This mid-term has 4 questions worth a total of 40 points. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page allotted to that question. Clearly indicate your derivations and circle your final answer.

Last Name: ______________________________

First Name: ______________________________

1. $\boxed{\text{10 points}}$ Signals and Systems

(a) $\boxed{\text{2 points}}$ What is the AC power of $x(t) = 2 + V_p \cos(2\pi f_o t)$?

$$\text{ac power} = \sigma_x^2(t) = \langle (x-\mu)^2 \rangle \qquad \mu = 2$$

$$= \frac{1}{T}\int_0^T V_p^2 \cos^2(2\pi f_o t)\,dt = \frac{V_p^2}{2}$$

(b) $\boxed{\text{3 points}}$ A binary random process has probability p_1 of sending a one and probability p_0 of sending a zero. What is the variance of this process? Show your work.

$$\sigma_x^2 = E\{X^2\} - m_x^2$$

$$= E\{X^2\} - (E\{X\})^2$$

$$= 0^2 \cdot p_0 + 1^2 \cdot p_1 - (0 \cdot p_0 + 1 \cdot p_1)^2$$

$$= p_1 - p_1^2$$

$$= p_1(1-p_1) = p_1 \cdot p_0$$

(c) $\boxed{\text{1 point}}$ Calculate $\int_{-\infty}^{\infty} \exp(-2x^3)\delta(x+2)\mathrm{d}x$.

$$e^{-2(-2)^3} = e^{-2(-8)} = e^{+16} = 8.89 \times 10^6$$

(d) $\boxed{\text{4 points}}$ A bandpass modulator transmits 5-mW of data through the air at a rate of 25-Mbps in square-wave NRZ form centred at 2-GHz. What is the bandwidth of the wireless channel needed to cleanly pass at least the 0.5-power bandwidth of this signal.

$$G_{NRZ}(f)\Big|_{baseband} \propto \mathrm{sinc}^2(f \cdot T)$$

$$\therefore f_{0.5\text{-power}} \text{ gives } \quad \frac{1}{2} = \mathrm{sinc}^2(f_{0.5\text{-power}} \cdot T)$$

$$\frac{1}{\sqrt{2}} = \mathrm{sinc}(f_{0.5\text{-power}} \cdot T)$$

$$\text{knowing } \quad \mathrm{sinc}\left(\frac{1.4}{\pi}\right) = \frac{1}{\sqrt{2}}$$

$$\therefore f_{0.5\text{-power}} \cdot T = \frac{1.4}{\pi}$$

$$f_{0.5\text{-power}} = \frac{1.4}{\pi} \cdot \frac{1}{T} = \frac{1.4}{\pi} \cdot 25 \times 10^6 = 11.14\,\text{MHz}$$

$\therefore$ at passband

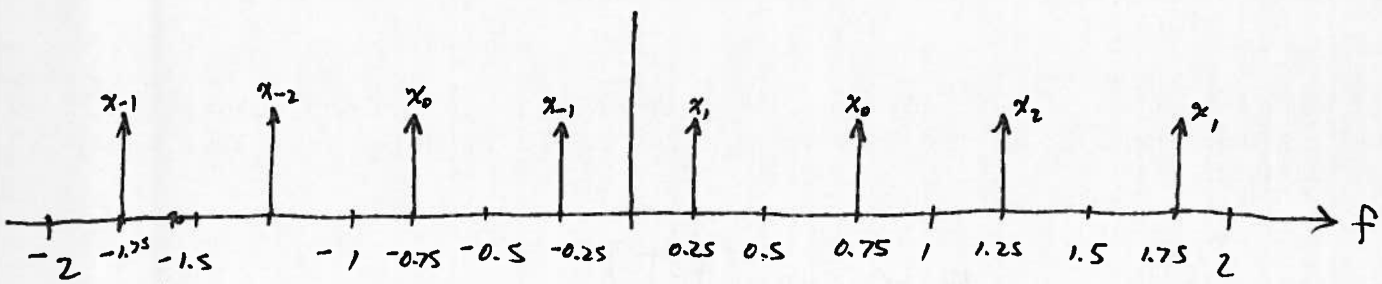
$$f_{0.5\text{-power}}\Big|_{passband} = 2 \times 11.14 = 22.3\,\text{MHz}$$

2. $\boxed{\text{10 points}}$ **Analog-to-Digital Conversion**

(a) $\boxed{\text{1 point}}$ **What is the mathematical expression for an ideal impulse sampling train (infinite extent) occurring every T_s seconds?**

$$x_s = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

(b) $\boxed{\text{3 points}}$ **$x(t) = A\cos(2\pi 0.75t)$ is subject to ideal impulse sampling at sampling frequency $f_s = 1$ Hz. Show the spectrum of the sampled signal between $\pm 2f_s$. Carefully label the frequency locations of your resulting spectrum (with numbers)**



