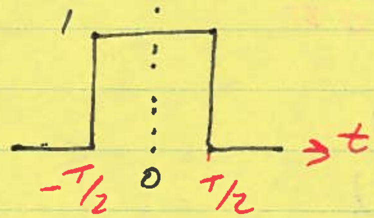
L4: Basic Network Calculations

4.1 Data Rate

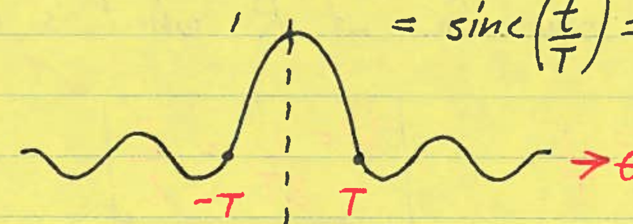
a) For bits made of:



$= rect\left(\frac{t}{T}\right)$

what is R? (data rate) $R = \frac{1}{T}$

b) How about...



$= sinc\left(\frac{t}{T}\right) = \frac{\sin(\pi t/T)}{\pi t/T}$

also $R = \frac{1}{T}$ ← aka "bandwidth"... "data bandwidth"

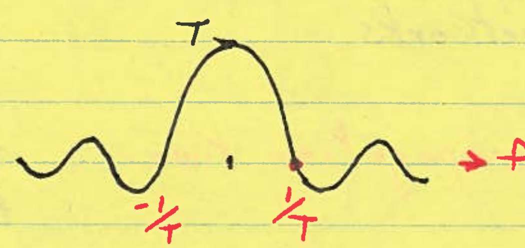
~~4.2 Bandwid~~ 4.2 Signal Bandwidth

- extent of signal energy or power in frequency
- for pulses with finite energy just ~~th~~take Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt$$

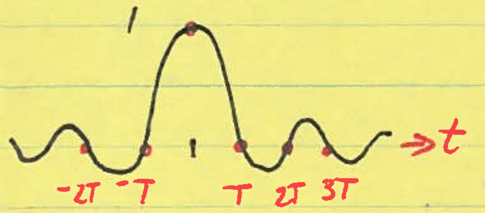
a)

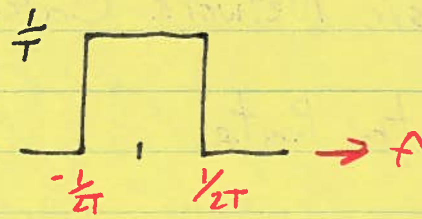
⟷



$$rect\left(\frac{t}{T}\right) \longleftrightarrow T \cdot sinc(fT) = T \cdot \frac{\sin(\pi \cdot f \cdot T)}{\pi \cdot f \cdot T}$$

b)





$\text{sinc}(t/T)$

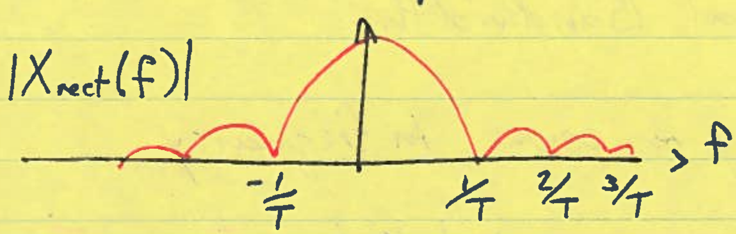
$= \dfrac{\sin(\pi t/T)}{\pi t/T}$ $\longleftrightarrow$ ~~$\frac{1}{T}\cdot \text{rect}\left(\frac{f}{2T}\right)$~~

$\dfrac{1}{T}\cdot \text{rect}\left(f \cdot T\right)$

- for sinc fn. at R what is W?

