

L15 Bandpass Detection

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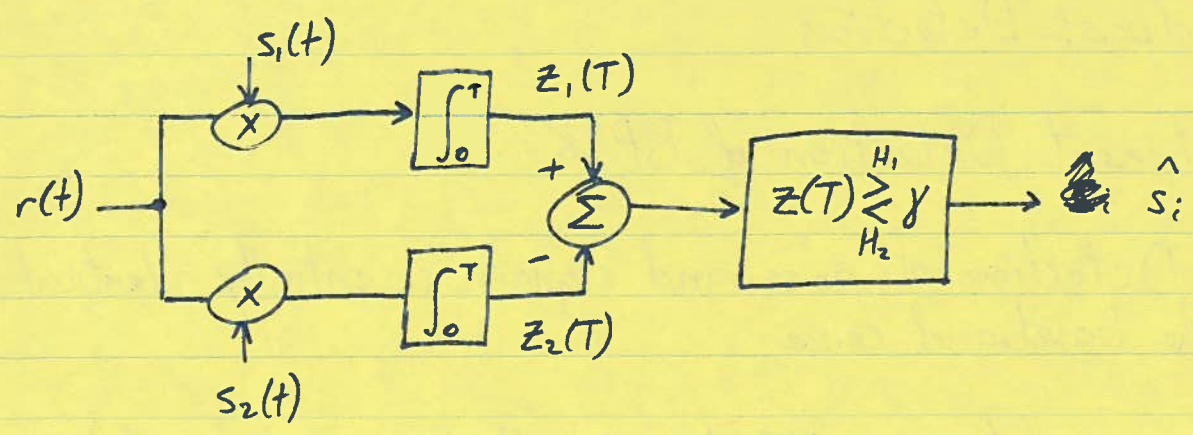
15.1 Coherent Detection of BPSK

- ~~Bas~~ Detection of passband signals essentially identical to the baseband case
- Use correlator or MF to map the received signal to $z(T)$ and make a decision
- What about the "coherent" part of the title? It means that you need to correctly align the phase of your matching signal ~~to~~ (i.e. in the receiver) to the phase of your incoming carrier. (~~that~~ the one that just flew through the channel into your receiver)... more on this in a moment
- For BPSK

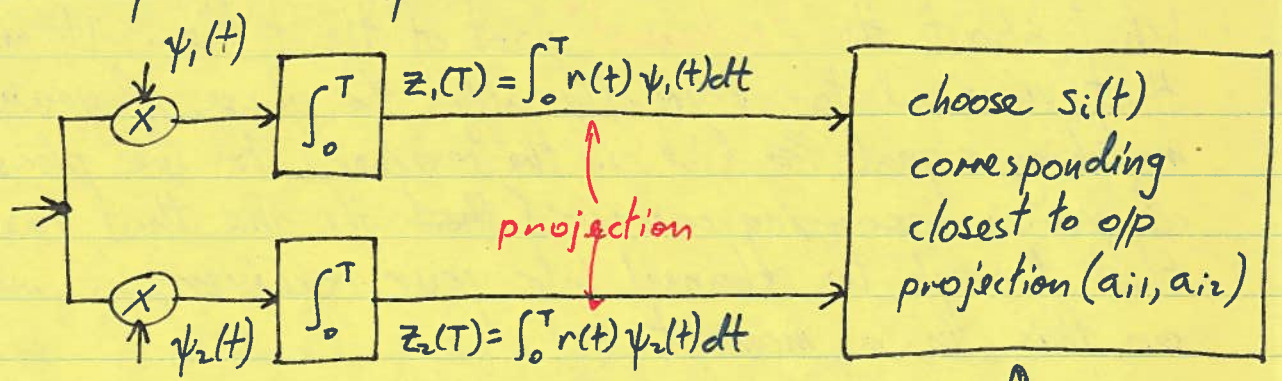
$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi) \quad 0 \leq t \leq T$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi + \pi) = -\sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi)$$

- as with baseband we can detect the digital information in these with...



