L4 Systems

- "SYSTEMS" → manipulate our signals to...
  - amplify
  - take out noise, distortion, etc.

  (filters, modulators)

- Again, need solid mathematical footing in handling these manipulations, i.e. SIGNAL TRANSFORMATIONS

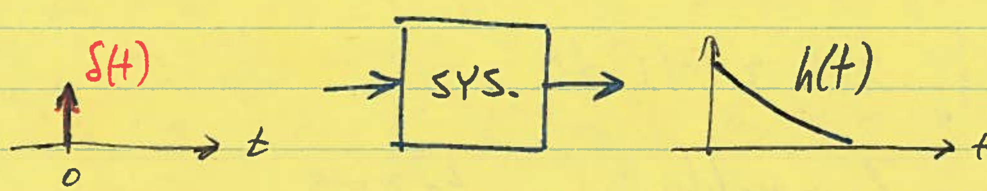
impulse response

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$X(f) \qquad H(f) \qquad Y(f)$$

4.1 Impulse Response

- system's response to $\delta(t)$ input: $h(t)$



- this can be used to find the response to an arbitrary input, $x(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

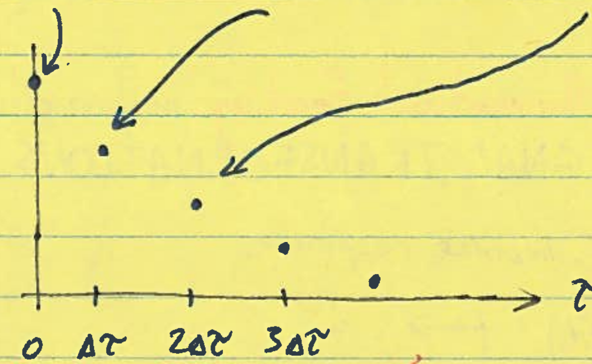
- since convolution is commutative...

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau$$

- for causal systems

$$y(t) = \int_0^{\infty} x(t-\tau)h(\tau)d\tau$$

$$y(t) = x(t)h(0)\Delta\tau + x(t-\Delta\tau)h(\Delta\tau)\Delta\tau + x(t-2\Delta\tau)h(2\Delta\tau)\Delta\tau + \ldots$$



how current i/p affects current o/p

the effect of past i/p on current o/p

if $x(t) = 0$ for $t<0$ then (still causal)

$$y(t) = \int_0^{t} x(t-\tau)h(\tau)d\tau$$

change of variables ... $t-\tau = s$

$$d(t-\tau) = -d\tau = ds$$

$$-\int_t^0 x(s)\,h(t-s)\,d\cancel{x} = \int_0^t x(t)h(t-s)ds$$

therefore

$$y(t) = \int_0^t h(t-\tau)\,x(\tau)\,d\tau$$

4.2 Frequency Response

- taking F.T. of $y(t) = x(t) * h(t)$

$$Y(f) = X(f) \cdot H(f)$$

↖ frequency response

- OR -

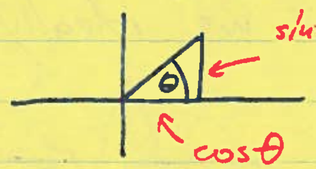
transfer function

- in general a complex fn.

$$H(f) = \underbrace{|H(f)|}_{\text{magnitude response}} e^{j\underbrace{\theta(f)}_{\text{phase response}}}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Euler's eqn., unity magnitude



$$\theta(f) = \operatorname{atan}\left[\frac{Im\{H(f)\}}{Re\{H(f)\}}\right]$$

$$\left\{ = \operatorname{atan}\left[\frac{\sin\theta}{\cos\theta}\right] = \operatorname{atan}\left[\tan\theta\right] \right\}$$ just some relations

- how to find $H(f)$?

1) F.T. of impulse response... "difficult" to make pure impulse

2) apply $x(t) = A\cos(2\pi f_0 t)$ & sweep f_0

output is $y(t) = A \cdot |H(f_0)| \cdot \cos\left[2\pi f_0 t + \theta(f_0)\right]$

4.3 Random Process Filtering

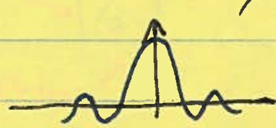
- simple, just compare i/p & o/p power spectra using

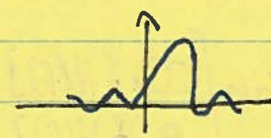
$$G_Y(f) = G_X(f) \cdot |H(f)|^2$$

- PSD devoid of phase component... averaged out during formation of autocorrelation which slides & multiplies the signals over time

4.4 Distortionless Transmission

- what we ideally want to achieve (to avoid ISI)





(distorted)

