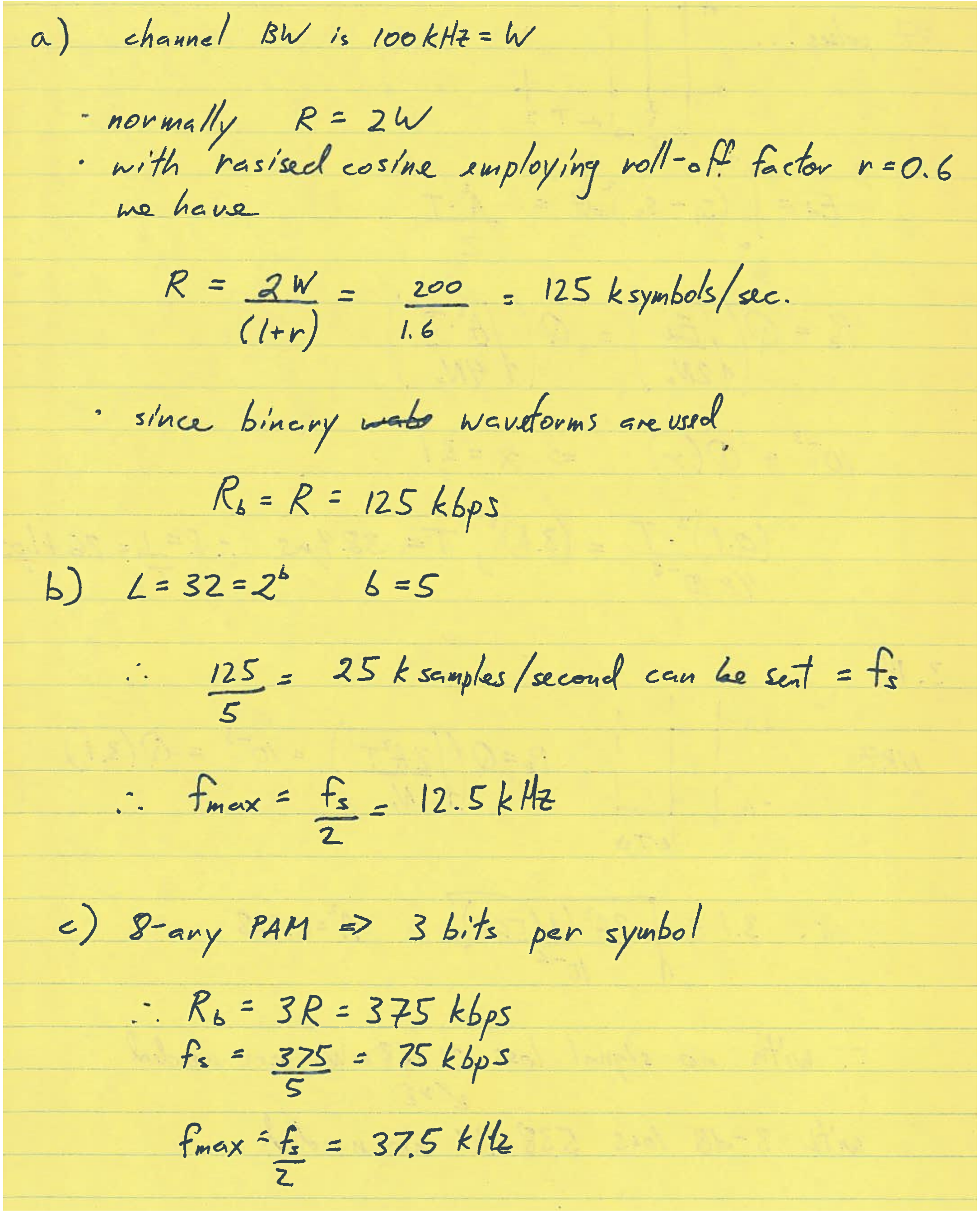
1. 3.12



a) channel BW is 100 kHz = W

- normally $R = 2W$
- with raised cosine employing roll-off factor $r = 0.6$ we have