• more generally we can express this in terms of the signal space concept



recall: $a_{ij} = \int_0^T s_i(t) \psi_j(t) dt$

projection of i th signal on j th basis

closest means $\arg \min_{s_i} (\bar{s}_i - \bar{z}_i(T))^2$

• for BPSK $\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t$ $0 \leq t \leq T$

$a_{11} = \int_0^T s_1(t) \psi_1(t) dt = \sqrt{E}$

$a_{21} = \int_0^T s_2(t) \psi_1(t) dt = -\sqrt{E}$



note: the effectiveness of this scheme is undermined if the phases of $s_i \neq \psi_i$ are not aligned

e.g. if ψ_1 is as before and $s_1 = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \frac{\pi}{2})$

$$s_2 = -\sqrt{\frac{2E}{T}} \cos(\omega_0 t + \frac{\pi}{2})$$

now $a_{11} = a_{21} = \phi!$ that's a big problem and hence requires some phase synchronization scheme

15.2 Coherent Detection of Multiple Phase-Shift Keying

- The natural progression from BPSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t - \frac{2\pi i}{M}) \quad 0 \leq t \leq T \quad i = 1, \dots, M$$

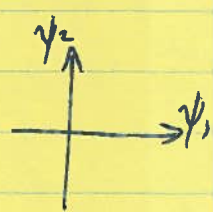
- Do we need an RX with M correlators ????
- No! Just 2 if we exploit ~~2~~ concepts from the signal-space perspective
- recall the basis functions we previously discussed with regards to M -ary PSK

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t$$

"in-phase"

$$\psi_2(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t$$

"quadrature"

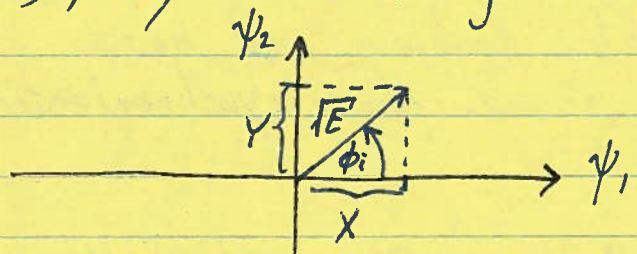


allowing us to express

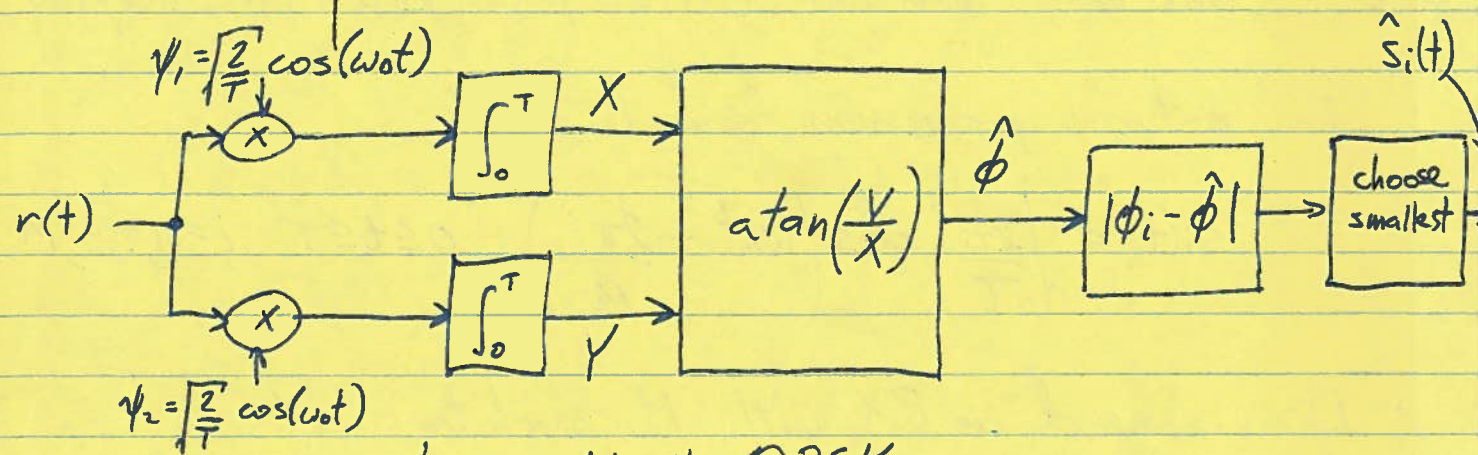
$$s_i(t) = a_{i1} \psi_1(t) + a_{i2} \psi_2(t)$$

$$= \underbrace{\sqrt{E} \cos\left(\frac{2\pi i}{M}\right)}_{\text{projection on } \psi_1 \text{ axis}} \psi_1(t) + \underbrace{\sqrt{E} \sin\left(\frac{2\pi i}{M}\right)}_{\text{projection on } \psi_2 \text{ axis}} \psi_2(t)$$