(c) $\boxed{\text{3 points}}$ **An A/D demonstrates a 20-dB SNR due to "normal" quantization for perfect clocking and a 10-dB SNR due to "normal" clocking (jitter) for perfect quantization. What is the total SNR for "normal" quantization AND clocking present together? Write your answer in dB.**

$$SNR_q = 100, \quad SNR_j = 10 \qquad SNR_T = \frac{SNR_q \, SNR_j}{SNR_q + SNR_j} = \frac{10 \times 100}{10 + 100} = \frac{1000}{110} = 9.09$$

$$SNR_T \big|_{dB} = 10 \cdot \log_{10}(SNR_T) = 9.58 \text{ dB}$$

(d) $\boxed{\text{2 points}}$ **Determine the minimum sampling rate needed to perfectly reconstruct the signal $x(t) = \sin(1.4 \times 10^3 t)/(1.4 \times 10^3 t)$.**

$$\frac{\pi}{T} = 1.4 \times 10^3 \quad \therefore \quad \frac{1}{T} = \frac{1.4 \times 10^3}{\pi} = 4.45 \times 10^2 = 2 \times f_m$$

$$\therefore \quad f_s = 2 f_m = 445 \text{ Hz}$$

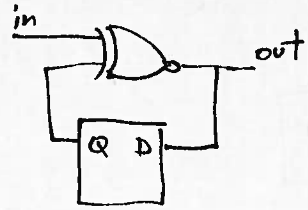
(e) $\boxed{\text{1 point}}$ **What's the average power of the noise associated with a 8-bit quantizer with a peak-to-peak range of 2.5 V?**

$$\sigma_n^2 = \frac{q^2}{12} \qquad q = \frac{2.5}{2^b} \qquad b = 8$$

$$\sigma_n^2 = \frac{1}{12}\left(\frac{2.5}{2^8}\right)^2 = 7.95 \times 10^{-6} \text{ W}$$

3. **10 points** Digital Transmitters

(a) **1 point** Sketch the digital circuit (like the one we talked about in class) that I can use to generate a differential line code. Clearly indicate the input and output ports.



(b) **2 points** A differential line decoder (not encoder) received the following signal: 00100. What must have been the data fed into the differential code encoder? Assume the initial decoder and encoder state is 0.

0 1 0 0 1 0 0

$\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$

1 0 0 0 1 0

$\boxed{0 \quad 0 \quad 1 \quad 1 \quad 0}$

(c) **2 points** What's the average power of a bipolar NRZ square signal with peak voltage (normalized to 1 Ω) of 1.3 V?

$$\sigma_x^2 = \frac{1}{2}(1.3)^2 + \frac{1}{2}(-1.3)^2 = (1.3)^2 = \boxed{1.69\,W}$$

(d) **2 points** Repeat the problem above for unipolar NRZ (peak of 1.3 V for a '1') where the chance of sending a '0' is 75%.

$$\sigma_x^2 = 0.25 \times (1.3)^2 = \boxed{0.423\,W}$$

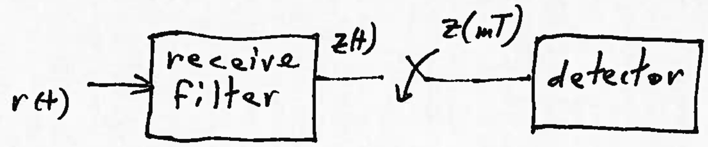
(e) **3 points** White noise with double-sided PSD of 10^{-9} W/Hz passes through an ideal LPF baseband transmit filter with gain 1 and bandwidth of 35 MHz. What is the autocorrelation of the filter output at $\tau = 0$?

$$R_x(\tau) = \int_{-\infty}^{\infty} G_x(f) e^{j2\pi f\tau} d\tau = \frac{N_0}{2} \int_{-f_b}^{f_B} e^{j2\pi f\tau} df = \frac{N}{2} \cdot 2f_b \cdot \frac{\sin(2\pi f_b \tau)}{2\pi f_b \tau}$$

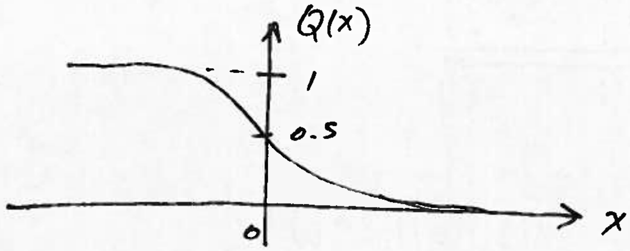
$$R_x(\tau) = \frac{N}{2} \cdot 2f_b = 10^{-9} \times 2 \times 35 \times 10^6 = 75 \times 10^{-3}\ W$$

4. $\boxed{\text{10 points}}$ Digital Receivers

(a) $\boxed{\text{1 point}}$ Draw the simplified digital receiver (ignore the channel) consisting of the part that figures out what information was actually sent and the part that makes the initially received noisy signal look good. Make sure to connect your blocks together correctly.