$$R = \frac{2W}{(1+r)} = \frac{200}{1.6} = 125 \text{ ksymbols/sec.}$$

- since binary ~~wavb~~ waveforms are used,

$$R_b = R = 125 \text{ kbps}$$

b) $L = 32 = 2^b$ $\quad b = 5$

$$\therefore \frac{125}{5} = 25 \text{ k samples/second can be sent} = f_s$$

$$\therefore f_{max} = \frac{f_s}{2} = 12.5 \text{ kHz}$$

c) 8-ary PAM ⇒ 3 bits per symbol

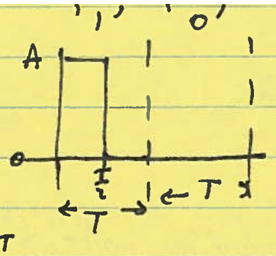
$$\therefore R_b = 3R = 375 \text{ kbps}$$

$$f_s = \frac{375}{5} = 75 \text{ kbps}$$

$$f_{max} = \frac{f_s}{2} = 37.5 \text{ kHz}$$

2. 3.13

RZ pulses . . .



$$E_d = \int_0^T (s_1 - s_2)^2 dt = \frac{A^2 \cdot T}{2}$$

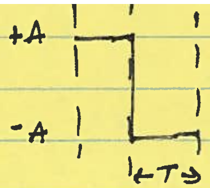
$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 \cdot T}{4N_0}}\right)$$

$$10^{-3} = Q(x) \quad \Rightarrow \quad x = 3.1$$

$$\frac{(0.1)^2 \cdot T}{4\times 10^{-8}} = (3.1)^2, \ T = 38.4\,\mu s \quad \therefore R = \frac{1}{T} = 26 \text{ kbps}$$

3. 3.14

NRZ



$$P_B = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) = 10^{-3} = Q(3.1)$$

$$\therefore \ 3.1 = \sqrt{\frac{2A^2(1/56k)}{10^{-6}}} \qquad A^2 = 0.268$$

∴ with no signal loss ~ 268 mW are needed

$\swarrow \times 2$

with 3-dB loss 538 mW are needed

4. 3.15

Nyquist min. bw is $\frac{1}{2T}$ where T is the symbol period

(implicit from Nyquist pulse shaping criterion)

⇒ PSD of random bipolar sequence $= T\left(\frac{\sin(\pi f T)}{\pi f T}\right)^2$

P_x = total area $= \int_{-\infty}^{\infty} T\left(\frac{\sin(\pi f T)}{\pi f T}\right)^2 df = ?$

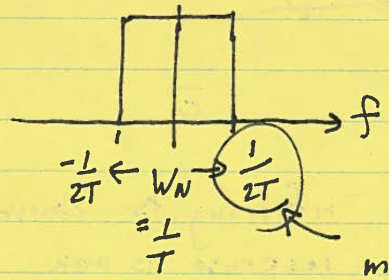
$$\int_0^\infty \frac{\sin^2 px}{x^2}dx = \frac{\pi p}{2}$$

∴ our integral of interest is $= 2\cdot T\cdot\frac{\pi}{2}\cdot\pi T\cdot\frac{1}{\pi^2 T^2}$

$P_x = 1$!

⇒ ∴ $W_N = \frac{P_x}{G_x(f)|_{pk}} = \frac{1}{T}$ (equiv. noise BW... double sided)

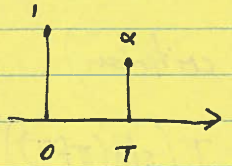
see Sklar pg. 48



matches Nyquist min. BW

5. 3.17

→ overall system impulse response is



$$h(t) = \delta(t) + \alpha\delta(t-T)$$

- the bigger (i.e. more taps) you make your ZFE (zero-forcing equalizer) the "better" will your final impulse response be closer to sigle $\delta(t)$

- e.g. imagine a 4 tap ZFE

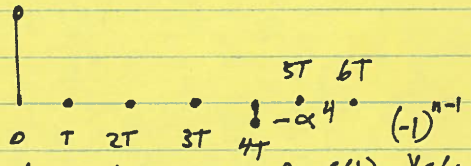
convolution matrix

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ 0 & \alpha & 1 & 0 \\ 0 & 0 & \alpha & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{\alpha} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$\bar{z} = \bar{X} \quad \bar{c}$$

$$\left.\begin{aligned} c_0 &= \underline{1} \\ c_1 &= -\alpha \cdot c_0 = -\alpha \\ c_2 &= -\alpha \cdot c_1 = +\alpha^2 \\ c_3 &= -\alpha \cdot c_2 = -\alpha^3 \end{aligned}\right\}$$

