Solutions to Chapter 4 Review Questions

Q1. The z-transform transfer function of an FIR filter is given by:

$$H(z) = 1 + 2z^{-1} + z^{-2}$$

Find the frequency response of the filter.

Solution:

$$H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = 1 + 2e^{-j\omega} + e^{-j2\omega}$$
$$= e^{-j\omega}e^{j\omega} + e^{-j\omega}e^{-j\omega} + 2e^{-j\omega}$$
$$= e^{-j\omega}\left[2\frac{\left(e^{j\omega} + e^{-j\omega}\right)}{2} + 2\right]$$
$$= 2e^{-j\omega}\left[1 + \cos(\omega)\right]$$

Q2. The impulse response of an FIR filter is given by:

$$h(n) = a_1\delta(n) + a_2\delta(n-1) + a_3\delta(n-2) + a_4\delta(n-3) + a_5\delta(n-4)$$

For what values of the impulse response samples will its frequency response have a linear phase?

Solution:

For a linear phase FIR filter, its frequency response is represented by:

$$H(e^{j\omega}) = e^{j\theta(\omega)}R(\omega) \qquad (1)$$

where $\theta(\omega) = -\alpha\omega + \beta$ and $R(\omega)$ is the amplitude response.

To find the values of impulse response samples that lead to a linear phase filter, we need to find the frequency response of the given filter first. Let's take the z-transform for the given filter, we have:

$$H(z) = a_1 + a_2z^{-1} + a_3z^{-2} + a_4z^{-3} + a_5z^{-4} \qquad (2)$$

Substituting $z = e^{j\omega}$ into (2), we have

$$
\begin{aligned}
H(e^{j\omega}) &= a_1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega} + a_4 e^{-j3\omega} + a_5 e^{-j4\omega} \\
&= \left(a_1 + a_5 e^{-j4\omega}\right) + \left(a_2 e^{-j\omega} + a_4 e^{-j3\omega}\right) + a_3 e^{-j2\omega} \\
&= e^{-j2\omega}\left(a_1 e^{j2\omega} + a_5 e^{-j2\omega}\right) + e^{-j2\omega}\left(a_2 e^{j\omega} + a_4 e^{-j\omega}\right) + a_3 e^{-j2\omega} \\
&= e^{-j2\omega}\left[(a_1 + a_5)\cos(2\omega) + (a_2 + a_4)\cos(\omega) + a_3\right] \\
&+ e^{-j2\omega} j\left[(a_1 - a_5)\sin(2\omega) + (a_2 - a_4)\sin(\omega)\right]
\end{aligned} \qquad (3)
$$

Comparing (3) with (1), it is obvious that the filter will have a linear phase if $a_1 = a_5$ and $a_2 = a_4$ or $a_1 = -a_5$, $a_2 = -a_4$ and $a_3 = 0$

Q3. The frequency response of a length-4 FIR filter with a real and symmetric impulse response has the following specific values:

$$H(e^{j0}) = 6, \text{ and } H(e^{j\frac{\pi}{2}}) = -1 - j$$

Determine $H(z)$.

Solution:

Since the filter is symmetric, its frequency response is given by:

$$
\begin{aligned}
& H\left(e^{j\omega}\right) = h(0) + h(1)e^{-j\omega} + h(1)e^{-j2\omega} + h(0)e^{-j3\omega} \\
& \therefore H\left(e^{0}\right) = 2h(0) + 2h(1) = 6 \rightarrow h(0) + h(1) = 3 \\
& H\left(e^{j\frac{\pi}{2}}\right) = h(0) + h(1)e^{-j\frac{\pi}{2}} + h(1)e^{-j\pi} + h(0)e^{-j\frac{3\pi}{2}} \\
& = h(0) + h(1)(-j) + h(1)(-1) + h(0)(j) \\
& = \left[h(0) - h(1) + jh(0) - jh(1)\right] = -1 - j \\
& \rightarrow h(0) - h(1) = -1 \\
& \therefore \begin{cases} h(0) + h(1) = 3 \\ h(0) - h(1) = -1 \end{cases} \rightarrow \begin{cases} h(0) = 1 \\ h(1) = 2 \end{cases}
\end{aligned}
$$

Q4.Determine the filter length for following FIR filters.

(a). Passband edge: 1 kHz, Stopband edge: 1.5 KHz, passband ripple is less than 0.01, and minimum stopband attenuation is 40dB. The sampling frequency is 5 KHz.

Solution :

$$Q\ N = \frac{-20\log(\sqrt{\delta_p \delta_s}) - 13}{14.6\Delta f} + 1$$

$$f_p = 1000\text{Hz}, \quad f_s = 1500\text{Hz}, f_{sample} = 5000\text{Hz},$$

$$\delta_p = 0.01, \quad \delta_s = -40\,\text{dB} = 0.01$$

$$\therefore \Delta f = (1500 - 1000)/5000 = 0.1$$

$$N = \frac{-20*(-2) - 13}{14.6*0.1} + 1 \approx 19$$

(b). Passband edge : $0.1f_{sample}$, Stopband edge: $0.12f_{sample}$, passband ripple is less than 0.001, and minimum stopband attenuation is 40dB.

Solution :

$$Q\ f_p = 0.1, \quad f_s = 0.12 \rightarrow \Delta f = 0.12 - 0.1 = 0.02$$

$$\delta_p = 0.001, \quad \delta_s = -40\,\text{dB} = 0.01$$

$$\therefore N = \frac{-20\log(\sqrt{\delta_p \delta_s}) - 13}{14.6\Delta f} + 1 = \frac{-20*(-2.5) - 13}{14.6*0.02} + 1 \approx 128$$

(c) The normalized passband and stopband edges are at 0.3 and 0.301, respectively. The passband and stopband ripple are 0.1 dB and -80dB, respectively.

Solution :

$$Q\ f_p = 0.3, \quad f_s = 0.301 \rightarrow \Delta f = 0.301 - 0.3 = 0.001$$

$$0.1\,\text{dB} = 1.0116 \rightarrow \delta_p = 0.1\,\text{dB} - 1 = 0.0116, \quad \delta_s = -80\,\text{dB} = 0.0001$$

$$\therefore N = \frac{-20\log(\sqrt{\delta_p \delta_s}) - 13}{14.6\Delta f} + 1 = \frac{-20*(-2.9678) - 13}{14.6*0.001} + 1 \approx 3176$$