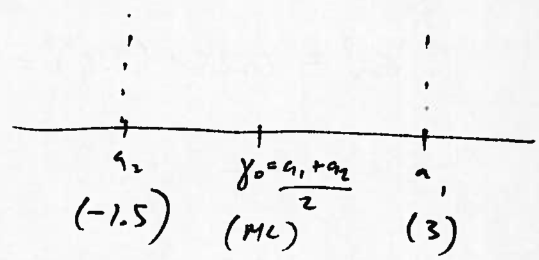
(b) $\boxed{\text{1 point}}$ Sketch $Q(x)$. Show and label the axes for clarity (otherwise no marks).



(c) $\boxed{\text{3 points}}$ The part of your receiver that figures out what information was actually sent is looking for levels at +3 and -1.5 V. The average noise power is 2 W. Quantitatively report on "how good" this part of your receiver assuming you employ the ML strategy. You should know what is meant by "how good".

$$P_B = P(H_2|s_1)\frac{1}{2} + P(H_1|s_2)\frac{1}{2}$$

BER



$$= \frac{1}{2}Q\left(\frac{a_1-\gamma_0}{\sigma_0}\right) + \frac{1}{2}Q\left(\frac{\gamma_0-a_2}{\sigma_0}\right)$$

$$= \frac{1}{2}Q\left(\frac{a_1-a_2}{2\sigma_0}\right) + \frac{1}{2}Q\left(\frac{a_1-a_2}{2\sigma_0}\right)$$

$$= Q\left(\frac{a_1-a_2}{2\sigma_0}\right) \qquad a_1 = 3,\ a_2 = -1.5 \ \& \ \sigma_0 = \sqrt{2}$$

$$= Q\left(\frac{4.5}{2\sqrt{2}}\right) = Q(1.59) \approx 0.054$$

(d) $\boxed{\text{1 point}}$ A binary digital communication scheme uses sinc-shaped symbols and works at a rate of 1.5-Mbps. The receiver samples signals of amplitude 0.7 (corresponding to a 1) and 0 V respectively from the matched filter. The system is subject AWGN with double-sided power spectral density of 10^{-7} W/Hz. The prior for a 0 is 0.6. What is the decision threshold assuming a ML strategy?

$$\gamma_0 = \frac{a_1 + a_2}{2} = \frac{0.7+0}{2} = 0.35\ V$$

(e) $\boxed{\text{4 points}}$ Repeat the problem above for a MAP strategy.

$$-\frac{(z-a_1)^2}{2\sigma_0^2} + \ln P(s_1) \underset{H_2}{\overset{H_1}{\gtrless}} -\frac{(z-a_2)^2}{2\sigma_0^2} + \ln P(s_2)$$

$$-(z^2 - 2a_1 z + a_1^2) + (z^2 - 2a_2 z + a_2^2) \underset{H_2}{\overset{H_1}{\gtrless}} 2\sigma_0^2 \ln \frac{P(s_2)}{P(s_1)}$$

$$2z(a_1 - a_2) + a_2^2 - a_1^2 \underset{H_2}{\overset{H_1}{\gtrless}} 2\sigma_0^2 \ln \frac{P(s_2)}{P(s_1)}$$

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1^2 - a_2^2}{2(a_1 - a_2)} + \frac{\sigma_0^2}{a_1 - a_2} \ln \frac{P(s_2)}{P(s_1)}$$

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \underbrace{\frac{a_1 + a_2}{2} + \frac{\sigma_0^2}{a_1 - a_2} \ln\left(\frac{P(s_2)}{P(s_1)}\right)}_{\text{threshold}}$$

$$\parallel$$

$$\frac{0.7+0}{2} + \frac{2\times10^{-7} \times 1.5\times10^{6}}{0.7-0} \cdot \ln \frac{0.6}{0.4}$$

$$= 0.35 + 0.43 \cdot 0.405 = \boxed{0.524}$$

Q1

Q2

Q3

Q4

Total