  $R = \dfrac{1}{T}$ $\boxed{W = \dfrac{1}{2T} = \dfrac{R}{2}}$

- and for rect? depends on what part of spectrum is ok to cut off



4.3 Delay

(3)

- Signals don't travel instantaneously across networks

- propagation time : amt. of time for physical signal to propagate

$$t_{prop} = \frac{d}{v} = \frac{d}{\sqrt{\epsilon_r} c}$$

$v = c$ in air

$v = \frac{2}{3}c$ in media (cables)

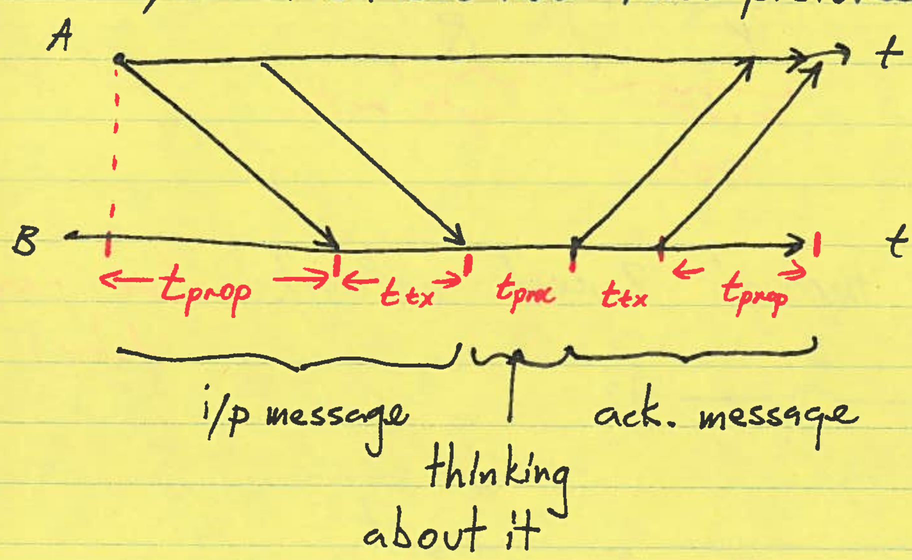
- transmission time: how long it takes to get a message/segment/packet/frame into a line

frame time in book

$$t_{tx} = t_f = t_{ack}$$

acknowledgment time

you often see ~~this~~ these pictures



total delay $= 2t_{prop} + t_f + t_{proc} + t_{ack}$

noting differences in "transmission time"

4.4 Delay of Packets Through Network

- ignore propagation delay
- message of : M bytes

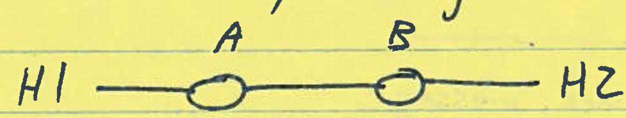
  data rate : R bps

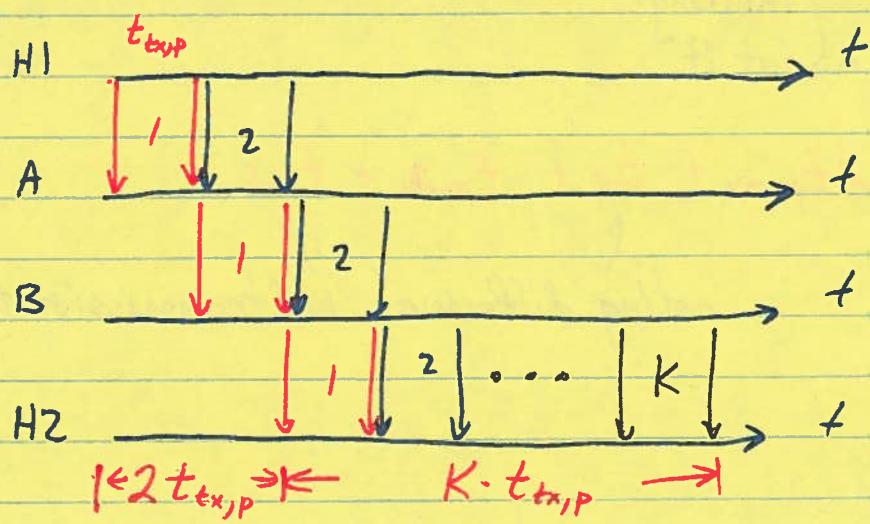
  packets : K (message split up into this)

  header : H bytes per packet

$\therefore \; t_{tx,p} = ? = \underbrace{\left(\frac{M}{K} + H\right)}_{\text{packet size}} \cdot \underbrace{\frac{8}{R}}_{\text{byte rate}}$