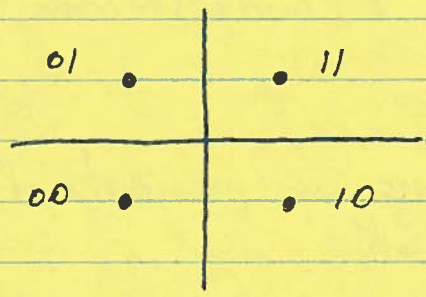
$r(t)$, my received signal (ignoring noise) looks like ...

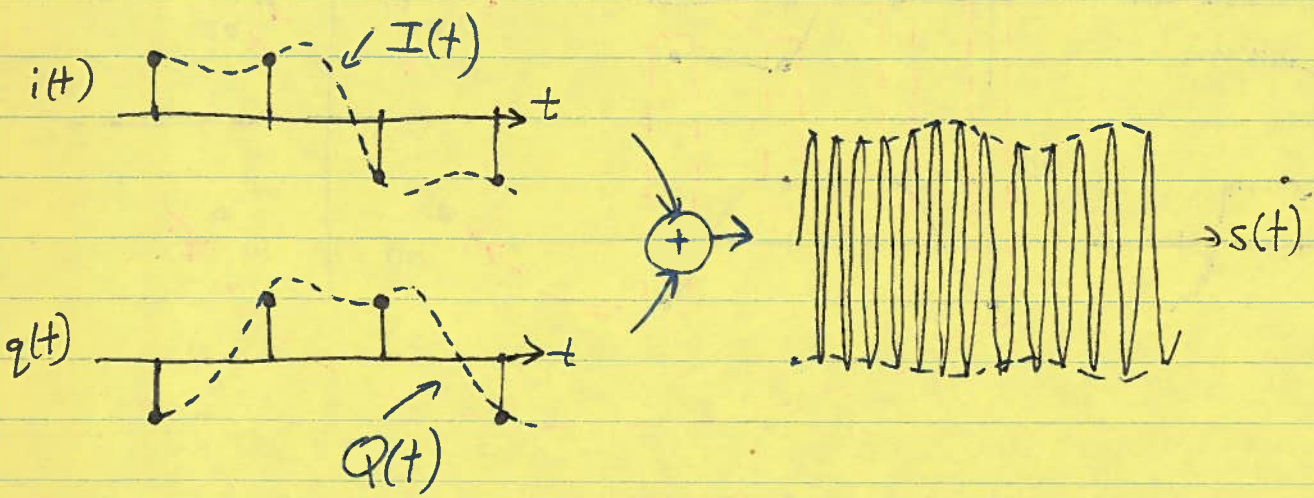
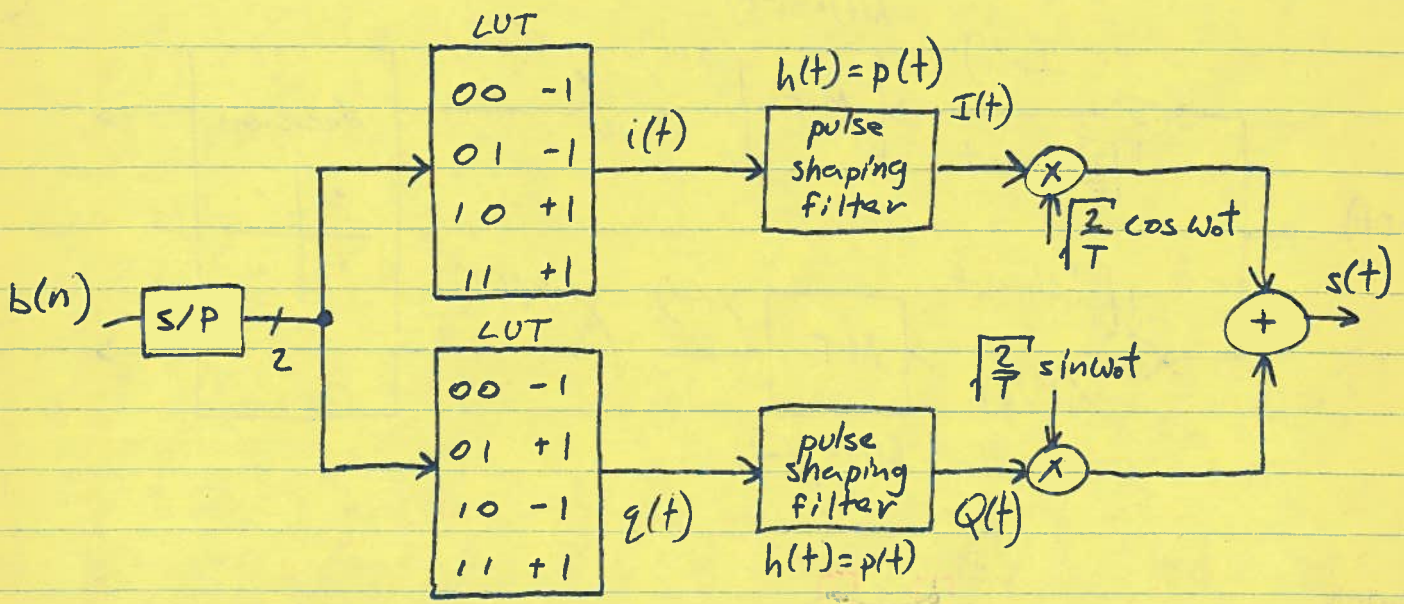


