

1. 1.20 (a)-(d)

a) $\frac{1}{2}$ power BW

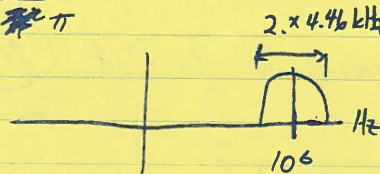
\Rightarrow at what f does $G_x(f)$ drop to $\frac{1}{2}$ its pk. val.

$$\frac{1}{2} = \left[\frac{\sin(\pi f_0 \times 10^{-4})}{\pi f_0 \times 10^{-4}} \right]^2 \quad (\text{forget about offset})$$

$$\frac{\sin \pi x}{\pi x} = \text{sinc}(x) = \frac{1}{\sqrt{2}} \quad \text{at } x \approx \frac{1.4}{\pi}$$

$$\therefore f_0 \times 10^{-4} = \frac{1.4}{\pi} \quad \text{represents } \frac{1}{2} \text{ power point}$$

$$f_0 = \frac{1.4 \times 10^4}{\pi} = 4.46 \text{ kHz}$$



$$\text{BW} \approx 2 \times 4.46 \approx 9 \text{ kHz}$$

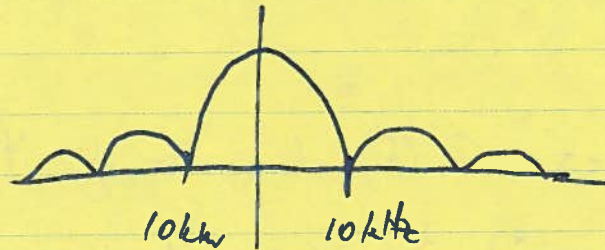
b) noise equiv BW :

$$W_N G_x(f_c) = P_x$$

$$W_N \cdot 1 = 2 \int_0^{\infty} \left[\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}} \right]^2 df = 2 \times 10^4 \int_0^{\infty} \left(\frac{\sin \pi x}{\pi x} \right)^2 dx$$

$$x = f \times 10^{-4} \quad = 10 \text{ kHz}$$

c) null-to-null BW



$$= 20 \text{ kHz}$$

d) 99% pwr. BW

$$0.99 = \frac{10^{-4} \int_0^{f_0} \text{sinc}^2(f \times 10^{-4}) df}{10^{-4} \int_0^{\infty} \text{sinc}^2(f \times 10^{-4}) df}$$

solve numerically to find 206 kHz

2. 2.2

$$a) 6400 \text{ bps} \quad b) 1600 \text{ symbols/s} = \frac{6400 \text{ bps}}{4 \text{ bits/symbol}}$$

3. 2.4

$$\begin{aligned}
 x_s(t) &= x(t) \cdot p(t) \\
 &= x(t) \left\{ \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t} \right\} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ \text{j-sin's cancel} \end{array} \\
 &= x(t) \cdot \left\{ c_0 + 2 \sum_{\substack{n=1 \\ n \neq m}}^{\infty} c_n \cos 2\pi n f_s t \right\} \\
 x_1(t) &= x_s(t) \cos 2\pi m f_s t \quad x_s \rightarrow \text{X} \rightarrow x_0(t) \\
 &= x(t) \cdot \left(\begin{array}{l} c_0 \cos 2\pi m f_s t \\ + 2 \sum_{\substack{n=1 \\ n \neq m}}^{\infty} c_n \cos 2\pi n f_s t \cos 2\pi m f_s t \\ + 2 c_m \cos^2 2\pi m f_s t \end{array} \right) \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{removed} \\ \text{by} \\ \text{LPF} \end{array} \\
 &\quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ \text{also removed} \\ \text{by LPF} \end{array} \\
 x_1(t) \rightarrow \text{LPF} \rightarrow x_0(t) &= x(t) \cdot 2 c_m \left(\frac{1}{2} + \frac{1}{2} \cos 4\pi m f_s t \right) \\
 &= c_m x(t)
 \end{aligned}$$

4. 2.5

$$a) \quad b \geq \log_2 L$$

↑
of bits needed for L levels

#. b ~~bits~~ bits need to be sent at least every T_s

$$R \geq b \cdot f_s$$

$$R \geq \log_2 L \cdot \frac{1}{T_s}$$

$$\frac{1}{R} = T \leq \frac{T_s}{\log_2 L}$$

b) equality is valid if L is a power of 2

5. 2.6

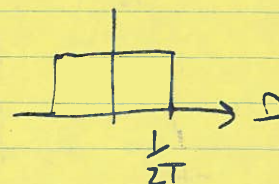
$$a) \quad 32 \quad b) \quad 256 \quad c) \quad 2^x$$