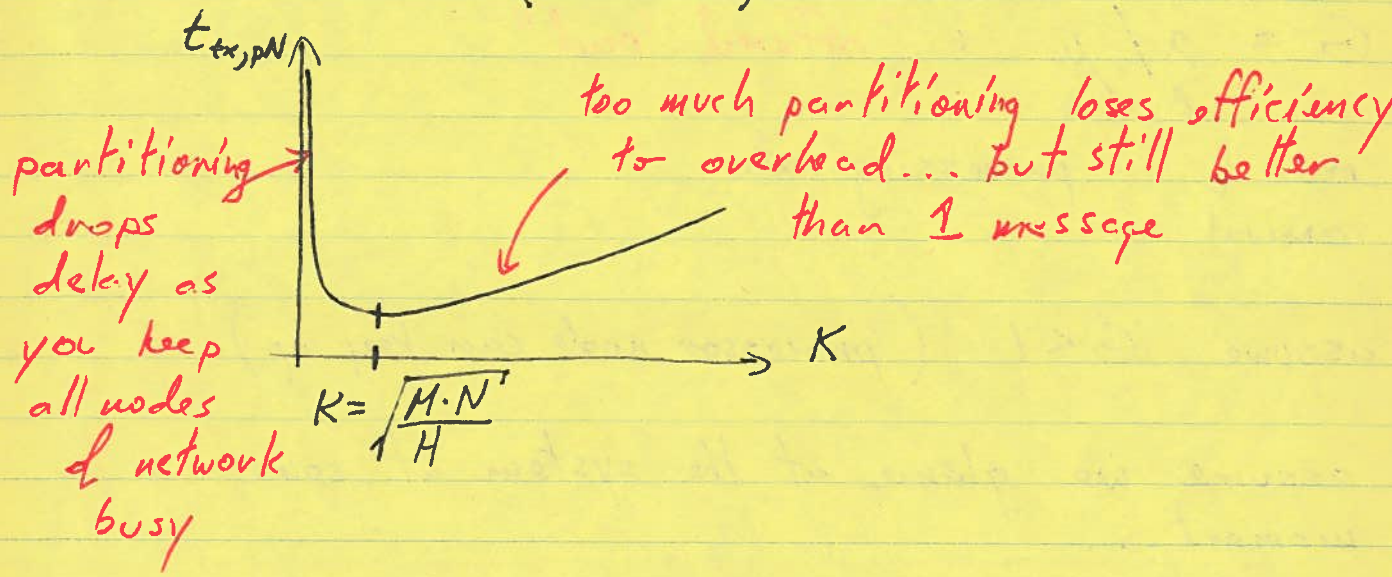
- total delay through 2 node network? (no propagation delay)





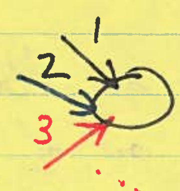
in general for N node network?

$$t_{tx,pN} = \frac{8}{R}\left(\frac{M}{K} + H\right)(N+K)$$



4.5 Queuing Delay

- Many different sources may be trying to squeeze through a node



- this will cause some average delay
- a basic estimation

λ : packets arriving per second (arrival rate)

$\bar{H}$ ~~t_{tx}~~ : avg. duration of packet (<u>like</u> transmission time)

$= \frac{L}{R}$

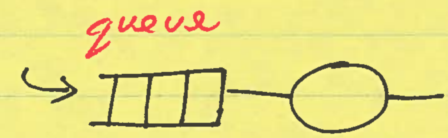
$\mu = \frac{1}{t_{tx} H}$: processing rate ... avg. time taken to process a packet by node

$G = \lambda / \mu$: "offered load"

(λ = rate of arrival; μ = processing rate)

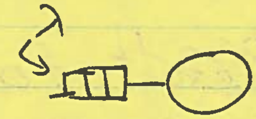
assume $G < 1$ (processor node can keep up)

- assume we glance at the system at some moment...



... and see some elements in the queue

- at what rate do I empty queue $= \mu - \lambda$
- $\therefore$ avg. time to empty queue $= T = \frac{1}{\mu - \lambda}$ -OR- avg. time to get out of node
- $\therefore$ avg. # of packets in que $= \lambda \cdot T = \frac{\lambda}{\mu - \lambda} = K$
- $\therefore$ avg. time to get to front of queue $= T - \frac{1}{\mu}$

e.g. $\lambda = 1000$ packets per sec. 

