Exercise 1. (a) Sample period T_{sample} of the given DFG is

$$T_{sample} = 4T \tag{3.1}$$

So the maximum achievable sample rate is

$$f_{sample} = \frac{1}{T_{sample}} = \frac{1}{4T} \tag{3.2}$$

(b) The pipelining levels are shown by the dashed lines in Figure 3.1. 9 registers are required.

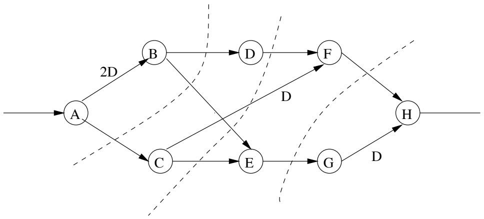

Fig. 3.1 Pipelining levels for Exercise 1(b)

Exercise 3. Let:

$$y_1(n) = a_1x_1(n) + a_2x_1(n-1) + a_3x_1(n-2) + a_4x_1(n-3) + a_5x_1(n-4) \quad (3.3)$$

$$y_2(n) = b_1x_2(n) + b_2x_2(n-1) + b_3x_2(n-2) + b_4x_2(n-3) + b_5x_2(n-4) \quad (3.4)$$

$$y(n) = y_1(n) + y_2(n) \quad (3.5)$$

Then transpose operation can be applied to $y_1(n)$ and $y_2(n)$ separately. The equivalent data-broadcast implementation is shown in Figure 3.3.

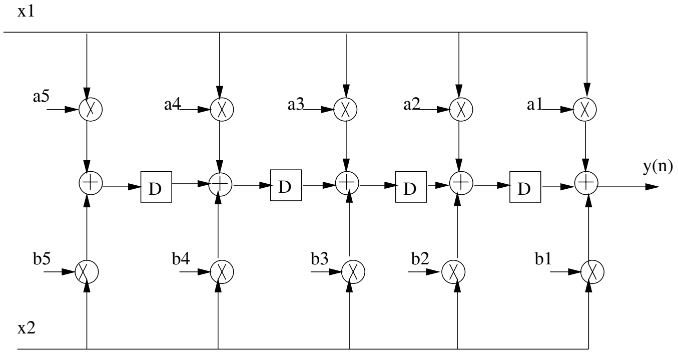

Fig. 3.3 Equivalent data-broadcast structure for Exercise 3

Exercise 7. (a). The critical path of one multiply-add time can be achieved by using the transpose FIR filter structure as shown in Figure 3.7(a).

(b). The block filter with block size 3 can be described by

$$\begin{aligned} y(3k) &= ax(3k) + bx(3k-4) + cx(3k-6) \\ y(3k+1) &= ax(3k+1) + bx(3k-3) + cx(3k-5) \\ y(3k+2) &= ax(3k+2) + bx(3k-2) + cx(3k-4). \end{aligned} \tag{3.7}$$

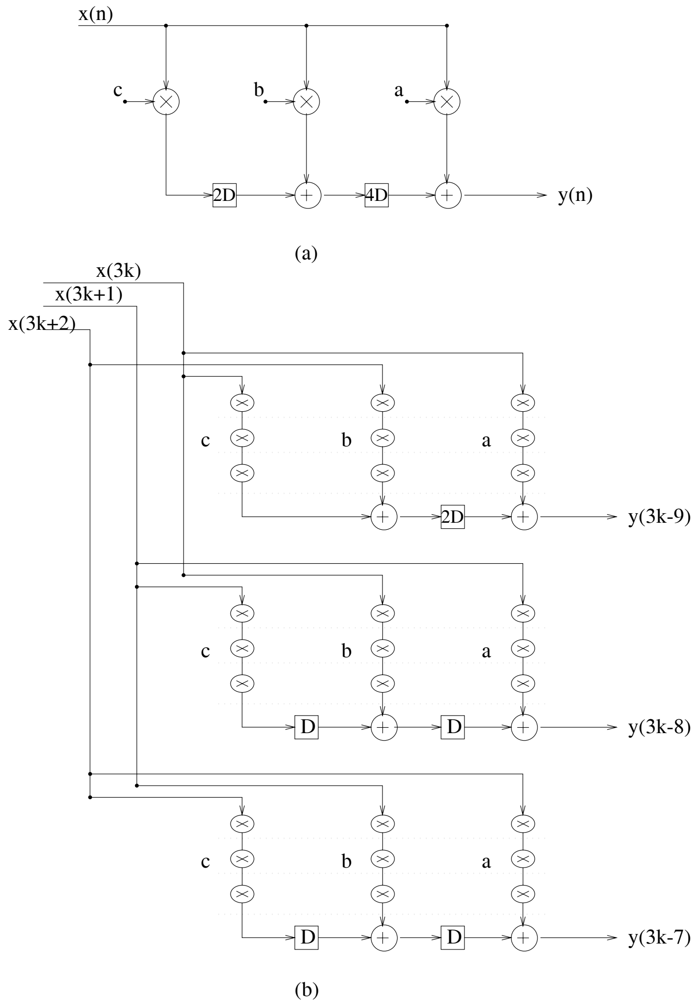

Fig. 3.7 (a):Transpose FIR filter for Exercise 7; (b):Pipelined Block Filter for Exercise 7

Exercise 9. Suppose M level pipelining is needed. Let β be the supply voltage reduction factor, i.e., the supply voltage can be reduced to βV_0 for the pipelined system. From text, the power reduction factor is β^2. Therefore, we have

$$\beta^2 \leq \frac{1}{5}. \tag{3.8}$$

Suppose that the pipelined system and the original system have the same sample rate, we have

$$M = \frac{\beta(V_0 - V_t)^2}{(\beta V_0 - V_t)^2}. \tag{3.9}$$

Take $\beta = \sqrt[2]{0.2} = 0.447$, then

$$M = \lceil \frac{0.447(5 - 0.4)^2}{(0.447 \times 5 - 0.4)^2} \rceil = \lceil 2.8 \rceil = 3. \tag{3.10}$$

Therefore, the system should be pipelined at 3 level. Substitute $M = 3$ into (3.9) and solve for $\beta = 0.427$. The supply voltage for pipelined system is $\beta V_0 = 2.14$ Volts. ■

Exercise 12. Since pipelining level M=4 and block size L=4,we have

$$16(\beta V_0 - V_t)^2 = \beta(V_0 - V_t)^2 \tag{3.15}$$

Substitute V_0=5 Volts and V_t=0.4 Volts in to (3.15), we have

$$\beta^2 - 0.2129\beta + 0.0064 = 0. \tag{3.16}$$

Solving for β, we get

$$\beta = 0.176675, \ or \ \beta = 0.03622. \tag{3.17}$$

$\beta = 0.03622$ is disgarded since $0.03622V_0 = 0.181$ Volts, which is less than the threshold voltage. Therefore, the supply voltage for the parallel pipelined system is $\beta V_0 = 0.883$ Volts. The power ratio is

$$Ratio = \beta^2 = 3.12\%. \blacksquare \tag{3.18}$$