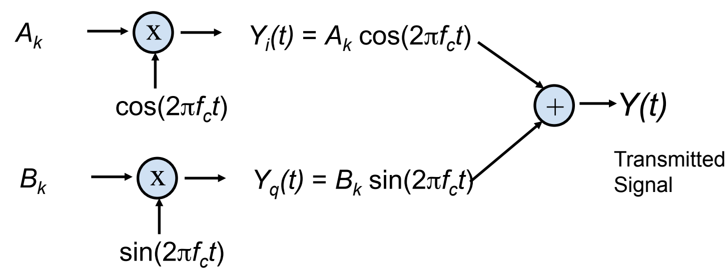
Modulated signal $x(t)\cos(2\pi f_c t)$ has bandwidth f_s Hz



- If bandpass channel has bandwidth f_s Hz,
  - It's baseband version has bandwidth of $f_s/2$ Hz, so…
  - …modulation system supports $f_s/2 \times 2 = f_s$ symbols/second
  - That is, f_s symbols/second per f_s Hz = 1 symbols/s•Hz
  - Recall baseband transmission system supports 2 symbols/s•Hz !!!

Quadrature Amplitude Modulation (QAM)

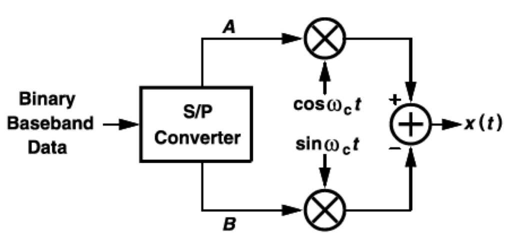
- QAM uses two-dimensional signaling
  - A_k modulates in-phase $\cos(2\pi f_c t)$
  - B_k modulates quadrature phase $\cos(2\pi f_c t + \pi/4) = \sin(2\pi f_c t)$
  - Transmit sum of in-phase & quadrature phase components

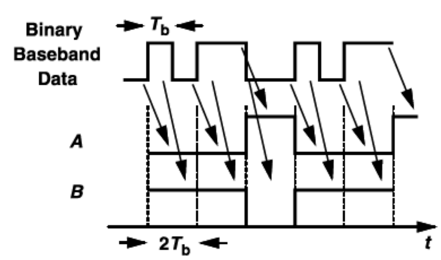


- $Y_i(t)$ and $Y_q(t)$ both occupy the bandpass channel
- QAM sends 2 symbols/s•Hz

QAM Signal Mapping

- Serial-to-parallel converter distributes input bits
- Two bits sent simultaneously





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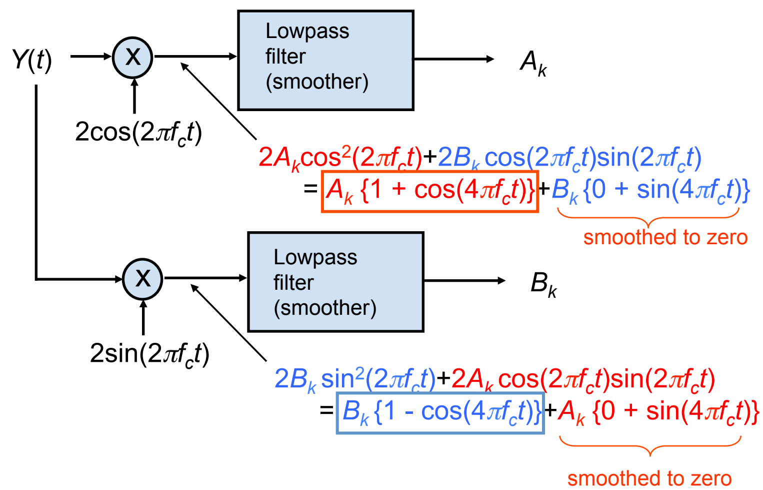
QAM Demodulation