$L = 1KB$ $\qquad R = 10$ Mbps

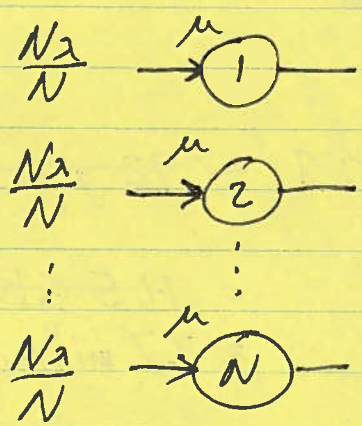
$\overline{H}$ ~~x~~ $= \frac{L \cdot 8}{R}$ $\qquad \therefore \mu = \frac{R}{8 \cdot L} = \frac{10^7}{8 \times 10^3} = 1{,}250$ packets/s

$\therefore T = \frac{1}{1250 - 1000} = 4ms \qquad K = \frac{\lambda}{\mu - \lambda} = \frac{1000}{1250-1000} = 4$

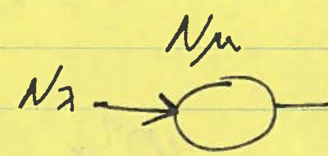
$\frac{1}{\mu} = 0.8ms$ ... spend avg. of 3.2 ms in queue before node starts processing me

4.6 Link Sharing

What's better as $\lambda \rightarrow N\lambda$

a) 

-OR-

b) 

$T_b = \frac{1}{N\mu - N\lambda} = \frac{T_a}{N}$

$T_a = \frac{1}{\mu - \lambda}$

everybody goes slow

- better to share a faster link than rely on a buch of slow parallel ones

4.7 Little's ~~Result~~ Formula

- another interesting result from ~~queu~~ queuing theory

$$N = \lambda \cdot T$$

- N ← avg. # of packets in system
- λ ← rate of arrival in system
- T ← avg. time spent in system

e.g.
- 1 billion users send packets into Internet
- avg. rate of 10 MB/day
- assume each packet spends 100 ms in internet

∴ avg. # of ~~packets~~ bits in Internet

$$10^9 \times \frac{~~8\times~~ 10^7 / 1500}{24 \times 60 \times 60} \times 0.1 = ~~92 \text{ giga bits}~~$$

= ~~11.5 GB~~

= 7.7 million

- also useful in approximating delays over large networks

4.8 Throughput

- ~~a couple of definitions~~ rate at which messages succesfully get through the network

$$S = P[\text{success}] \cdot G$$ ← normalized to avg. duration of packet

S: carried load, carried traffic intensity, throughput

G: offered load, offered traffic intensity

$$S = P[\text{success}] \cdot \lambda \cdot \overline{H}$$

λ: packet arrival rate

$\overline{H}$: avg. packet duration time (e.g. $\overline{H} = \frac{L}{R}$)

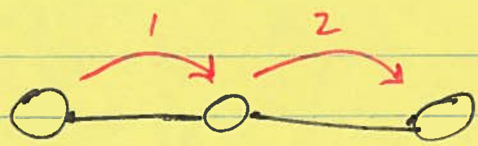
$$S = P[\text{success}] \cdot \frac{\lambda}{\mu}$$

- more simply, if you notice it takes 3MB 2 minutes to download

$$S = \frac{8 \times 3 \times 10^6}{120 \text{ sec}} = 2 \times 10^5 \text{ bps} = 200 \text{ kbps}$$

4.9 Errors

- just an error example
- Have message of $L = 10^6$ bits
- And want to send it over 2 hops



but each hop has:
error rate of $p = 10^{-6}$

How many bits do we have to send?
What's the probability that the message gets through on 1 hop?

$$P_c = \underbrace{(1-p)}_{\text{no error in any bit}}{}^{L} \approx e^{-Lp} = e^{-1} \approx 1/3$$

$\therefore$ only 1/3 chance ... error free per hop
$\therefore$ it will take on avg. 3 tries per hop
$\therefore$ need to send 6 Mbits over 2 hops

What if ... message split into 10^5-bit packets

$P_c' \approx e^{-1/10} \approx 0.9$ $\therefore$ need to send each packet 1.1 times on avg.

$\therefore$ 2.2 Mbits over 2 hops