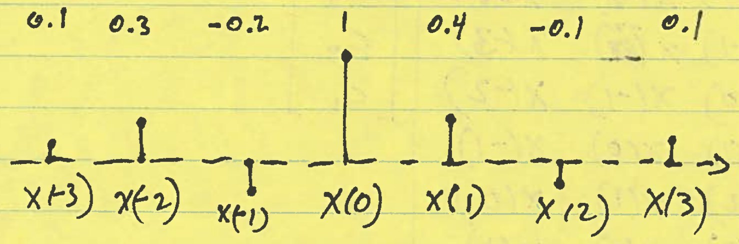
referring to convolution matrix response is now



clearly n-tap ZFE results in net impulse response of $\delta(t) \overset{(-1)^{n-1}}{+} \delta(t-nT)\alpha^n$

$$h_{total} = \delta(t) + (-1)^{n-1}\delta(t-nT)\cdot\alpha^n$$

6. 3.18



$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

inverting the convolution matrix

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0.8752 & 0.2593 & -0.2107 \\ -0.3079 & 0.8347 & 0.2593 \\ 0.2107 & -0.3079 & 0.8752 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$c_{-1} = 0.2593$ $c_0 = 0.8347$ $c_1 = -0.3079$

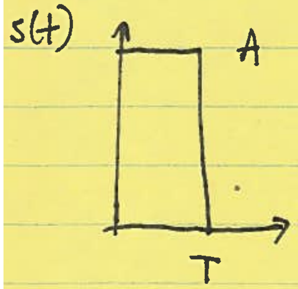
$$\begin{bmatrix} z(-3) \\ z(-2) \\ z(-1) \\ z(0) \\ z(1) \\ z(2) \\ z(3) \end{bmatrix} = \begin{bmatrix} x(-2) & x(-3) & 0 \\ x(-1) & x(-2) & x(-3) \\ x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \\ x(3) & x(2) & x(1) \\ 0 & x(3) & x(2) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

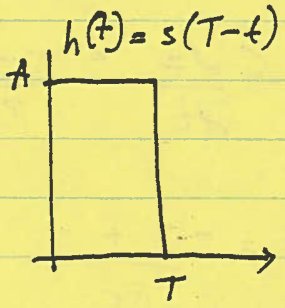
$z(k) =$ 0.1613, 0.1678, 0.0, 1.0, 0.0, -0.1807, 0.1143

largest sample magnitude contributing to ISI = 0.1807

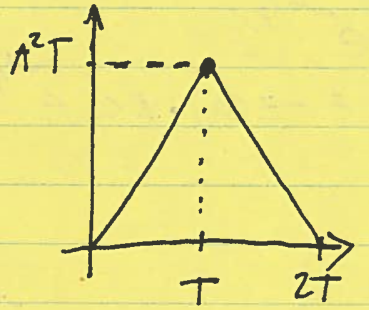
sum of ISI magnitudes = 0.6241

7. MF output with rectangular pulse inputs.





$$z(t) = s(t) * h(t) = \int_0^t s(\tau)\, h(t-\tau)\, d\tau$$

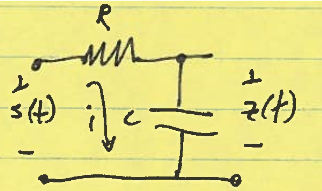


$$z(t) = \begin{cases} A^2 t & 0 \le t \le T \\ -A^2 t + 2A^2 T & T \le t \le 2T \\ 0 & \text{otherwise} \end{cases}$$

$$SNR_T = \frac{A^2 T}{N_0/2} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{N_0/2}$$

$$= \frac{2A^2 T}{N_0}$$

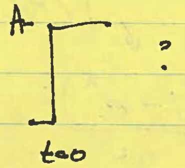
8. MF replaced with RC.



$$i = C\frac{dz}{dt} = \frac{s-z}{R}$$

$$\frac{dz}{dt} = \frac{s-z}{RC}$$

step response

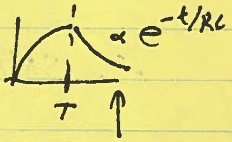


?