X & Y are projections (i.e. my a_{ij} 's)



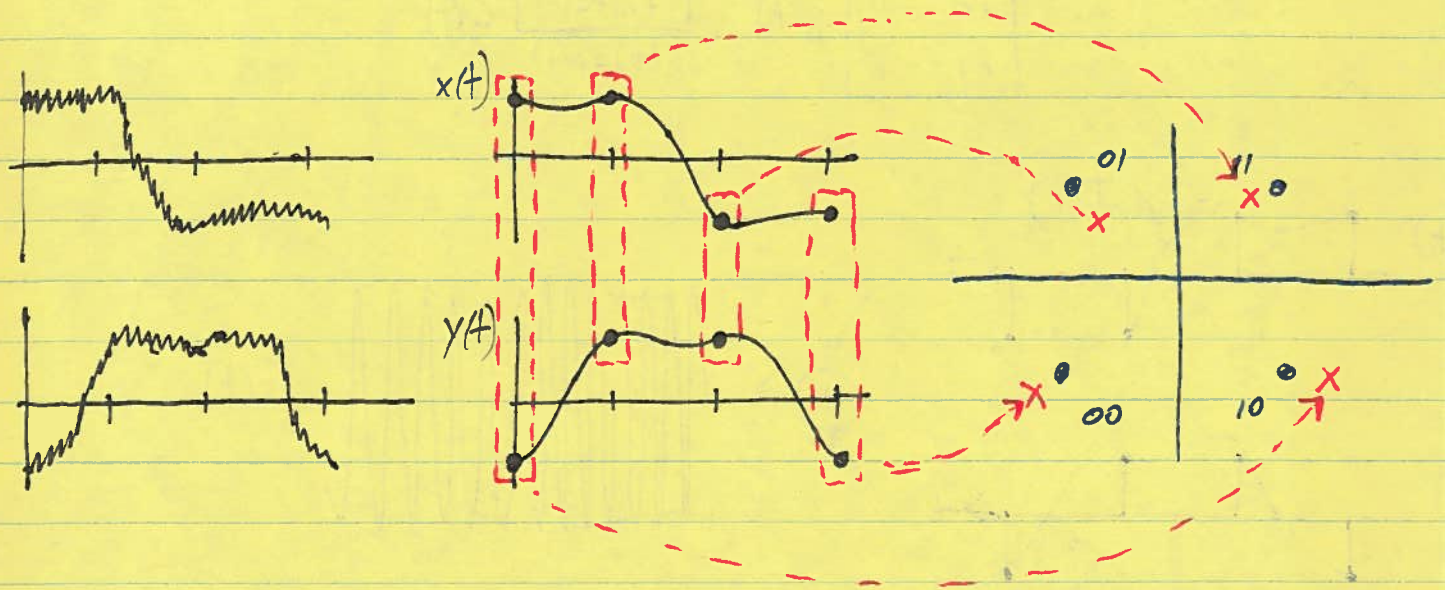
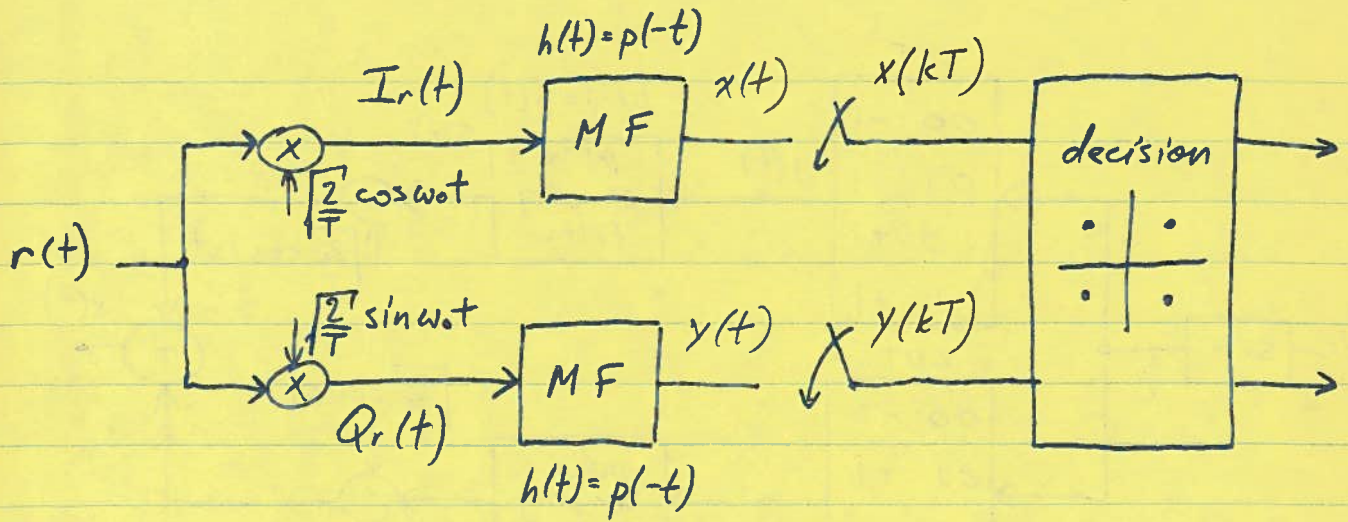
a common variant is $M=4$ QPSK





| | | | | | | | | | |
|----------|---|----|---|----|---|----|---|----|---------|
| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| $b(n)$ | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 01 11 |
| k | | 0 | | 1 | | 2 | | 3 | |
| $a_1(k)$ | | +1 | | +1 | | -1 | | -1 | |
| $a_2(k)$ | | -1 | | +1 | | +1 | | -1 | 00 10 |

and the receiver...



15.3 Noncoherent Detection of DPSK

We can get by without necessarily knowing the phase in general: $s(t) = \sum_k A p(t - kT) \cos(\omega_0 t + \theta_k)$

$$\theta_k = \text{atan} \left(\frac{a_1(k)}{a_2(k)} \right) \begin{matrix} \text{in phase} \\ \text{quadrature} \end{matrix} \leftarrow \text{phase} \leftrightarrow \text{data map}$$

e.g. $\theta_k = 0, \pi$ in BPSK

$\theta_k = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ in QPSK

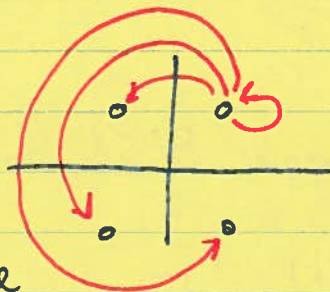
... instead of mapping data to phase ...