- distortionless means... linearly proportional & delayed

$$y(t) = \underbrace{K}_{\text{gain}}\, x(t - \underbrace{t_0}_{\text{time delay}})$$

constants

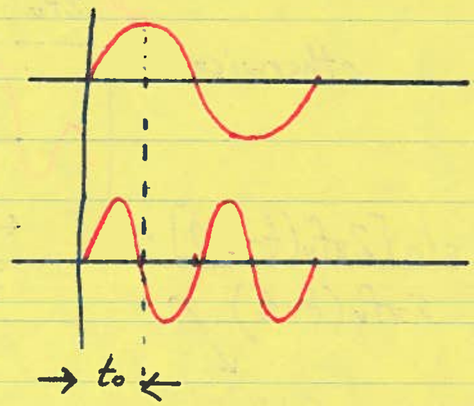
F.T.: $Y(f) = K e^{-j2\pi f t_0} X(f)$ } • scaling in amplitude
• linearly dependent (on frequency) phase shift

- so the higher the freq. the bigger the phase delay (if you want a distortionless system)

- e.g. two signal components

f_1 — over t_0 accumulates: $\pi/2$

$f_2 = 2f_1$ — " " " : π



- clearly the higher frequencies accumulate ~~phase~~ phase faster & so for the same t_0 must be shifted more rads ~~that~~ for higher freqs. relative to lower freqs.

$$\boxed{\frac{\theta}{2\pi} = \frac{t_0}{T}} = f \cdot t_0 \longleftarrow \text{phase \& time shifts relative to } 2\pi \text{ \& } T$$

$$t_0 = \frac{\theta}{2\pi f} \leftarrow \text{need } \theta \text{ to track } f \text{ for const. delay over frequencies}$$

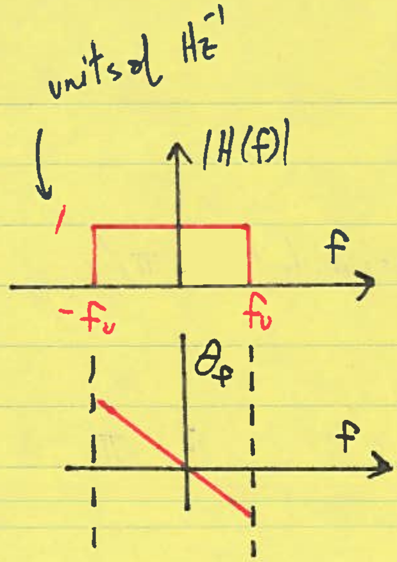
- group delay $\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$ } another filter metric

4.5 Ideal Filters

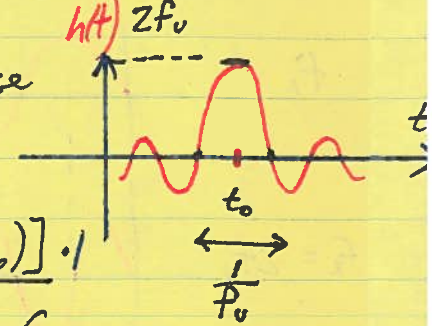
- Filters with ABRUPT transitions, and linear phase are useful approximations, that simplify arithmetic

- 3 main varieties

LPF



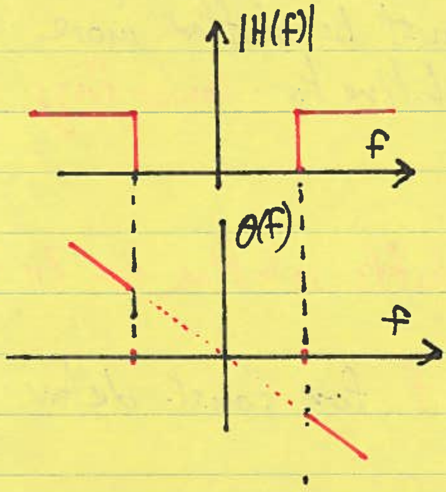
$$H = \begin{cases} e^{-j2\pi f t_0} & \text{for } |f| \leq f_0 \\ 0 & \text{otherwise} \end{cases}$$



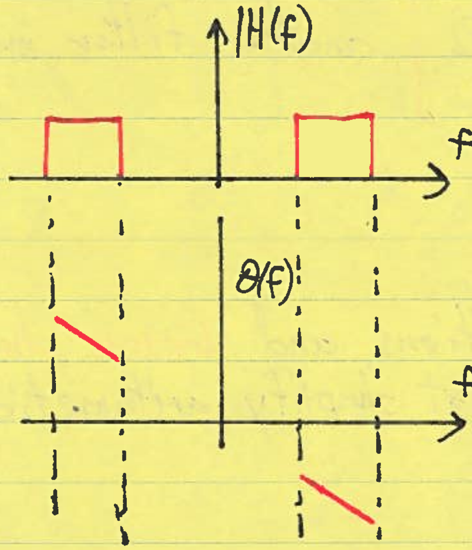
$$h(t) = \underbrace{2f_0}_{[Hz]} \cdot \underbrace{\frac{\sin[2\pi f_0(t-t_0)]}{2\pi f_0(t-t_0)}}_{[Hz^{-1}]} \cdot 1$$