do Laplace $$sZ = \frac{A}{sRC} - \frac{Z}{RC}$$

$$Z(s) = \frac{A}{s(1+sRC)} = \frac{A}{s} - \frac{A\cdot RC}{(1+sRC)}$$

$$z(t) = \begin{cases} A(1-e^{-t/RC}) & 0 \le t \le T \\ A(1-e^{-T/RC})e^{-\frac{(t-T)}{RC}} & T \le t \end{cases}$$



same idea for decay

max. at T: $z(T) = A(1-e^{-T/RC})$

avg. noise o/p power is $$\int_{-\infty}^{\infty} \frac{N_0}{2} \cdot \frac{df}{1+(2\pi f RC)^2}\, df = \frac{N_0}{4RC}$$

$$\therefore \quad SNR_T = \frac{A^2(1-e^{-T/RC})^2}{N_0/4RC}$$

max. SNR_T w.r.t. RC

$$SNR_T = \frac{4A^2 \cdot T}{N_0} \cdot \frac{(1-e^{-T/RC})^2}{T/RC}$$

$$\downarrow$$

$$\frac{(1-e^{x})^2}{x^2}$$

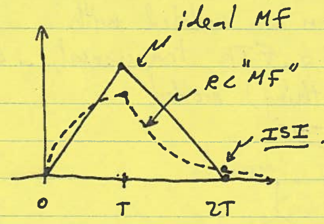
$$\frac{d}{dx}\left[\frac{(1-e^{x})^2}{x^2}\right] = 0 = \frac{2x\, e^{-x}(1-e^{-x}) - (1-e^{-x})^2}{x^2} \ldots$$

...gives $1 + 2x = e^{x} \Rightarrow x \approx 1.257 = \frac{T}{RC}$

$$\therefore \; SNR_T \approx 0.815 \cdot \frac{A^2 T}{N_0/2}$$

$\approx$ 80% of optimal M.F. design ... not bad

- what is ignored however is the ISI introduced by such a filter

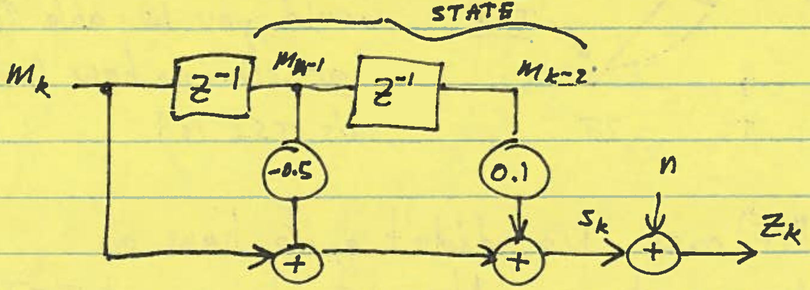


... would you be able to characterize how bad this ISI is?

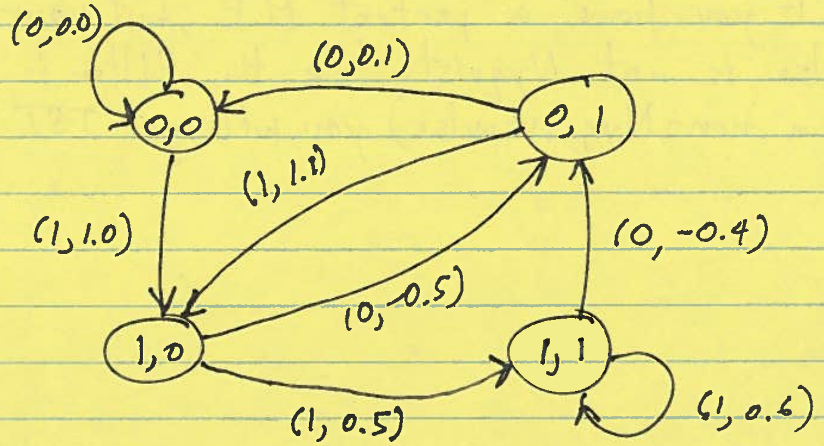
- in this (RC "MF") case we didn't quite have a perfect MF to operate on a Nyquist pulse & got ISI
- conversley if you have a perfect M.F., but your working pulse is not Nyquist (i.e. the filter is matched to a non-Nyquist pulse) you will get ISI as well

9. Sequence detector ideas.

1.) our net channel can be modeled with the shift-register (i.e. FIR, transversal, ... bunch of names for the same thing) model.

