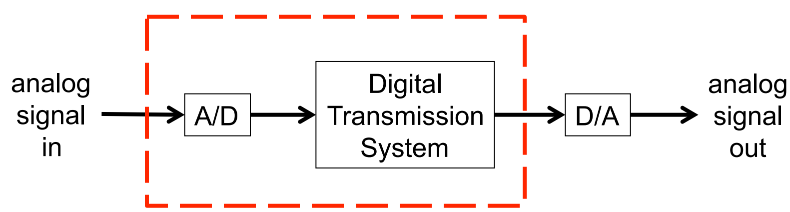
EECS Fall 2014

L9: Digitization

Sebastian Magierowski
York University

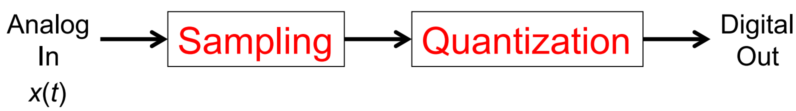
Outline



- How are analog signals digitized?
- How are digital signals transmitted/communicated?
  - Subject of following lecture
- Describe these mostly from the conceptual level

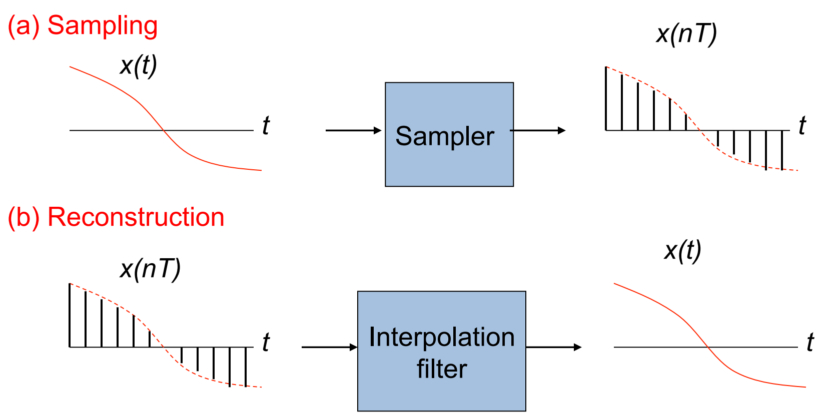
Digitization of Analog Signals

1. Sampling: obtain samples of $x(t)$ at discrete time intervals
2. Quantization: map each sample into an approximation value of finite precision
   - Pulse Code Modulation: telephone speech
   - CD audio