6. 2.7

$$x(t) = \frac{\sin 6280t}{6280t} = \frac{\sin \pi t/T}{\pi t/T}$$

$$\therefore T = 5 \times 10^{-4} \text{ s}$$

$$f_m = \frac{1}{10 \times 10^{-4}} = 1 \text{ kHz}$$



$$\therefore f_s = 2000 \text{ samples/second}$$

7. 2.8

$$a) \quad \text{SNR}_q = 3L^2 \quad (\text{eqn. 2.20 in text})$$

$$10 \log(3L^2) \geq 30 \text{ dB}$$

$$L = \lceil 18.26 \rceil = 19 \quad \left. \begin{array}{l} \text{min number of} \\ \text{levels} \end{array} \right\} \text{ quantization levels}$$

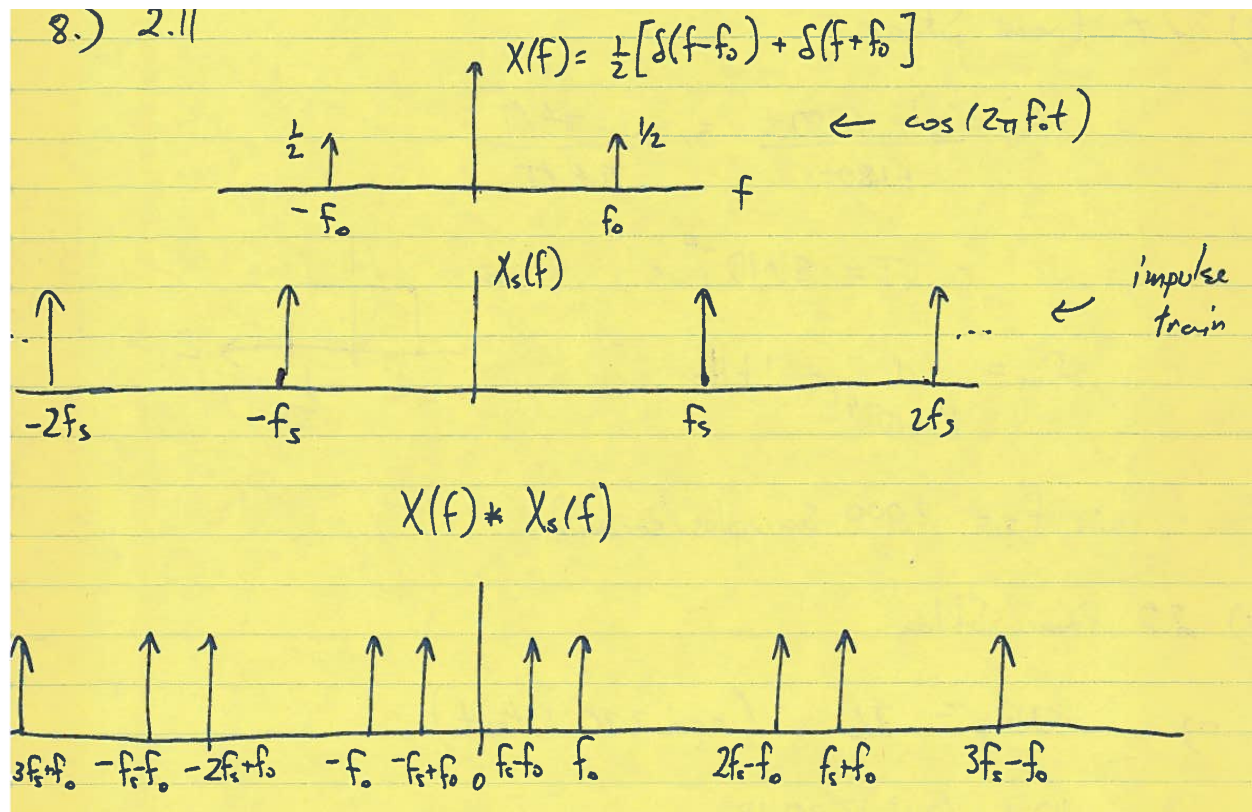
$$l = \lceil \log_2 L \rceil = 5 \text{ bits/sample}$$

$$b) \quad T_b = \frac{T_s}{l} = \frac{1}{l f_s} = \frac{1}{5 \times 8k} = 25 \mu\text{s}$$

$$W = \frac{1}{T_b} = \frac{1}{25 \mu\text{s}} = 400 \text{ kHz} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{1st lobe of rect pulse}$$