$$= 2f_0 \cdot \text{sinc}[2f_0(t-t_0)]$$

HPF

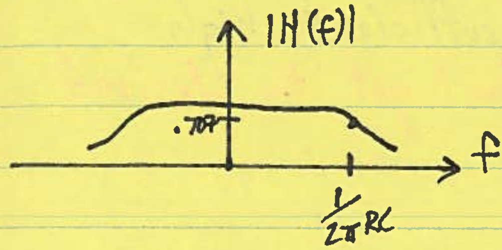


BPF



4.6 Real Filters

- Real filters aren't abrupt e.g. RC



$$H(f) = \frac{1}{1+j2\pi fRC} = \frac{1 \cdot e^{-j\theta(f)}}{\sqrt{1+(2\pi fRC)^2}}$$

$$\theta(f) = \operatorname{atan}(2\pi fRC)$$

when $2\pi fRC = 1 \Rightarrow |H(f)| = \frac{1}{\sqrt{2}} = 0.707$

$$\downarrow$$

$$\boxed{|H(f)|^2 = \frac{1}{2} \text{ @ } f = \frac{1}{2\pi RC}}$$

- ½ power BW $\left(f = 1/2\pi RC\right)$ ... sometimes called "filter BW"
- but this designation is arbitrary
- generally we express input-output power relations on logarithmic scale

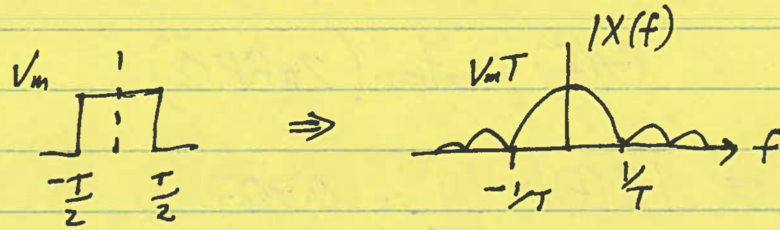
$$\underbrace{|H(f)|_{dB}}_{\text{new symbol}} = 10\log\left(\frac{P_{out}(f)}{P_{in}(f)}\right) = 10\log\left(\frac{V_{out}^2(f)}{V_{in}^2(f)}\right)$$

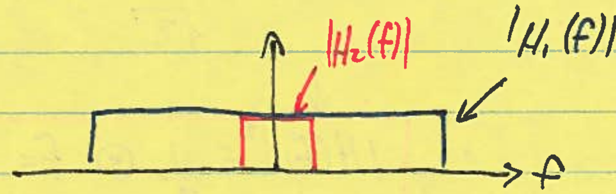
$$= 20\log\left(\frac{V_{out}(f)}{V_{in}(f)}\right) = 20\log|H(f)|$$

$\therefore$ ½ power point is $20\log 0.707 \simeq -3\,\text{dB}$ point

4.7 Filter Effects

- looking at a signals $|X(f)|$ it is clear that to pass signals undistorted a sufficiently high BW is needed (otherwise you don't get the desired constant K gain)

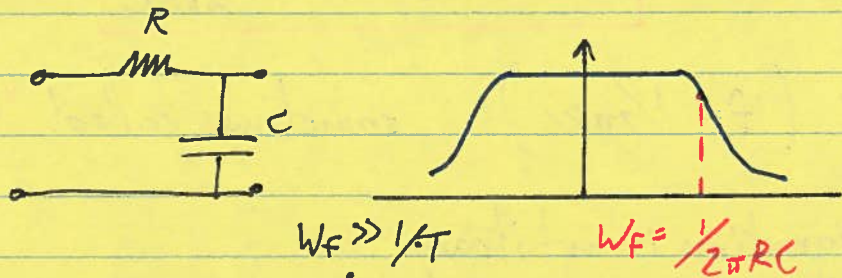




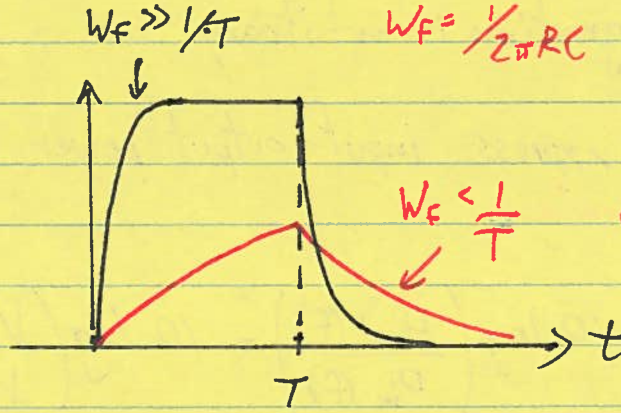
$H_1(f)$: no distortion

$H_2(f)$: some distortion

for



if W_F



distortion is clear