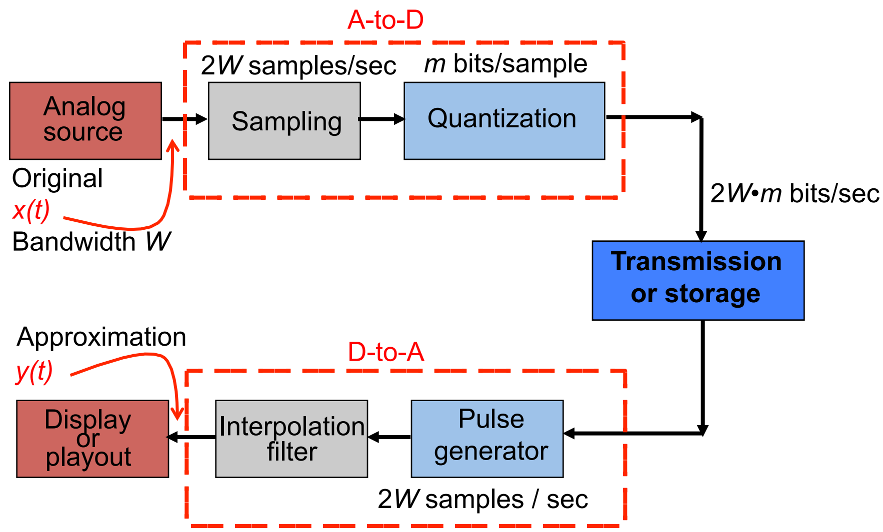


Sampling Theorem

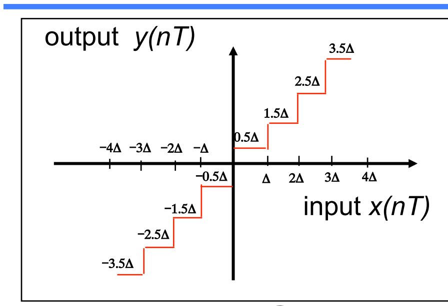
Nyquist: Perfect reconstruction if sampling rate $1/T \geq 2W_s$



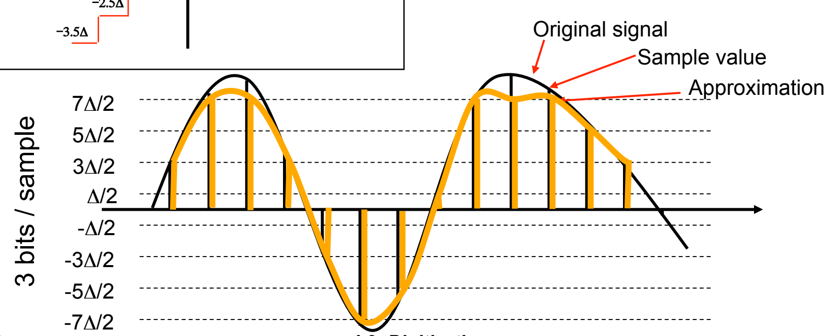
Digital Transmission of Analog Information



Quantization of Analog Samples

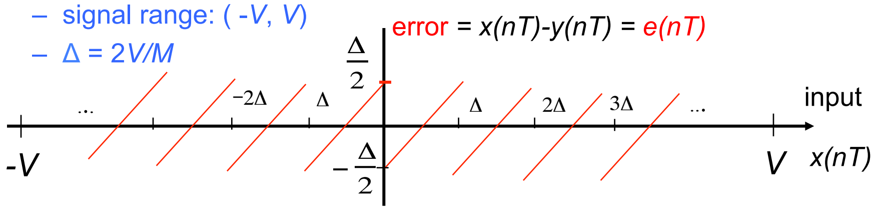


- Quantizer maps input
  - closest of 2^m representation values
- Quantization error
  - "noise" = $x(nT) - y(nT)$



Quantizer Performance

- $M = 2^m$ levels
  - signal range: $(-V, V)$
  - $\Delta = 2V/M$



- If M is large, then error is roughly uniformly distributed between $(-\Delta/2, \Delta/2)$
- Average Noise Power = Mean Square Error:

$$\sigma_e^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e(x)^2 dx = \frac{\Delta^2}{12}$$

Quantizer Performance

- Figure of Merit:
  - Signal-to-Noise Ratio = Avg signal power / Avg noise power
  - Let σ_x^2 be the signal power, then

$$SNR = \frac{\sigma_x^2}{\Delta^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3\left(\frac{\sigma_x}{V}\right)^2 M^2 = 3\left(\frac{\sigma_x}{V}\right)^2 2^{2m}$$

- The ratio $V/\sigma_x \approx 4$
- The SNR is usually stated in decibels:
- SNR [dB] = $10 \log_{10} \sigma_x^2/\sigma_e^2 = 6 + 10 \log_{10} 3\sigma_x^2/V^2$
- SNR [dB] = $6m - 7.27$ dB for $V/\sigma_x = 4$.

Example: Telephone Speech

W = 4 kHz, so Nyquist sampling theorem

⇒ 2W = 8000 samples/second

Suppose error requirement = 1% error

SNR = 10 log$(1/.01)^2$ = 40 dB

Assume $V/\sigma_x = 4$, then

40 dB = 6m – 7

⇒ m = 8 bits/sample

PCM ("Pulse Code Modulation") Telephone Speech:

Bit rate= 8000 x 8 bits/sec= 64 kbps