8. 2.11

8.) 2.11



9. 2.15

$R = 8000 \text{ samples/s} \times 6 \text{ bits/sample} = 48 \text{ kbps}$
 $W = \frac{1}{T_b} = R = 48 \text{ kHz}$
 $\text{SNR}_q = 3L^2 = 3(64)^2 = 12,288 \approx 41 \text{ dB}$

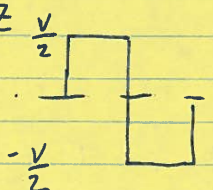
Binary case

$R_s = \frac{48 \text{ kbps}}{2 \text{ bits/symbol}} = 24 \text{ k symbols/s}$
 $W = \frac{1}{T} = R_s = 24 \text{ kHz}$
 $\text{SNR}_q = 41 \text{ dB}$

4-level case

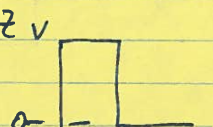
10. 2.17

bipolar NRZ



$$\frac{V^2}{4} = \frac{1}{2} \left(\frac{V}{2} \right)^2 + \frac{1}{2} \left(\frac{V}{2} \right)^2$$

unipolar NRZ



$$\frac{V^2}{2} = \frac{1}{2} (V^2) + \frac{1}{2} (0)^2$$

- bipolar needs half the power
- more sophisticated design needed to produce symmetrical wave form

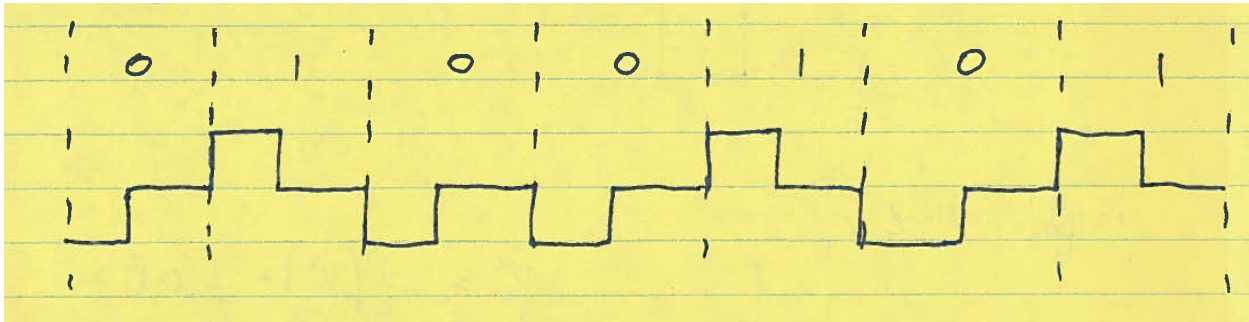
11. 2.18

$$T1 \text{ data rate} = 24 \frac{\text{samples}}{\text{frame}} \times 8 \frac{\text{bits}}{\text{sample}} \times 8000 \frac{\text{frames}}{\text{s}} + 1 \frac{\text{bit}}{\text{frame}}$$

$$= 193 \frac{\text{bits}}{\text{frame}} \times 8000 \frac{\text{frames}}{\text{s}} = 1.544 \times 10^6 \text{ bps}$$

$$\text{bandwidth efficiency } \eta = \frac{R}{W} = \frac{1.544 \times 10^6}{386 \times 10^3} = 4 \frac{\text{bps}}{\text{Hz}}$$