Signal Constellations

- Convenient to write modulated signals in "quadrature" form…

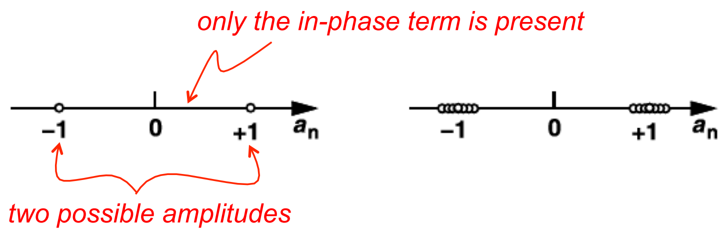
$$y(t) = y_i(t)\cos(2\pi f_c t) + y_q(t)\sin(2\pi f_c t)$$

  – "in-phase" and "quadrature" components
- And to plot it as a signal constellation in the complex place…
  – Plot samples of in-phase terms along real axis
  – Plot samples of quadrature terms along imaginary axis

BPSK Constellation

- For example…

$$y_{BPSK}(t) = a_n\cos(\omega_c t),\ a_n = \pm 1$$



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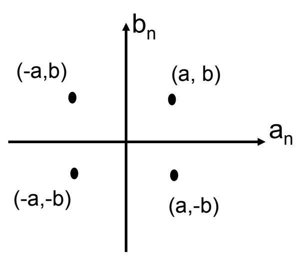
4-QAM Constellation

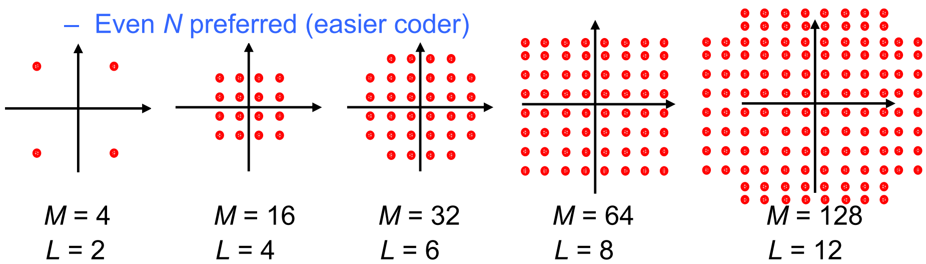
- Both terms present
  - In-phase and quadrature

$$y_{QAM}(t) = a_n\cos(\omega_c t) + b_n\sin(\omega_c t)\ ,\ a_n,b_n = \pm 1$$



Larger QAM Constellations

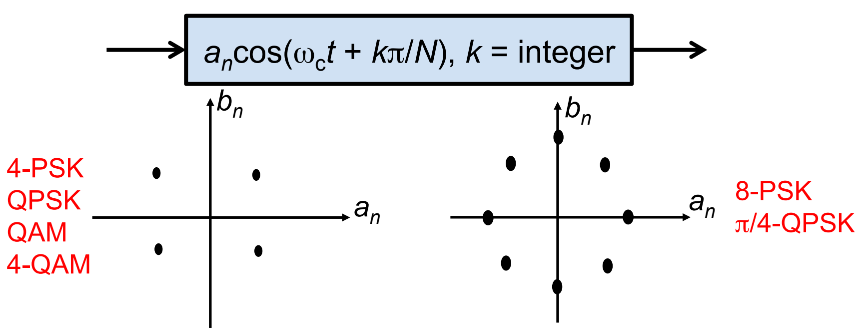
- With 4-QAM
  - bits represented per symbol: $N = 2$
  - Constellation points: $M = 2^N = 4$
- Many other possibilities (rectangular constellation)
  - $N = 3,4,5,...$
  - Even N preferred (easier coder)



- Increasing N requires more power

Phase-Shift Keying (PSK)

- A common variant
- Conceptually, vary phase of signal based on symbol (already saw this)



- Constant amplitude

π/4-QPSK Modulator

- Just 2 QAM modulators
  - phase shifted relative to each other
  - top modulator has both cos/sin always active
  - bottom modulator has either cos/sin active