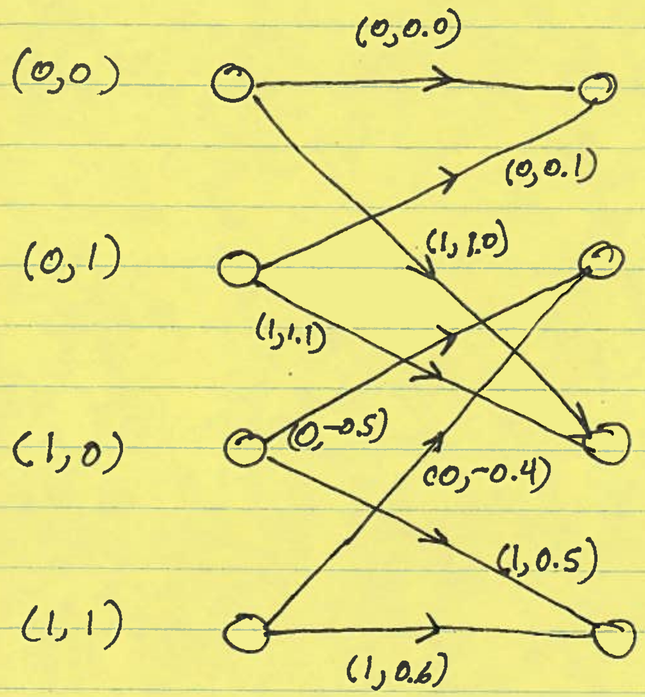


the state transition diagram is

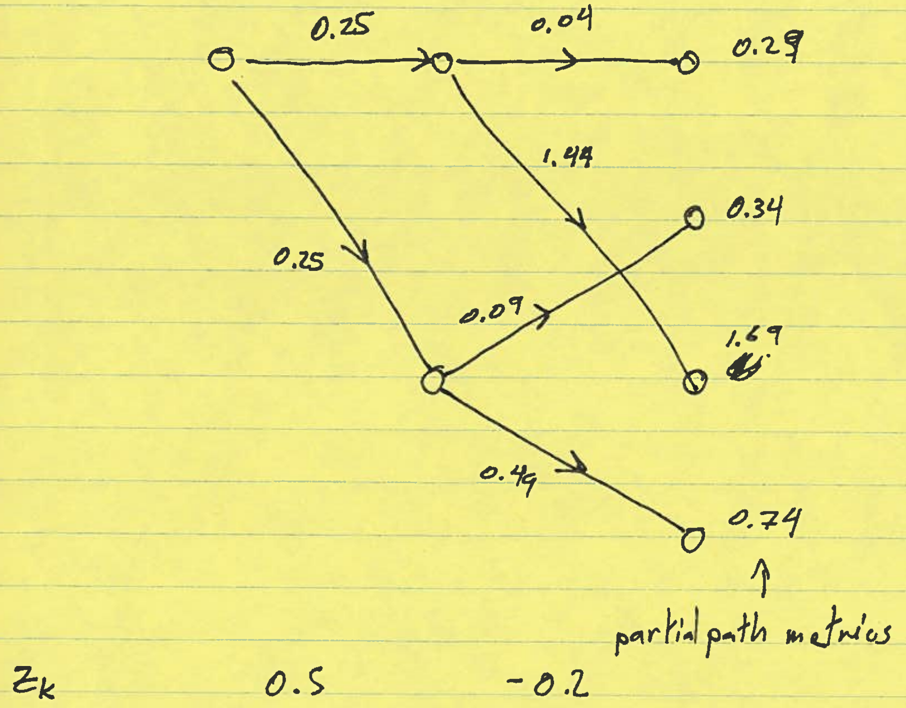


labels: m_{k-1}, m_{k-2} | (m_k, s_k)

2)



3)



say $z_3 = 0.9$

would you be able to draw the next stage of the trellis showing just the survivor paths?