... map it to **PHASE SHIFT**

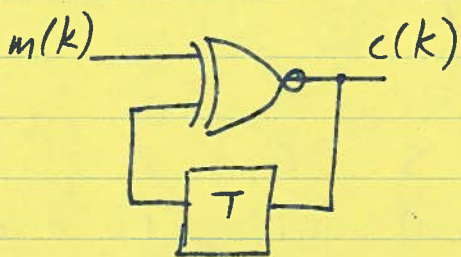
$$\theta_k = \theta_{k-1} + \Delta\theta_k$$

$$\Delta\theta_k = 0, \pi \text{ in BPSK}$$

$$\Delta\theta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \text{ in QPSK}$$



• this is basically our differential line encoder again

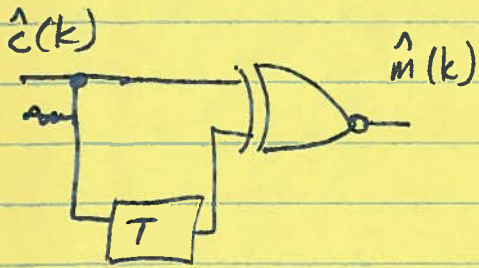


| $m(k)$ | $c(k-1)$ | $d(k)$ |
|--------|----------|--------|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

if current i/p $m(k)$ & past o/p have same phase send a 1
 if different phase send a 0

• can send $\bar{c}(k)$ as well,

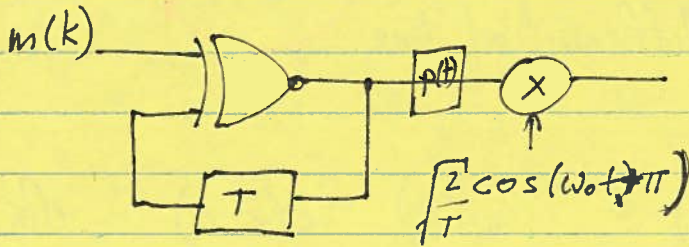
... decoder should look pretty familiar as well



| $\hat{c}(k-1)$ | $\hat{c}(k)$ | $\hat{m}(k)$ |
|----------------|--------------|--------------|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

if there's no difference between the last two bits the info is a one

a simple BPSK looks like

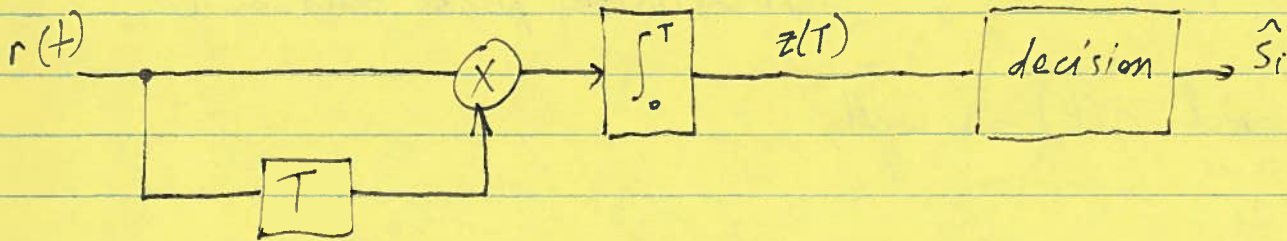


| k = | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------|-------|-------|-------|---|---|-------|-------|-------|---|
| m(k) | | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| c(k) | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| $\theta(k)$ | π | π | π | 0 | 0 | π | π | π | 0 |

and the receiver is

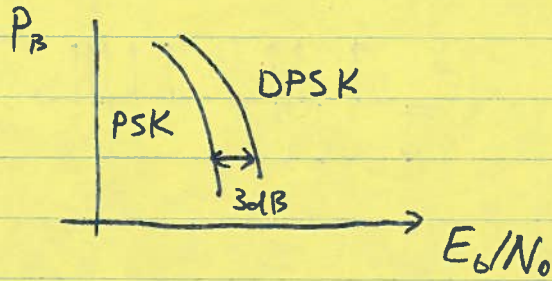
$$\frac{2}{T} \int_0^T \cos(\omega_0 t + \pi) \cos(\omega_0 t + \pi) dt = +1$$