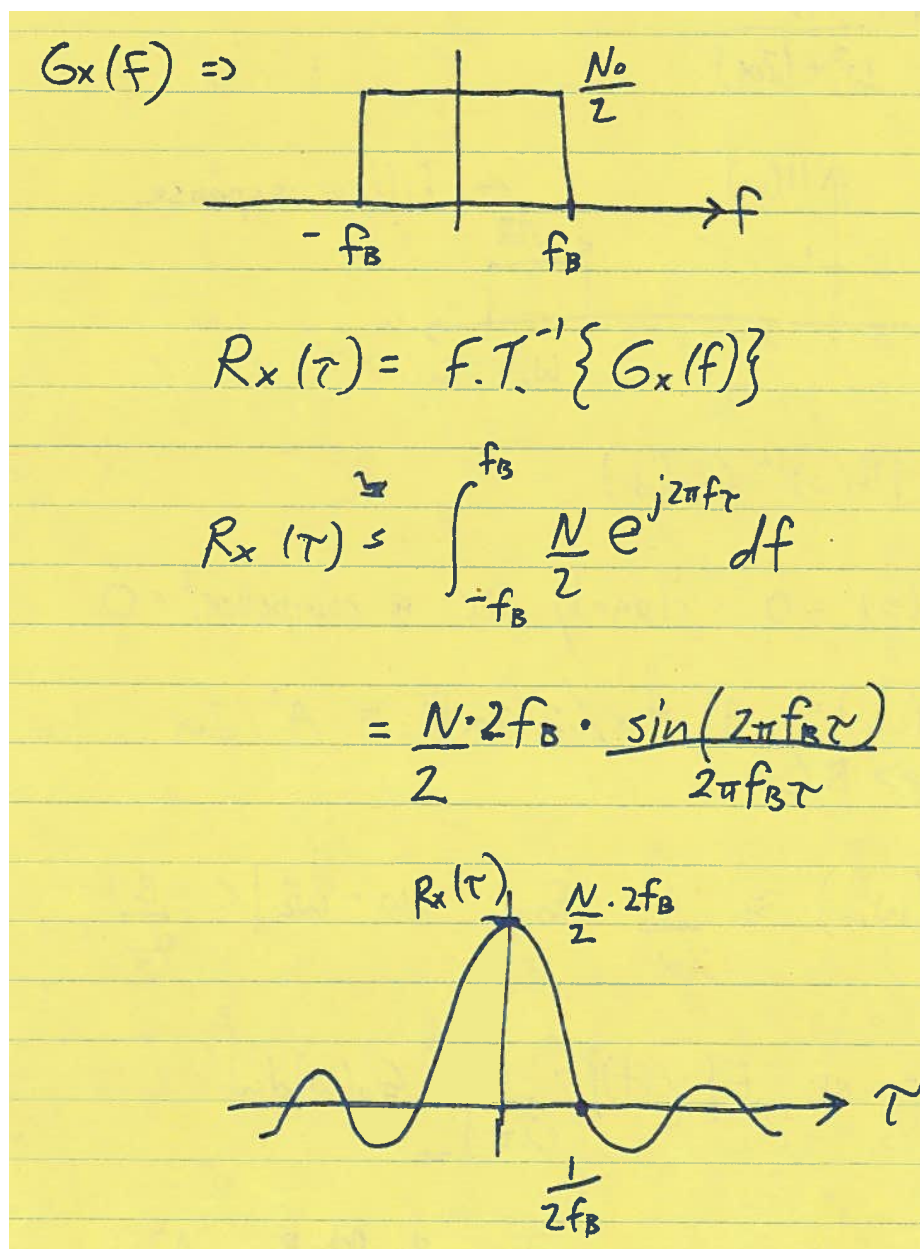
12. Bipolar RZ



13. Gaussian RV

$$1 - 2Q\left(\frac{6}{3}\right) = 1 - 2Q(2) = 1 - 2 \times 0.0728 = 0.9545$$

14. Filtered white noise



15. BPF'd noise

$$G_x(\omega) = A^2 \frac{4\alpha}{\omega^2 + (2\alpha)^2}$$

← filter response

$$G_y(\omega) = |H(\omega)|^2 G_x(\omega)$$

since $H(0) = 0$ clearly DC component = 0

at ω_c $G_x(\omega_c) = A^2 \cdot 4\alpha / 2(2\alpha)^2 = A^2 / 2\alpha$
 around $\omega_c \gg B/2\pi$

filter output $\Rightarrow G_y(\omega_c) \approx \frac{A^2}{2\alpha}$ for $|\omega - \omega_c| < \frac{B}{4\pi}$

avg. output power $E\{y^2(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_y(\omega) d\omega$

$$= \frac{1}{2\pi} \cdot 2 \cdot \frac{B}{2\pi} \cdot \frac{A^2}{2\alpha}$$

$$= \frac{A^2 \cdot B}{(2\pi)^2 \alpha}$$

16.3.4

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\frac{1 - (-1)}{2}\right)$$

$$= Q(1) = 0.1587$$

17.3.5

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad E_b = A^2 T \text{ for bipolar signalling}$$

