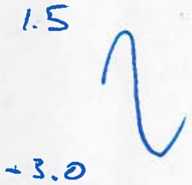
LE/EECS 4214 Digital Communication Fall 2014
Quiz #2, Thurs. Oct. 16, 2014 Name: ____________________

1. (*3 points*) A quantizer is capable of processing signals between +1.5 V and -3.0 V what is the maximum AC signal power of a sine wave that can be linearly processed by such a quantizer?



$V_{pp} = 4.5 \quad V_p = 2.25V$

$x(t) = V_p \cos(2\pi t/T)$

$$P_{AC} = \sigma_x^2 = \langle x^2(t) \rangle = \frac{1}{T}\int_0^T \left[V_p \cos\left(\frac{2\pi t}{T}\right)\right]^2 dt$$

$$= \frac{V_p^2}{T}\int_0^T \left\{\frac{1}{2} + \frac{1}{2}\cos\left(\frac{4\pi t}{T}\right)\right\} dt$$

$$= \frac{V_p^2}{T} \cdot \frac{T}{2} = \frac{(2.25)^2}{2} = \boxed{2.53W}$$

2. (*3 points*) An analog signal has been quantized to 16 levels, but the resulting SNR is 25 dB too low. How many bits should I be using in my quantizer to meet the SNR spec?

$25 = 6.02\,b$

$b = 4.15$

$\therefore$ need 5 more bits

to get 16 levels needed 4 bits

$\therefore$ quantizer needs $\boxed{9 \text{ bits}}$

3. (*2 points*) A 32-kHz signal has been quantized to 15-bit resolution. What is the minimum time duration of one bit needed to transmit this data? (Assuming only 2-level binary transmission).

$f_m = 32$ kHz

$f_s \geq 2f_m = 64$ kHz

$T_s \leq \frac{1}{f_s} = 15.625\mu s$

$T_{bit} \leq \frac{T_s}{15} = $ **1.04 μs**

4. (*2 points*) Determine the minimum sampling rate necessary to sample and perfectly reconstruct the signal $x(t) = \text{sinc}(12345t)$ where t is in seconds (s).

$\frac{\sin(\pi 12345t)}{\pi 12345t} = \frac{\sin(\pi t/T)}{\pi t/T}$

$\frac{\pi}{T} = 12345$ Hz

$T = \frac{\pi}{12345} = 2.544 \times 10^{-4}$ s

f_m of this signal is

$f_m = \frac{1}{2T}$

$f_{s,min} = 2f_m = \frac{1}{T} = 39.3$ kHz

39.3 kSps

$c = 3 \times 10^8$ m/s (in free space), $c = 2 \times 10^8$ m/s (in media), 1 km $= 10^3$ m, 1 ms $= 10^{-3}$ s, 1 Mb $= 10^6$ b

$$\mathcal{F}\{\text{rect}(t/T)\} = T\text{sinc}(fT) = T\sin(\pi fT)/\pi fT$$
$$\mathcal{F}\{\text{sinc}(t/T)\} = T\text{rect}(fT)$$
$$\mathcal{F}\{1 - |\tau|/T\} = T\text{sinc}^2(fT)$$
$$\sin(a+b) = \sin a\cos b + \cos a \sin b, \cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\sin(a \pm b) = \sin a\cos b \pm \cos a \sin b, \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\sin 2a = 2\sin a \cos a, \cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1$$
$$\cos a = (e^{ja} + e^{-ja})/2, \sin a = (e^{ja} - e^{-ja})/j2, \tan a = \sin a/\cos a$$
$$\psi_x(f) = |X(f)|^2, G_x(f) = \sum |c_n|^2 \delta(f - nf_o), G_x(f) = \lim_{T\to\infty} \tfrac{1}{T}|X_T(f)|^2$$
$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)\text{d}t, R_x(\tau) = \lim_{T\to\infty} \tfrac{1}{T}\int_{-\infty}^{\infty} x(t)x(t+\tau)\text{d}t$$
$$c_n = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi n f_o t)\text{d}t$$
$$\text{SNR [dB]} = 10\log(\text{SNR}), SNR_{q,dB} = 6.02b + 10.8 + 10\log(\sigma_x^2/V_{pp}^2), SNR_j = 3/(\sigma_t^2 + f_H^2)$$