$$\frac{2}{T} \int_0^T \cos(\omega_0 t) \cos(\omega_0 t + \pi) dt = -1$$



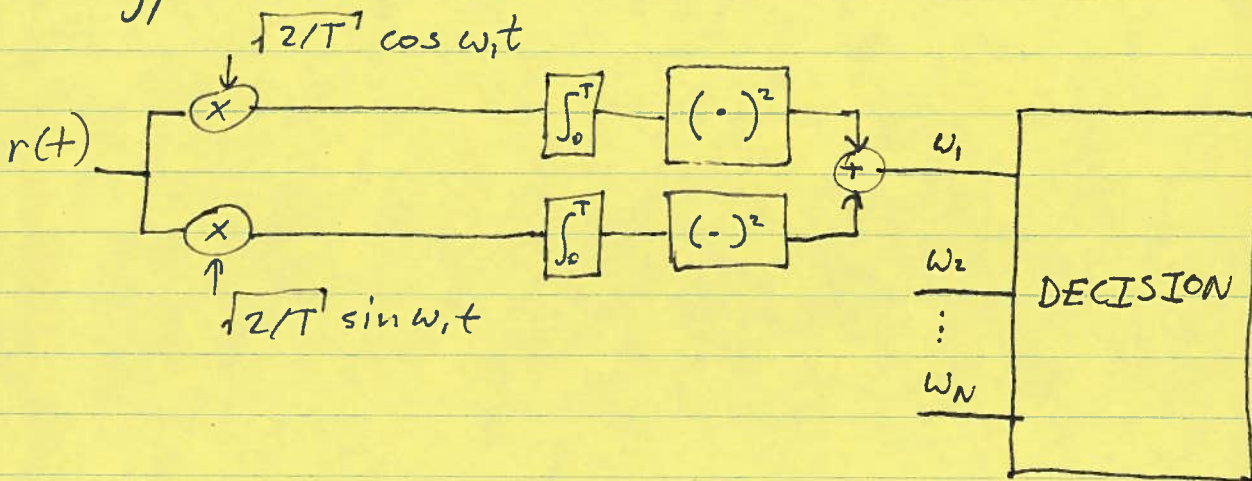
| z(T) | \sqrt{E} | \sqrt{E} | $-\sqrt{E}$ | $+\sqrt{E}$ | $-\sqrt{E}$ | \sqrt{E} | \sqrt{E} |
|------|------------|------------|-------------|-------------|-------------|------------|------------|
| | 1 | 1 | 0 | 1 | 0 | 1 | 1 |

- in coherent signaling we demodulate the noisy received signal with a clean reference
- in DPSK we use a noisy reference as well so we have 2x the noise (i.e. signal + ref.) ∴ we expect

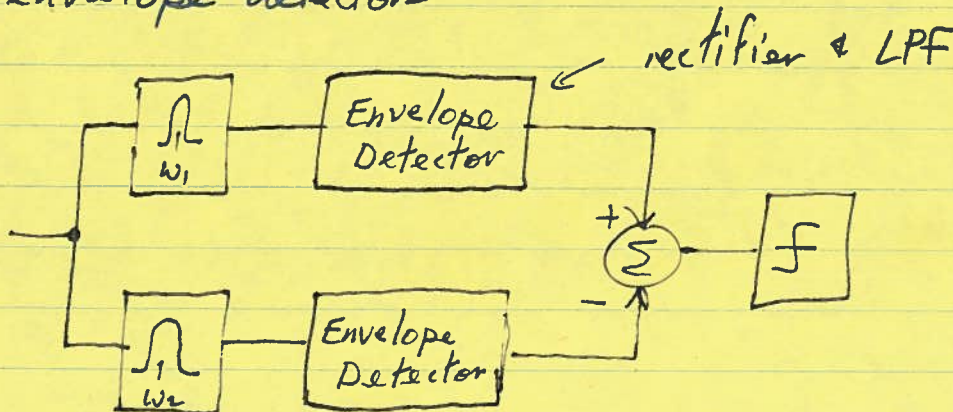


15.4 Noncoherent FSK

- Energy detector method



- envelope detectors



or even