$$\neq A=1 \therefore E_b = T$$

$$P_B = Q(x) \leq 10^{-3}$$

$$x = \sqrt{\frac{2E_b}{N_0}} = 3.09$$

$$\frac{E_b}{N_0} = 4.77 \quad ; \quad \frac{N_0}{2} = 10^{-3} \text{ (given)}$$

$$E_b = T = 4.77 \times 10^{-3} \times 2$$

$$\therefore R = \frac{1}{T} \leq 104.9 \text{ bps}$$

18.3.6

$$a) \quad P(s_1) = P(s_2) = 0.5$$

$$\frac{p(z|s_1)}{p(z|s_2)} \stackrel{H_1}{\geq} \frac{P(s_2)}{P(s_1)} \quad \int_0^T dt \quad \int_0^T -1 dt$$

$$\text{for } P(s_1) = P(s_2) \quad \gamma_0 = \frac{a_1 + a_2}{2} = \frac{T + (-T)}{2} = 0$$

$$b) P(s_1) = 0.7 \Rightarrow P(s_2) = 0.3$$

using B.12

$$\frac{z(a_1 - a_2)}{\sigma_0^2} \stackrel{H_1}{\underset{H_2}{\geq}} \ln \frac{P(s_2)}{P(s_1)}$$

$$z \stackrel{H_1}{\underset{H_2}{\geq}} \frac{\sigma_0^2}{a_1 - a_2} \ln \frac{P(s_2)}{P(s_1)} = \gamma_0$$

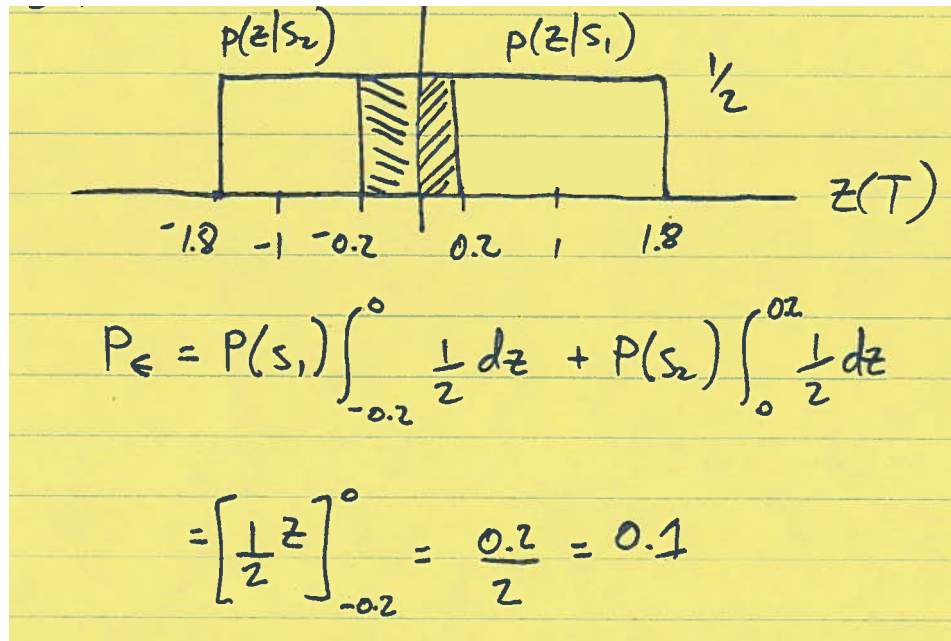
$$\gamma_0 = \frac{0.1}{2T} \ln \left(\frac{0.3}{0.7} \right)$$

$$= \frac{-0.04}{T}$$

$$c) \gamma_0 = \frac{0.1}{2T} \ln \left(\frac{0.8}{0.2} \right) = \frac{0.07}{T}$$

d) ~~a priori~~ a priori probabilities pull the decision threshold to the more likely possibility (to ~~take place~~ happen in the first place)

19.3.7



20.3.8

a) 16 levels = $M = 2^k$ $k = 4$ bits/symbol

$$R_s = \frac{R}{\log_2 M} = \frac{10 \text{ Mbps}}{4 \text{ b/sym}} = 2.5 \text{ M symbols/s}$$

$$\text{min BW} = R_s/2 = 1.25 \text{ MHz}$$

c) Using eqn. 3.80 $W = \frac{1}{2}(1+r)R_s$

$$1.375 \text{ MHz} = (1+r) \cdot 1.25 \text{ MHz}$$

$$r = 0.1$$